Candidate Name	Centre Number	Candidate Number
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GCSE

SOLUTIONS

185/09

MATHEMATICS
HIGHER TIER
PAPER 1

A.M. TUESDAY, 9 November 2010

2 hours

CALCULATORS ARE NOT TO BE USED FOR THIS PAPER

For Examiner's use only

Question	Maximum Mark	Mark Awarded
1	5	
2	9	
3	9	
4	5	
5	7	
6	4	
7	8	
8	10	
9	6	
10	6	
11	2	
12	5	
13	9	
14	11	
15	4	
TOTAL MARK		

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

Take π as 3·14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

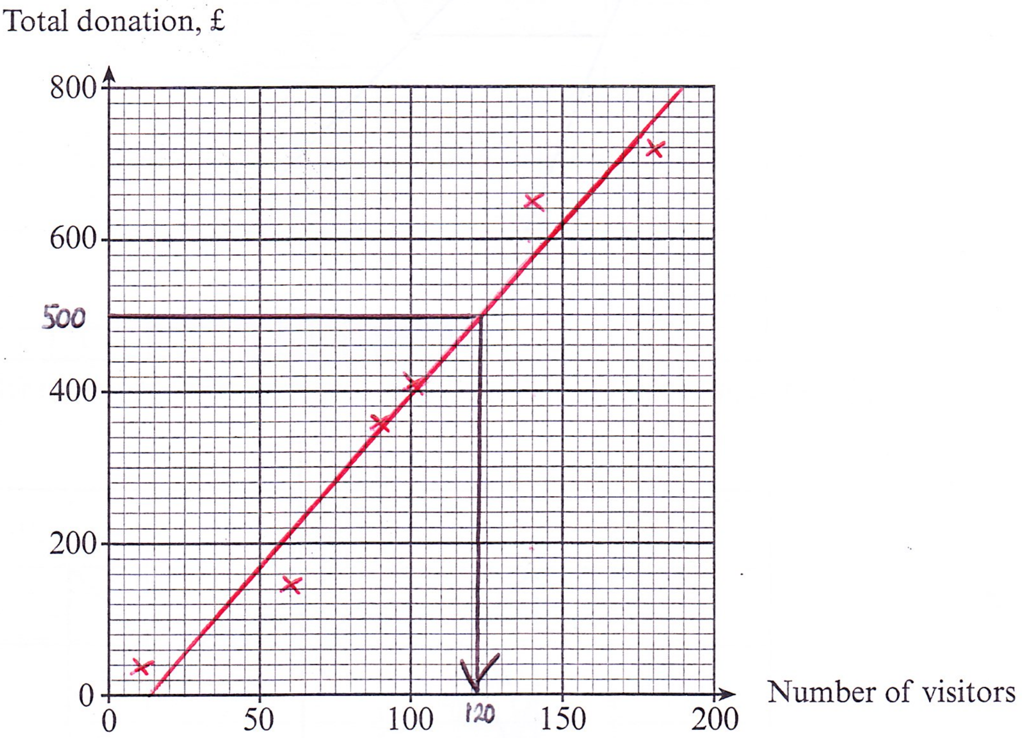
Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

1. The number of visitors to a historical site and the total donation given were recorded each weekend for 6 weeks. The table below shows the results.

Number of visitors	90	140	10	60	100	180
Total donation, £	360	650	40	150	410	700

(a) On the graph paper provided draw a scatter diagram of these results. [2]



(b) Describe the correlation between the number of visitors and the total donation.

POSITIVE

[1]

(c) Draw, by eye, a line of best fit on your scatter diagram. [1]

(d) Use your line of best fit to find an estimate for the number of visitors to the historical site on a weekend when the total donation was £500.

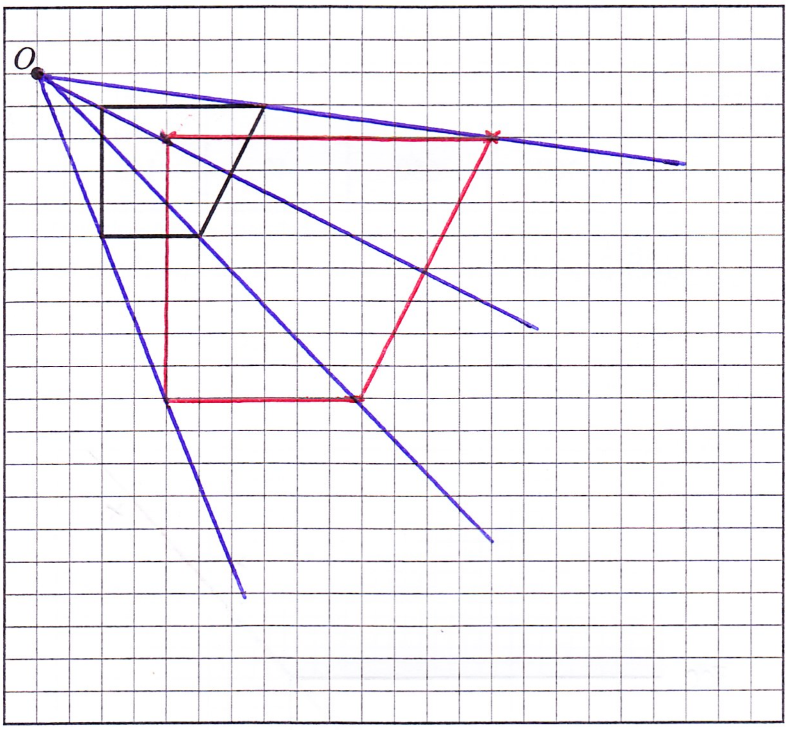
120 visitors

[1]

