

Surname
Other Names

Centre Number

Candidate Number
0



GCSE

185/09

SOLUTIONS

MATHEMATICS

HIGHER TIER

PAPER 1

P.M. WEDNESDAY, 9 November 2011

2 hours

**CALCULATORS ARE
NOT TO BE USED
FOR THIS PAPER**

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

Take π as 3.14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1	5	
2	6	
3	10	
4	8	
5	7	
6	4	
7	7	
8	5	
9	10	
10	7	
11	5	
12	4	
13	7	
14	11	
15	4	
TOTAL MARK		

1. Tomas has one spin of the circular spinner shown below.
Two of the lines shown on the diagram are diameters of the circle.

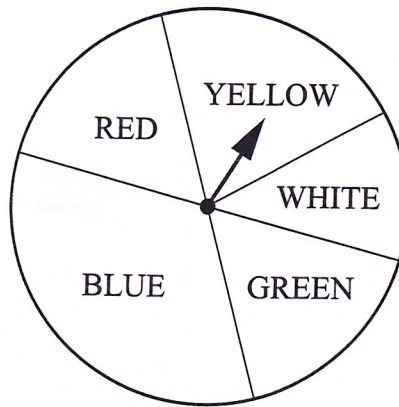


Diagram not drawn to scale

Red and
Green are the
same size!!

- (a) The table below shows the probabilities of Tomas obtaining YELLOW, WHITE and GREEN with one spin of the spinner.
Complete the table.

Colour	YELLOW	WHITE	GREEN	RED	BLUE
Probability	0.2	0.12	0.18	0.18	0.32

$$0.2 + 0.12 + 0.18 + 0.18 + 0.32 = 1$$

- (b) Find the probability of obtaining either WHITE or GREEN on the spinner. *They must all add to 1 which is a certainty!* [3]