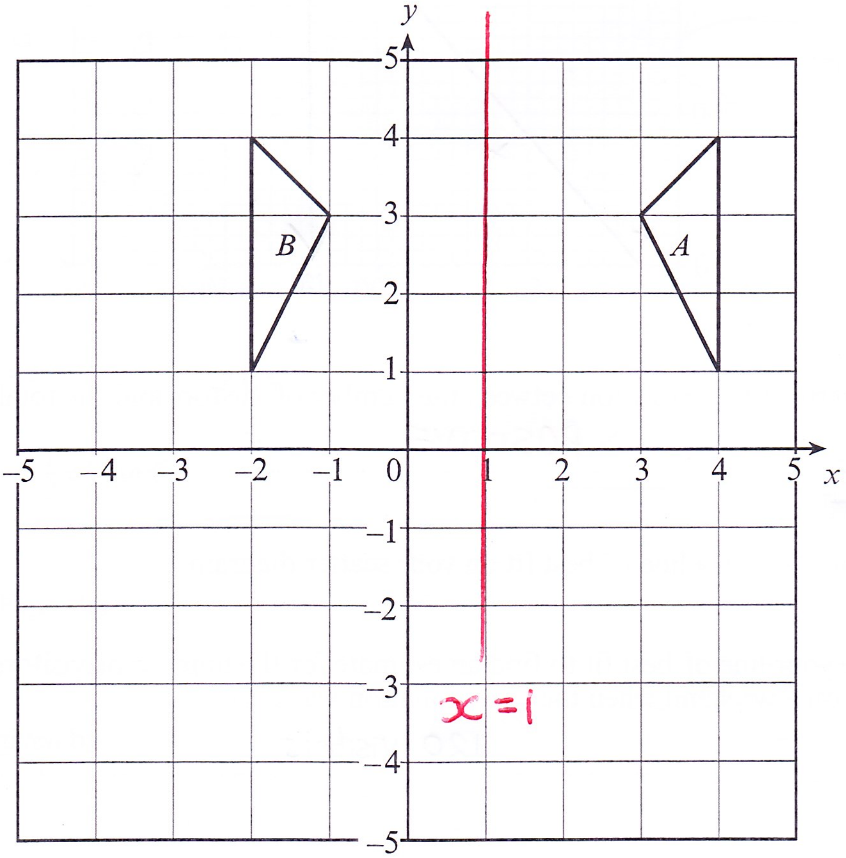
Twice as big,
twice as far from O

2. *(a)* On the grid below, draw the enlargement of the given shape using a scale factor of 2 and centre *O*.

[3]



(b) Describe fully the transformation that transforms triangle *A* into triangle *B*.

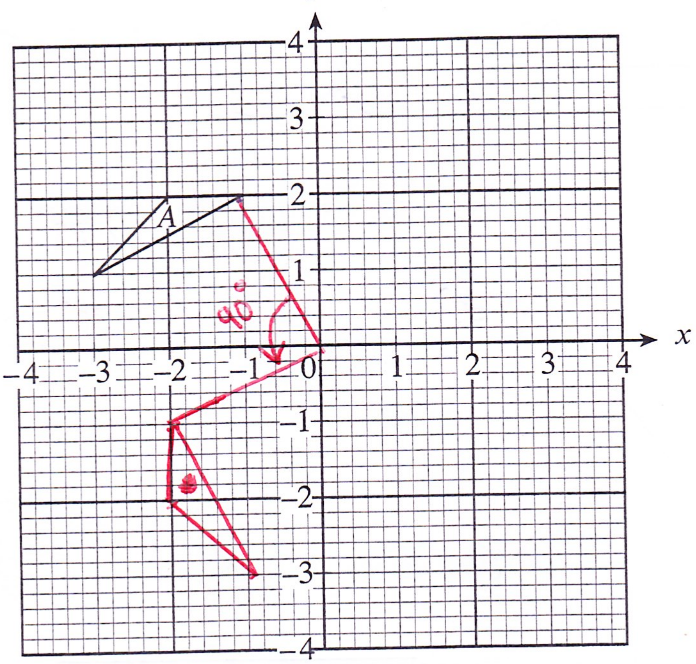


Reflection in the line x = 1

[2]

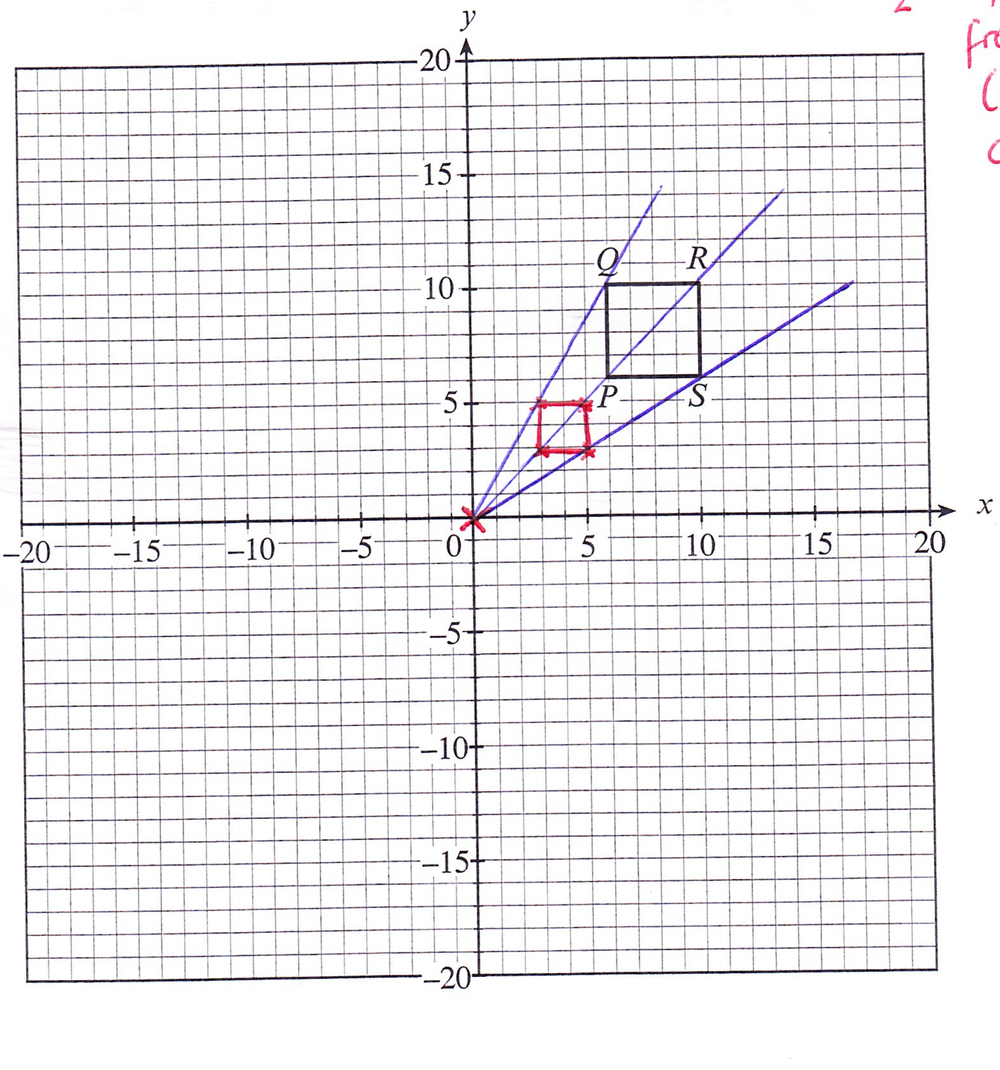
(c) Rotate the triangle *A* through 90° anticlockwise about the origin. [2]

means (0,0)



(d) Enlarge the square *PQRS* using centre (0, 0) by a scale factor of ½. [2]

½ the size and ½ as far from (0,0) centre



Turn over.