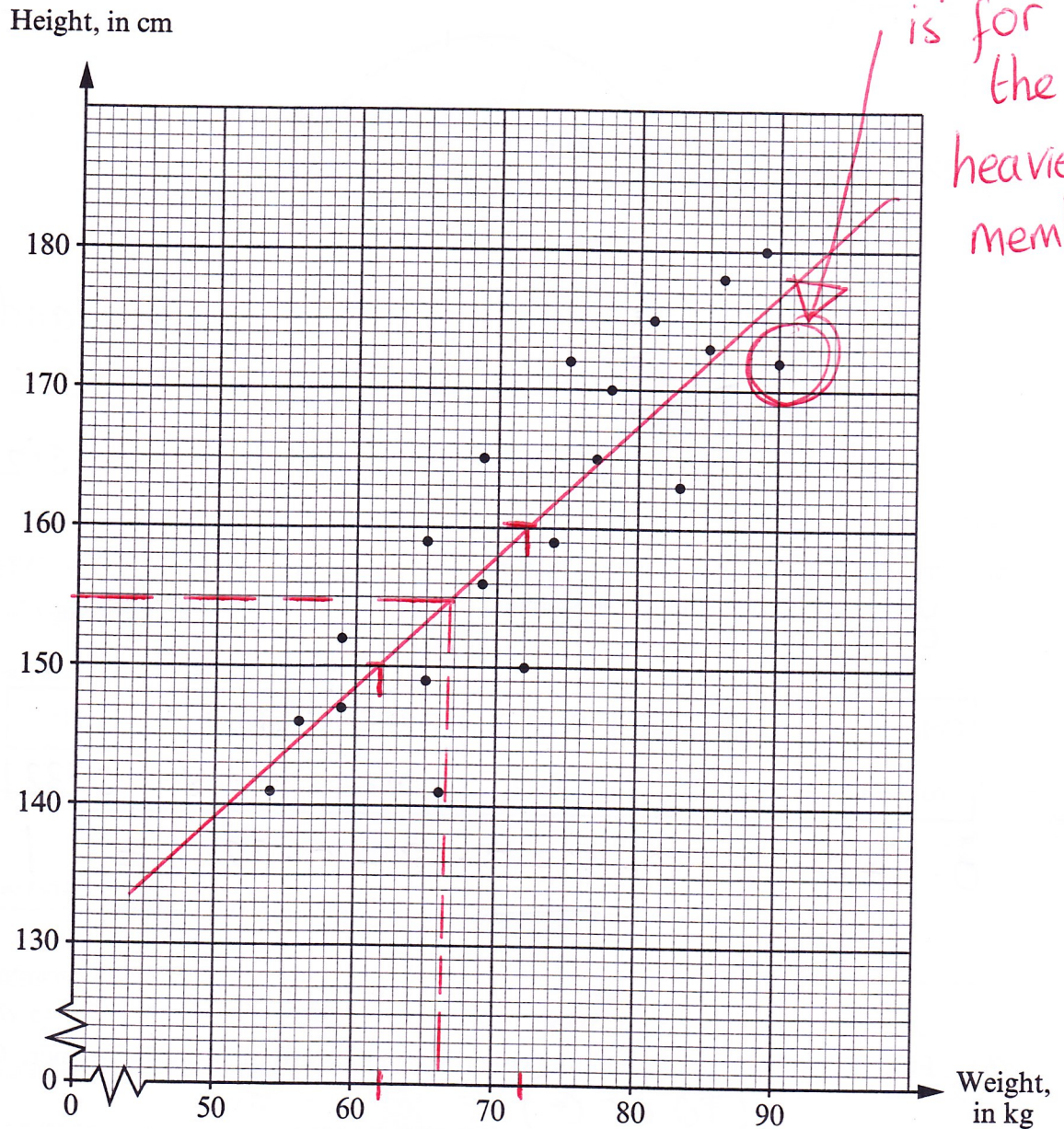
$$P(W \text{ or } G)$$

$$= 0.12 + 0.18$$

$$= 0.3$$

[2]

2. The scatter diagram shows the height, in cm, and the weight, in kg, for each of 20 members of a sports club.



- (a) Write down the height and weight of the **heaviest** of the 20 members of the sports club.

Weight 90 kg

Height 172 cm
[2]

- (b) Write down the type of correlation shown by the scatter diagram.

POSITIVE

[1]

- (c) Draw, by eye, a line of best fit on the scatter diagram.

DONE ON GRAPH

[1]

- (d) Estimate the weight of a person of height 155 cm.

67 kg

[1]

- (e) Is it possible to estimate the weight of a person with a height of 210 cm from the scatter diagram? You must give a reason for your answer.

YES. eg. You could use 160 cm
and $(150 \text{ cm} \div 3)$.

$$160 \text{ cm} = 72 \text{ kg}$$

$$150 \div 3 = 50 \text{ cm} \approx 63 \text{ kg} \div 3 = 21 \text{ kg}$$

[1]

$$210 \text{ cm} = 72 + 21 \\ = 93 \text{ kg}$$

3. (a)

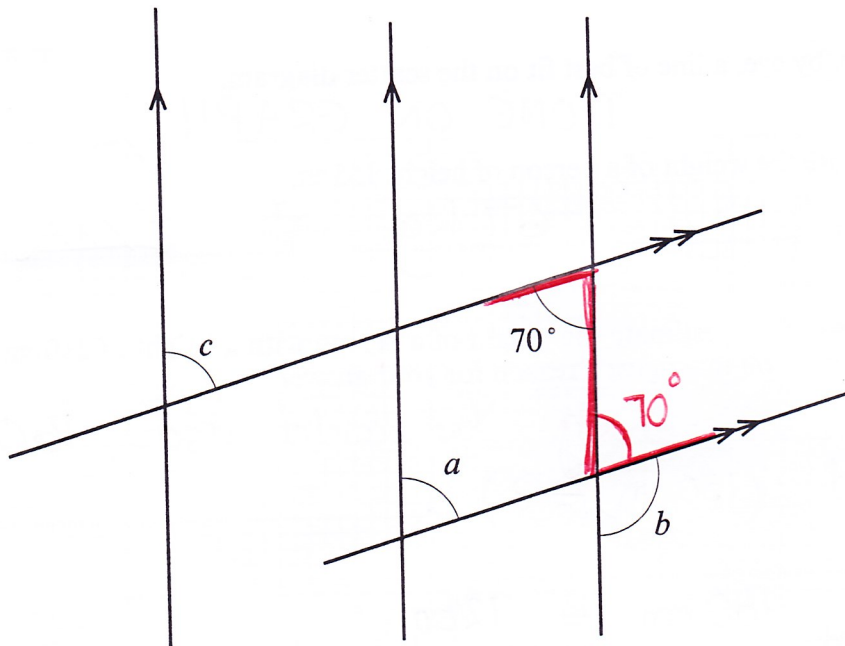


Diagram not drawn to scale

Find the size of each of the angles marked a , b and c .Use alternate and or corresponding angles
Z F $a = 70^\circ$ from alt