55+38 = 93°
180 − 93 = 87°

3. *(a)*

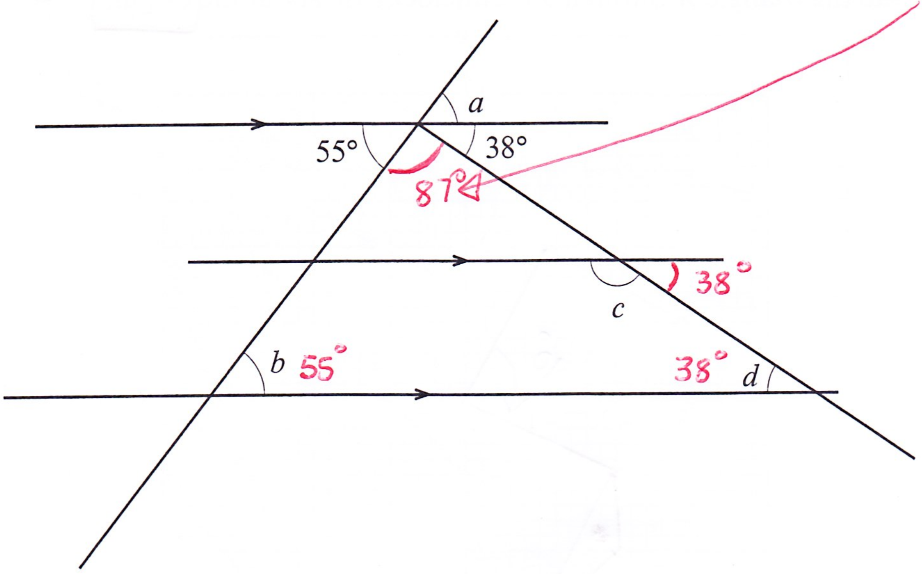


Diagram not drawn to scale

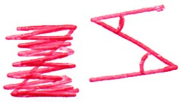
Find the size of each of the angles marked *a, b, c* and *d*.

a = 180 − 38 − 87
a = 55°

c = 180 − 38
c = 142°

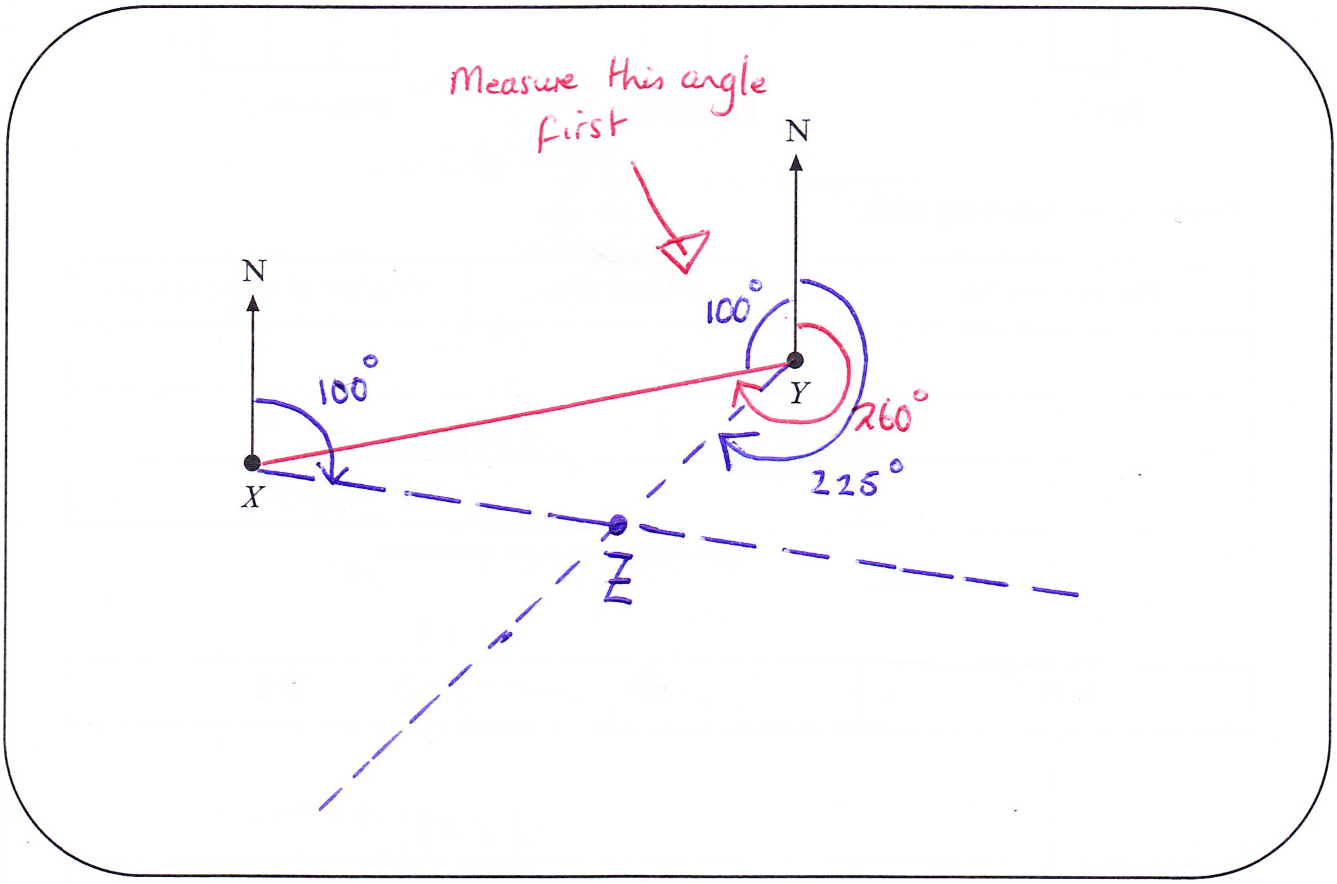
a = 55 ° b = 55 ° c = 142 ° d = 38 °

[4]



(b) This plan is drawn to scale.

Scale: 1 cm represents 20 m



clockwise from Y

(i) Write down the bearing of the point *X* from the point *Y*.

360 - 100 = 260°

(ii) A point *Z* is to be plotted on the above plan.
The bearing of *Z* from *X* is 100°, and the bearing of *Z* from *Y* is 225°.
Find and mark the position of *Z* on the above plan.

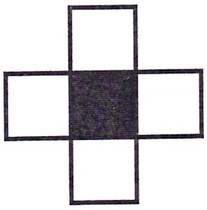
(iii) How far is *Z* from *X* in metres?

XZ = 4·9 cm

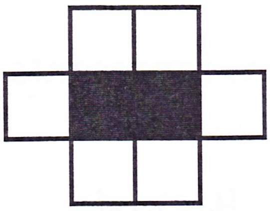
= 4·9 x 20 m [5]

= 98 m

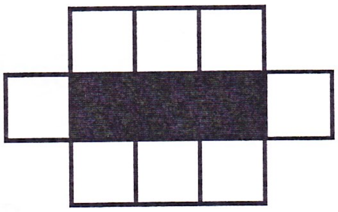
4.



Pattern 1



Pattern 2



Pattern 3

Complete the following table.

Pattern number	Number of black squares	Number of white squares
1	1 $\xrightarrow{\times 2 + 2}$	4
2	2	6
3	3 $\xrightarrow{\times 2 + 2}$	8
15	15 $\xrightarrow{\times 2 + 2}$	32
100	100 $\xrightarrow{\times 2 + 2}$	202
n	n	$2n+2$

[5]

5. *(a)* Given that

$$\boxed{234 \times 789 = 184626}$$

ALSO

$\frac{184626}{234} = 789$

find

same digits but 3 decimal places

(i) 23·4 × 7·89,

= 184·626

NOTICE

(ii) $\frac{18462 \cdot 6}{2 \cdot 34}$, $\begin{matrix} \times 100 \\ \times 100 \end{matrix}$ $= \frac{1846260}{234}$

$= 7890$

(iii) 117 × 789.

↑ This is half of 234

so answer is half of the given answer

$2\overline{)184626}^{\,92313} = 92313$

[3]

(b) Share £385 in the ratio 2 : 9.

11 ratio parts = 385

1 ratio part = $11\overline{)38^{8}5^{5}}^{\,35}$ = £35

∴ 2 ratio parts = 2 × 35 = £70

9 ratio parts = 9 × 35 = £315

$\begin{array}{r} 35 \\ \times\, _4 9 \\ \hline 315 \end{array}$

[2]

(c) Estimate the value of $\frac{607 \cdot 45}{0 \cdot 2498}$.

$\approx \frac{600}{0 \cdot 2} \approx \frac{6000}{2} \approx 3000$

[2]

6. *(a)*

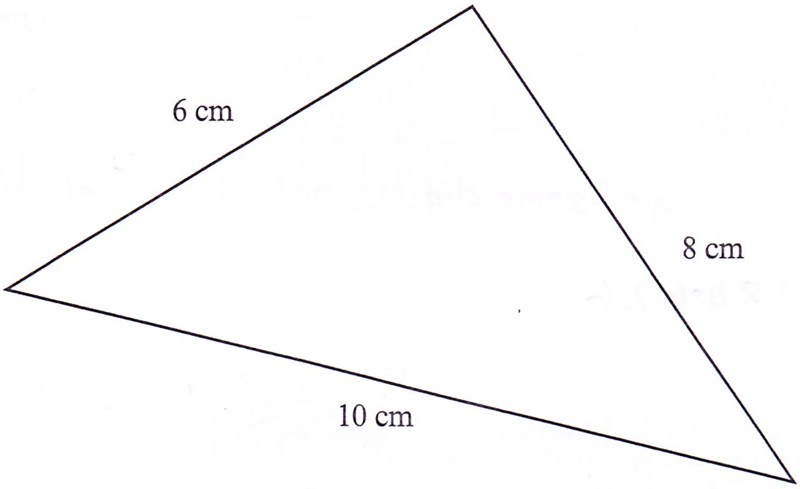


Diagram not drawn to scale

Show, by calculation, that the triangle drawn above is a right-angled triangle.

If it is right-angle Pythagoras must work

$h^2 = a^2 + b^2$

$10^2 = 6^2 + 8^2$

$100 = 36 + 64$

YES This is true

so it must be right-angled

[2]

(b) Calculate the area of a semicircle with a radius of 20 cm using π = 3·14.

$A = \frac{\pi r^2}{2}$

$A = \frac{3 \cdot 14 \times 20 \times 20}{2}$

If you x20 then ÷ 2 it is the same as x 10

[2]

$A = 3 \cdot 14 \times 20 \times 10$

$A = 628 \text{ cm}^2$

7. *(a)* Three bananas and one apple cost a total of 65p.
Seven bananas and two apples cost a total of £1.49.
How much does one apple cost?

set up simultaneous equations

×2 ① — $3b + 1a = 65$

×1 ② — $7b + 2a = 149$ ← get in pence

$6b + 2a = 130$

$-7b \overset{-}{+} 2a \overset{-}{=} 149$ change sign bottom line

ADD $-b = -19$

$19 = b$ banana 19p

eqn ① ⇒ $3b + 1a = 65$

$3(19) + 1a = 65$

$57 + a = 65$

$a = 65 - 57$

$a = 8p$

Put brackets in

[4]

(b) Solve $\frac{(2x+3)}{6} + \frac{(x-1)}{3} = \frac{7}{12}$.

Make all fractions 12

$\frac{2(2x+3)}{12} + \frac{4(x-1)}{12} = \frac{7}{12}$ leave alone

× top and bottom by 2 × top and bottom by 4

NOW × all by 12

$2(2x+3) + 4(x-1) = 7$

$4x + 6 + 4x - 4 = 7$

$8x + 2 = 7$

$8x = 7 - 2$

[4]

$8x = 5$

$x = \frac{5}{8}$

0185
090011

8. *(a)* Simplify $\frac{a^6 \times a^8}{a^7}$.

$= \frac{a^{14}}{a^7}$

$= a^7$

[1]

(b) Expand $x(x + 2)$.

$x^2 + 2x$

[1]

(c) Make w the subject of the following formula.

$$6(w + y) = 10y - 7$$

$6w + 6y = 10y - 7$

$6w = 10y - 6y - 7$

$6w = 4y - 7$

$w = \frac{(4y-7)}{6}$

[3]

(d) Factorise $x^2 + 1x - 12$ and hence solve the equation $x^2 + x - 12 = 0$.

$x^2 + x - 12$

$= (x + 4)(x - 3)$

×	+
−12	+1
(+4)(−3)	

so $(x+4)(x-3) = 0$

either $x + 4 = 0$ or $x - 3 = 0$

$x = -4$ $x = 3$

[3]

(e) Solve $3n + 5 > 9 + n$.

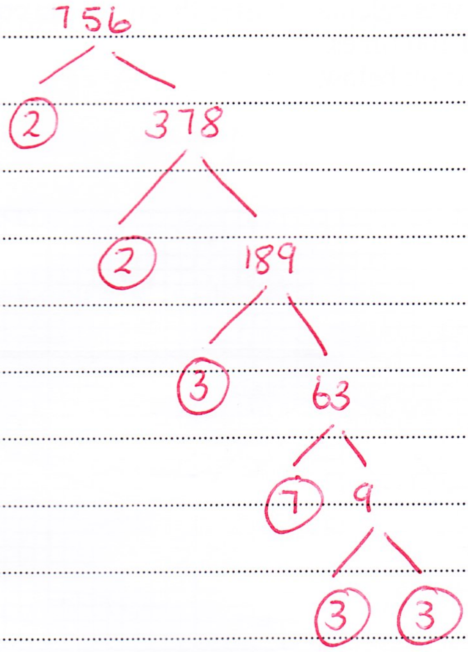
$3n - n > 9 - 5$

$2n > 4$

$n > 2$

[2]

9. *(a)* Express 756 as a product of prime numbers in index form.