$$a = 70^\circ \quad b = 110^\circ \quad c = 70^\circ$$

↑
straight line

[3]

(b) Calculate the size of the interior angle of a regular 10 sided polygon.

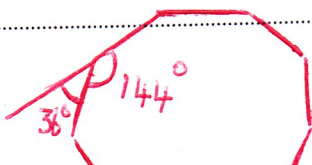
Method 1

$$\begin{aligned} 10 \text{ exterior angles} &= 360^\circ \\ 1 \text{ ext angle} &= 36^\circ \\ \text{So interior angle} &= 180 - 36 \\ &= 144^\circ \end{aligned}$$

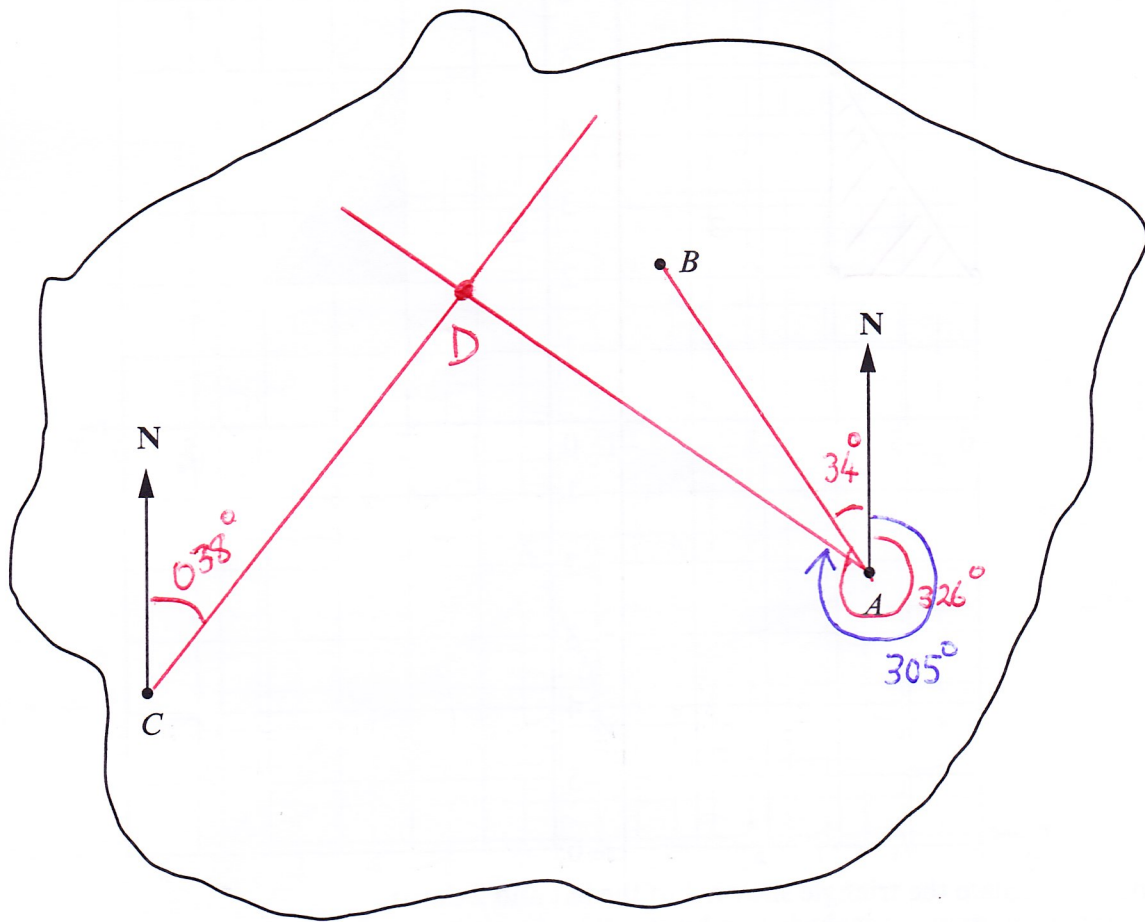
Method 2