$\begin{array}{r} 378 \\ 2\,\overline{)\,7^{1}5^{1}6} \end{array}$

$\begin{array}{r} 189 \\ 2\,\overline{)\,3^{1}7^{1}8} \end{array}$

$\begin{array}{r} 63 \\ 3\,\overline{)\,189} \end{array}$

$756 = 2^2 \times 3^3 \times 7$

[3]

(b) Evaluate the following. Express your answer in standard form.

$$\frac{2^8 \times 5^2}{2^2}$$

same as 2^6

$= 2^6 \times 5^2$

$= 64 \times 25$

$= 1600$

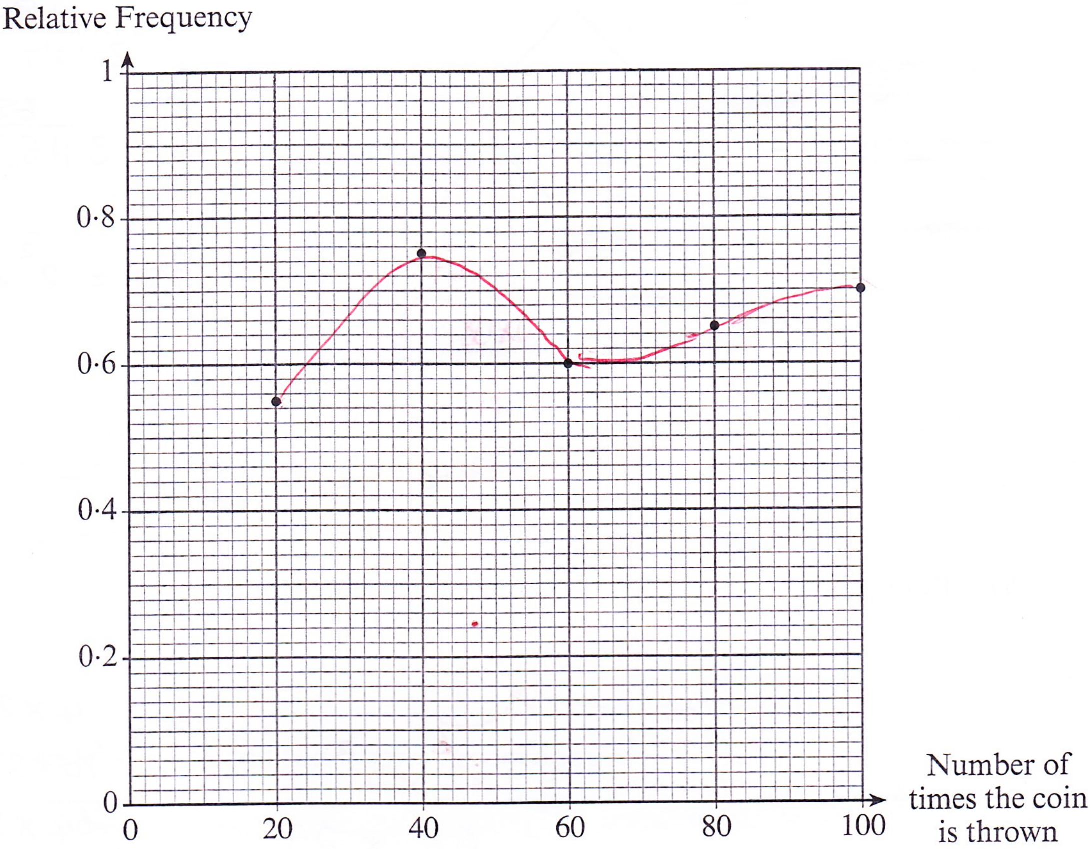
$4 \times 25 = 100$

$16 \times 25 = 400$

$64 \times 25 = 1600$

[3]

10. An experiment was carried out to investigate the probability of obtaining a tail when a biased coin is thrown.
The relative frequency of a tail was calculated after throwing the coin a total of 20 times, 40 times, 60 times, 80 times and 100 times.
The results are plotted on the graph below.



(a) Which reading do you think gives the best estimate for the probability of obtaining a tail? You must give a reason for your answer.

Either : 100 throws because the more times it is thrown the more likely it is to be giving results that settle at a relative frequency the same as the probability

OR

From the oscillating pattern above it may appear that the 80 throws relative frequency is likely to be the probability value. [2]

(b) Using the graph, find how many

(i) tails were obtained in the first 60 throws,

0·6 × 60

= 6 × 6

= 36

(ii) heads were obtained in the 100 throws.

TAILS = 0·7 × 100

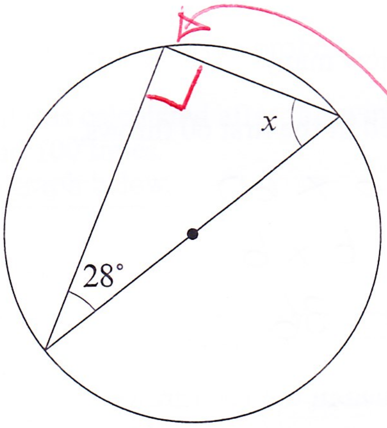
= [illegible] 70

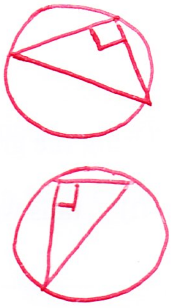
= [illegible]

SO HEADS = 30

[4]

11.





Angle in semi-circle = 90°

Diagram not drawn to scale

Find the size of the angle marked x.

$x = 180 - 90 - 28$

$x = 62°$

[2]

12. Given that y is inversely proportional to x^2, and that $y = 4$ when $x = 5$,

(a) find an expression for y in terms of x,

$y = \frac{k}{x^2}$

$4 = \frac{k}{5^2}$ $\quad$ $\therefore y = \frac{100}{x^2}$

$4 = \frac{k}{25}$

$100 = k$

[3]

(b) use the expression you found in *(a)* to complete the following table.

x	–1	5	10
y	100	4	1

If $x = -1$

$y = \frac{100}{(-1)^2}$

$y = \frac{100}{1}$

$y = 100$

If $y = 1$

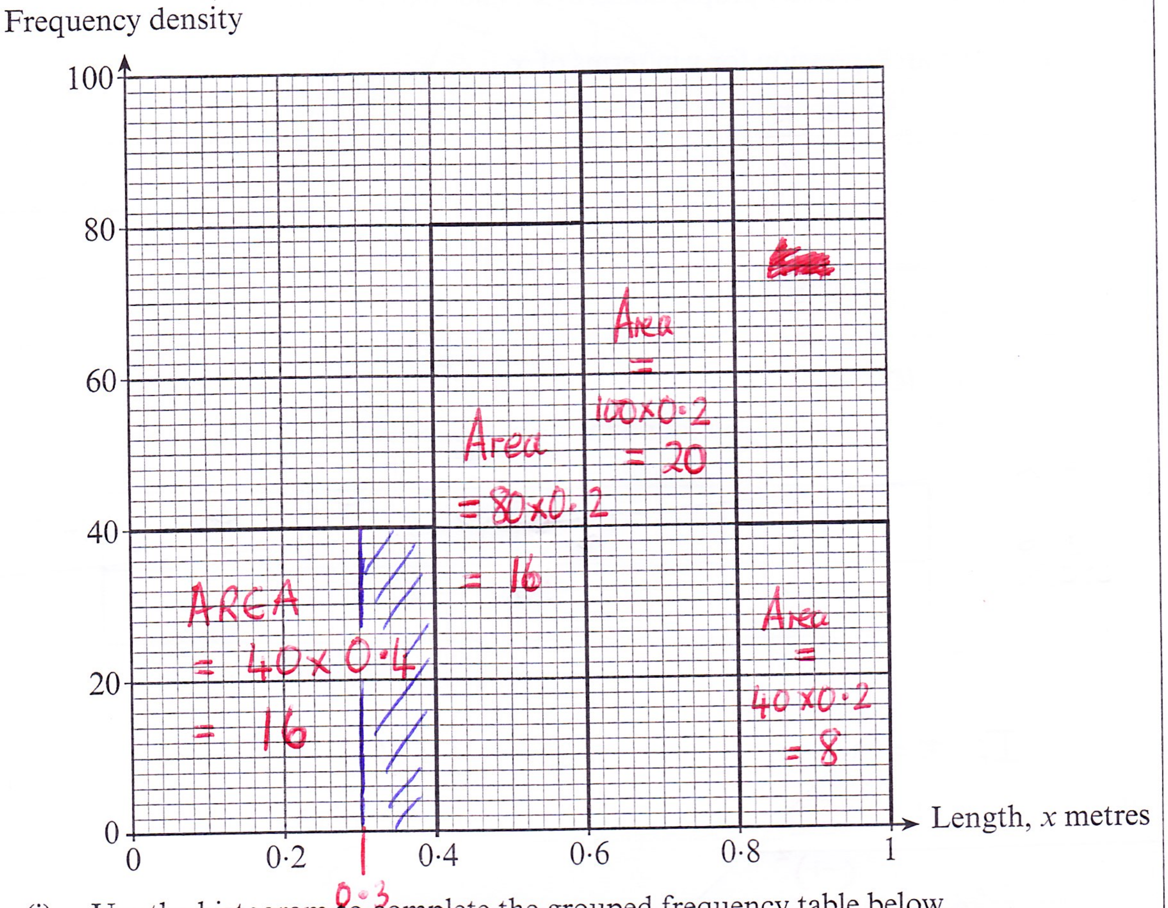
$1 = \frac{100}{x^2}$

$x^2 = 100$

$x = 10$

[2]

13. *(a)* The lengths of logs in a pile were recorded.
The histogram below illustrates the results.



(i) Use the histogram to complete the grouped frequency table below.

Length, x metres	$0 < x \leqslant 0{\cdot}4$	$0{\cdot}4 < x \leqslant 0{\cdot}6$	$0{\cdot}6 < x \leqslant 0{\cdot}8$	$0{\cdot}8 < x \leqslant 1$
Frequency	16	16	20	8

(ii) Calculate an estimate of the number of logs with lengths greater than 0·3 m.

$8 + 20 + 16 + (40 \times 0{\cdot}1)$

$= 44 + 4$

$= 48$

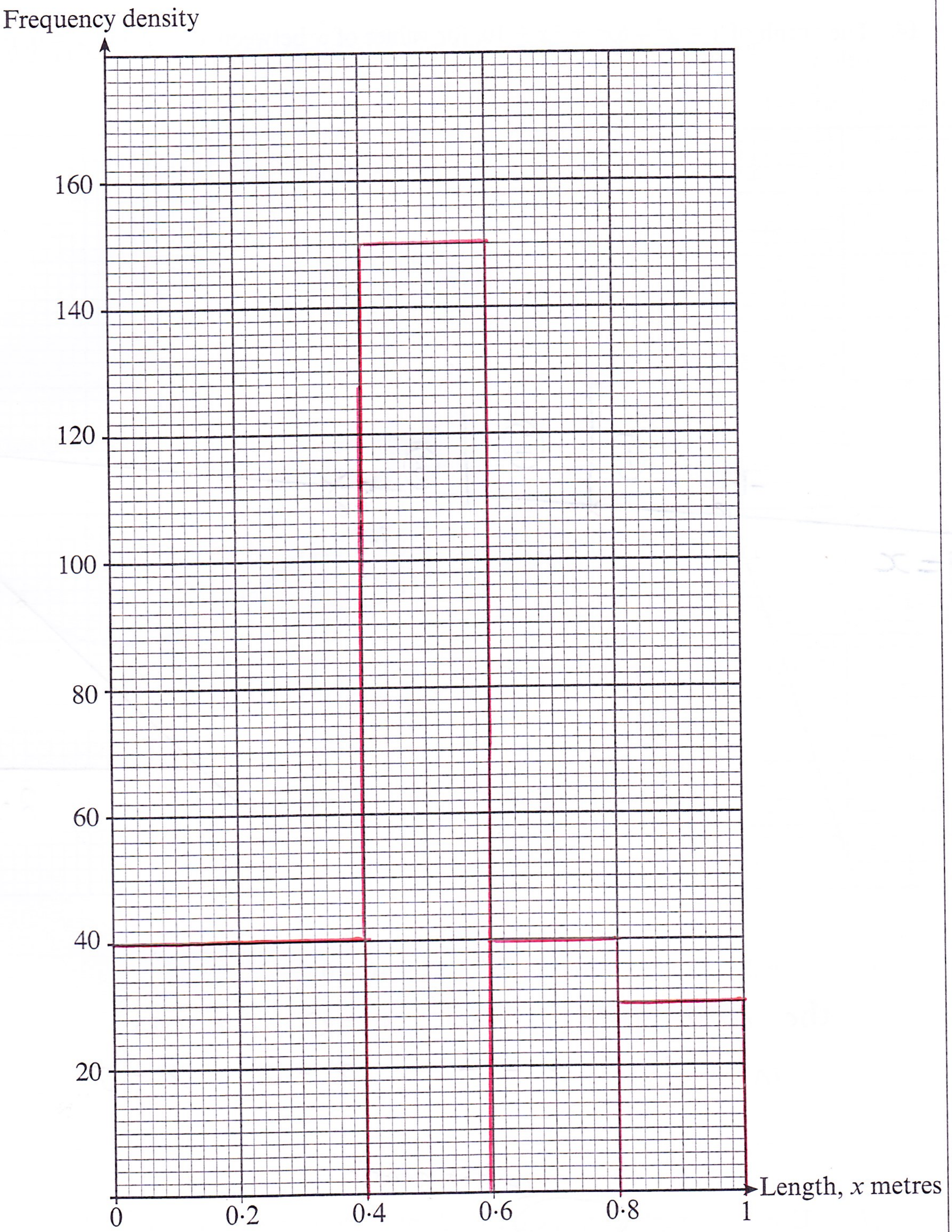
[4]

(b) The lengths of logs in a different pile were recorded.
The results are summarised in the grouped frequency distribution below.

Length, x metres	$0 < x \leqslant 0{\cdot}4$	$0{\cdot}4 < x \leqslant 0{\cdot}6$	$0{\cdot}6 < x \leqslant 0{\cdot}8$	$0{\cdot}8 < x \leqslant 1$
Frequency	16	30	8	6
Frequency density	$\frac{16}{0{\cdot}4} = 40$	$\frac{30}{0{\cdot}2} = 150$	$\frac{8}{0{\cdot}2} = 40$	$\frac{6}{0{\cdot}2} = 30$

Complete the frequency density row in the table and draw a histogram.

[3]



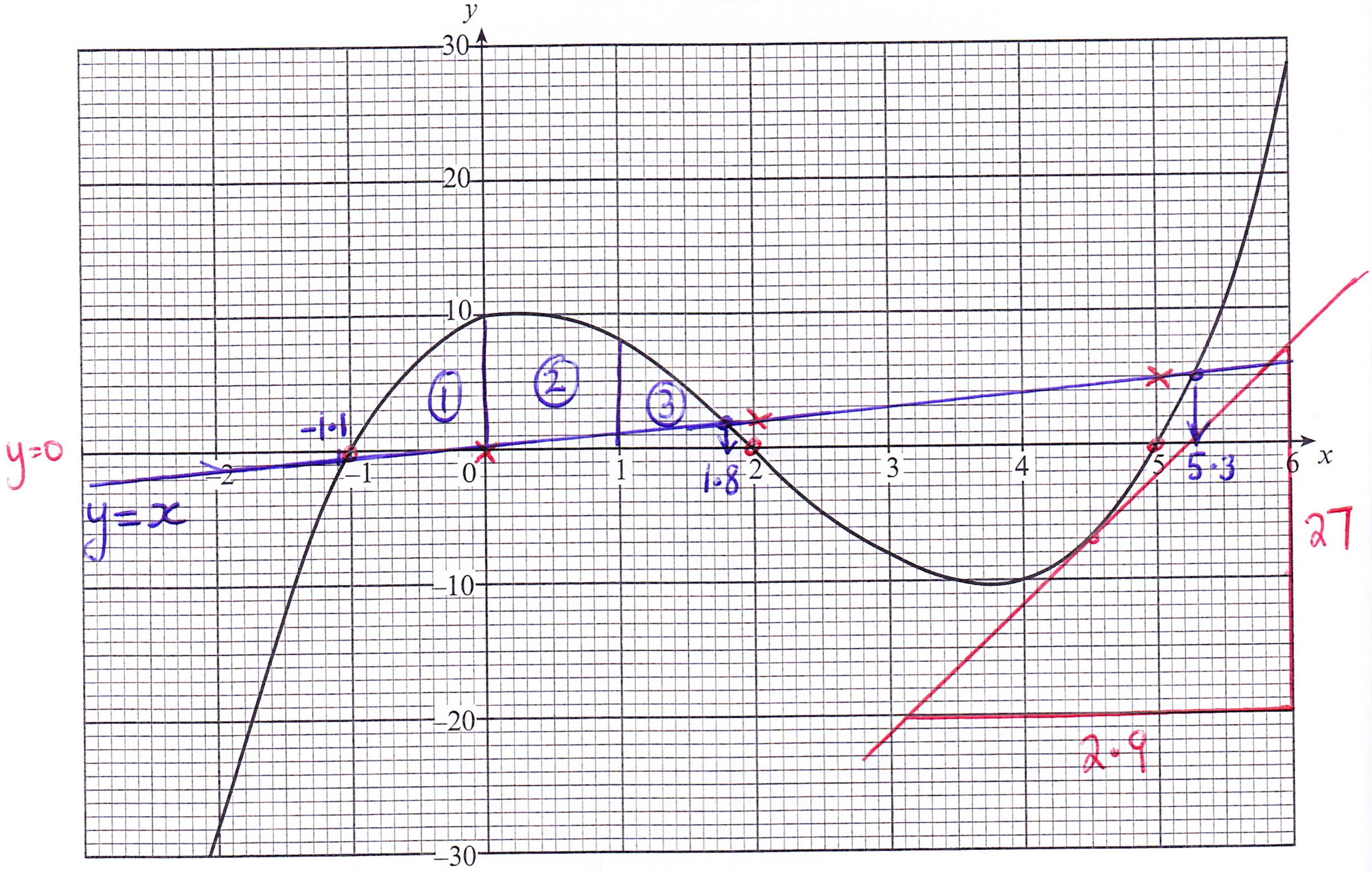
(c) Thomas compares the two histograms.
He states that the mean length of the logs in the first pile is greater than the mean length of the logs in the second pile.
Is Thomas correct? You must give a reason for your answer.

2nd histogram sees mean length in tallest bar 0·4 - 0·6 m
1st histogram mean length appears to be more likely to be 0·6 to 0·8 . ∴ Yes .

[2]

Turn over.

14. The graph of $y = x^3 - 6x^2 + 3x + 10$, for values of x between $x = -2$ and $x = 6$, is drawn below.



(*a*) Use the graph to solve $x^3 - 6x^2 + 3x + 10 = 0$.

Use curve given and $y = 0$ (x axis)

intercepts $x = -1$ $x = 2$ $x = 5$

[1]

← not exact

(*b*) Using the graph, estimate the gradient of the curve $y = x^3 - 6x^2 + 3x + 10$ when $x = 4{\cdot}5$.

Draw tangent at $x = 4{\cdot}5$

Gradient of tangent $= \frac{\text{height}}{\text{base}} = \frac{27}{2{\cdot}9} \approx 10$

[3]

(c) By drawing an appropriate straight line on the graph, solve the equation

$$x^3 - 6x^2 + 2x + 10 = 0$$

Add x to both sides

$x^3 - 6x^2 + 2x + x + 10 = 0 + x$

$x^3 - 6x^2 + 3x + 10 = x$

Use curve and $y = x$

$y = x$

x	0	2	5
y	0	2	5

$x = -1{\cdot}1$ $x = 1{\cdot}8$ $x = 5{\cdot}3$

[3]

(d) Use the trapezium rule with three strips of equal width to estimate the area of the region enclosed by the curve $y = x^3 - 6x^2 + 3x + 10$ and the x-axis between $x = -1$ and $x = 2$.

Area = ① + ② + ③

= triangle + trapezium + triangle

$= \frac{bh}{2} + \frac{(a+b)h}{2} + \frac{bh}{2}$

$= \left(\frac{1\times10}{2}\right) + \frac{(10+8)1}{2} + \left(\frac{1\times8}{2}\right)$

$= 5 + 9 + 4$

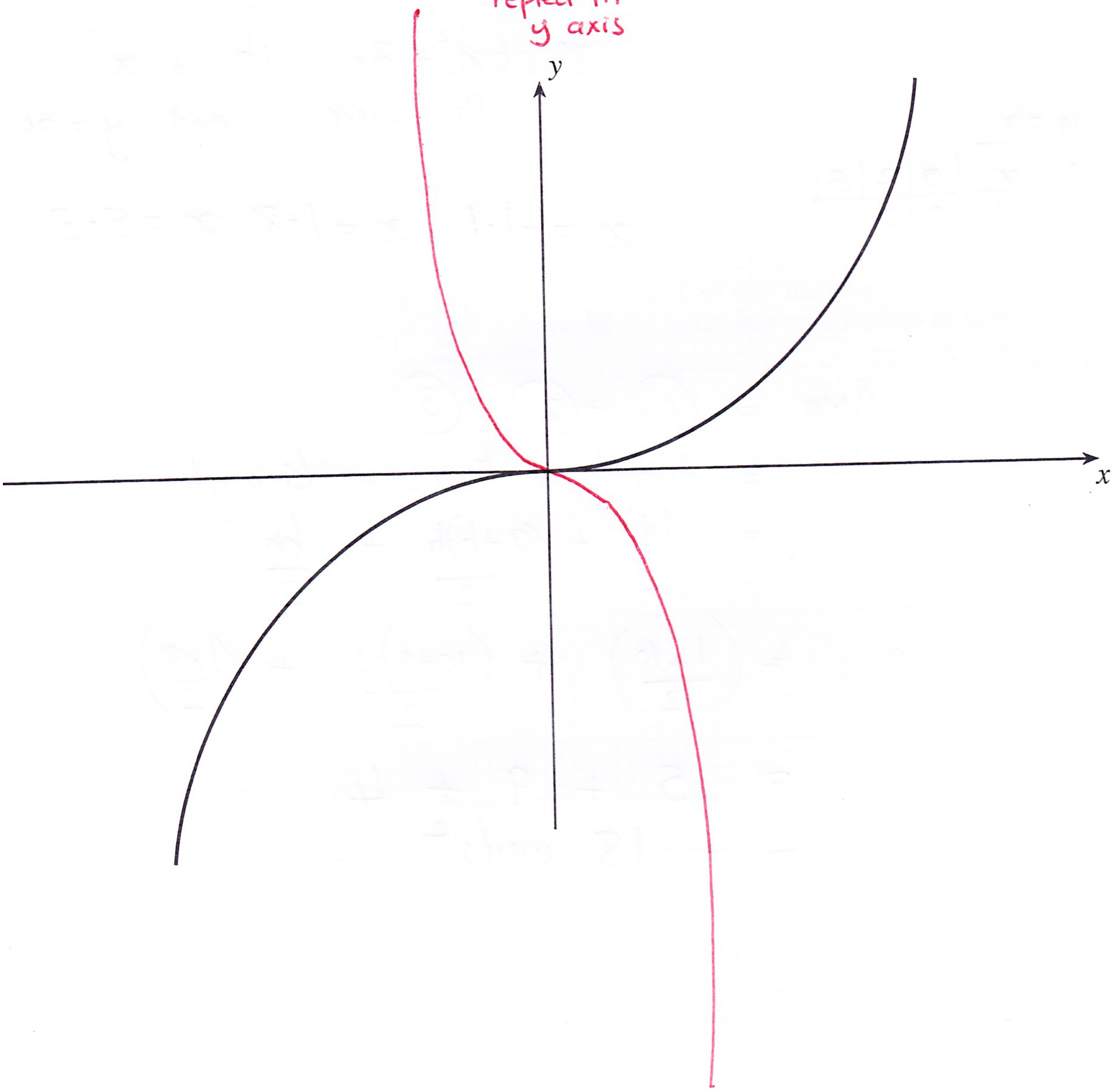
$= 18$ units^2

[4]

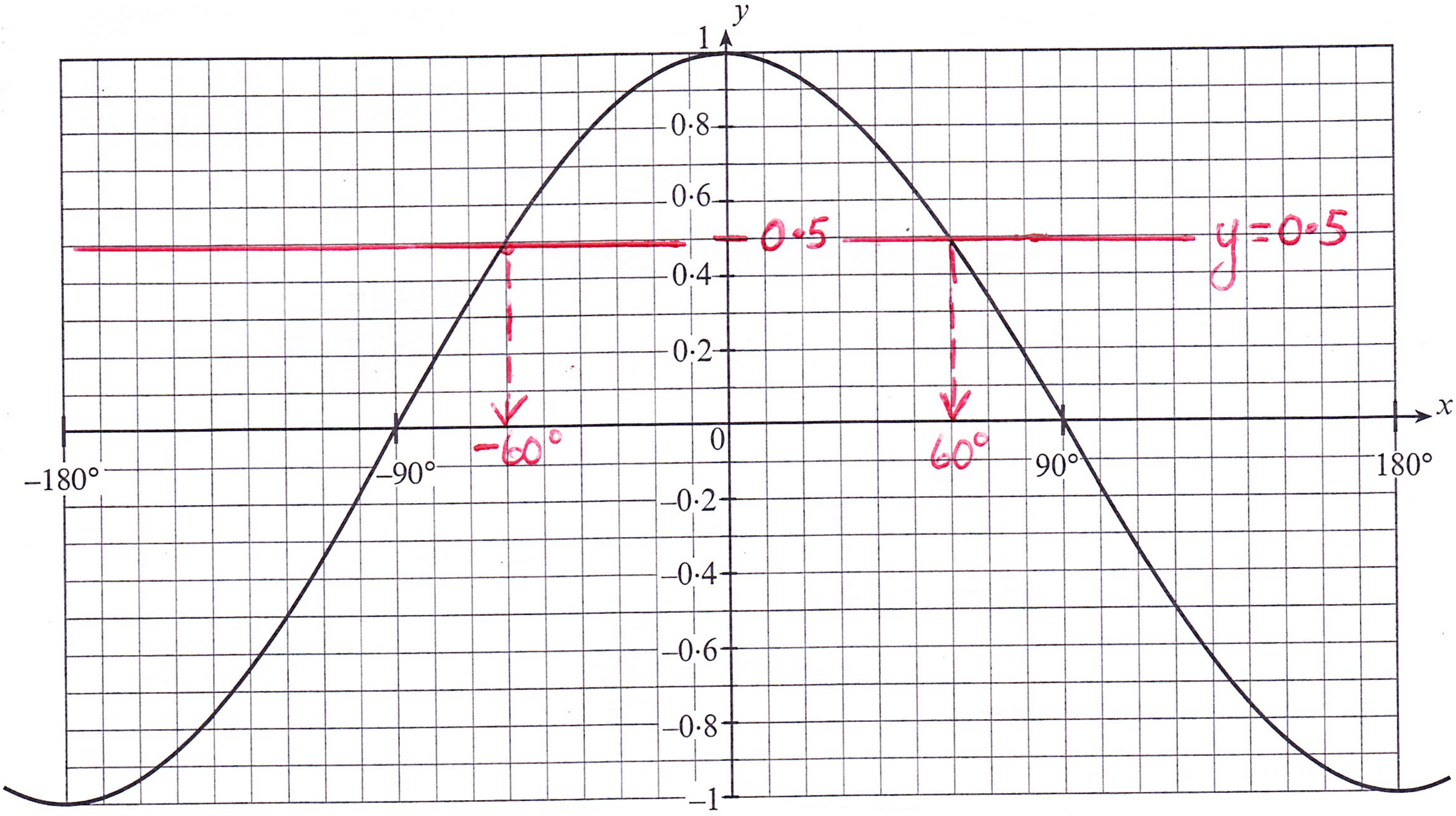
steeper

15. *(a)* The diagram shows a sketch of $y = x^3$.
On the same axes draw a sketch of $y = -4x^3$. [2]

reflect in y axis



(b) The diagram shows a sketch of $y = \cos x$.



Find the values of x in the range $-180° \leqslant x \leqslant 180°$ which satisfy the equation $\cos x = 0·5$.

Use $y = 0·5$

Use given $y = \cos x$ curve

Because the graph is accurately drawn and not a sketch, no calculator is needed

[2]

From crossing points

$x = -60°$ or $x = 60°$