$$\begin{aligned} 10 \text{ angles so } 8 \text{ triangles} \\ 10 \text{ angles} &= 8 \times 180 = 1440^\circ \\ 1 \text{ interior} &= \frac{1440}{10} \\ &= 144^\circ \end{aligned}$$

[3]



(c)



- (i) Write down the bearing of point B from point A .

326°

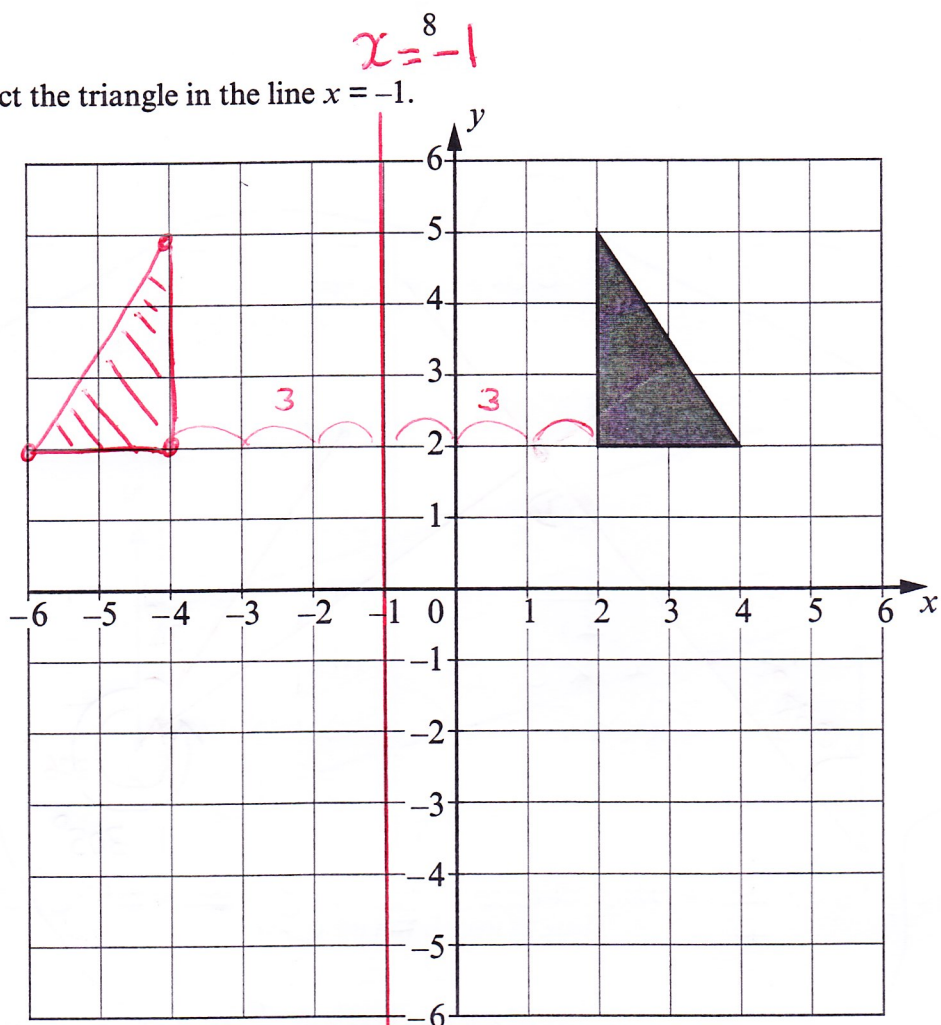
- (ii) A point D is to be plotted on the above plan.
The bearing of D from C is 038° , and the bearing of D from A is 305° .
Find and mark the position of D on the above plan.

[4]

* Remember
Bearings are
clockwise angles
from North lines!

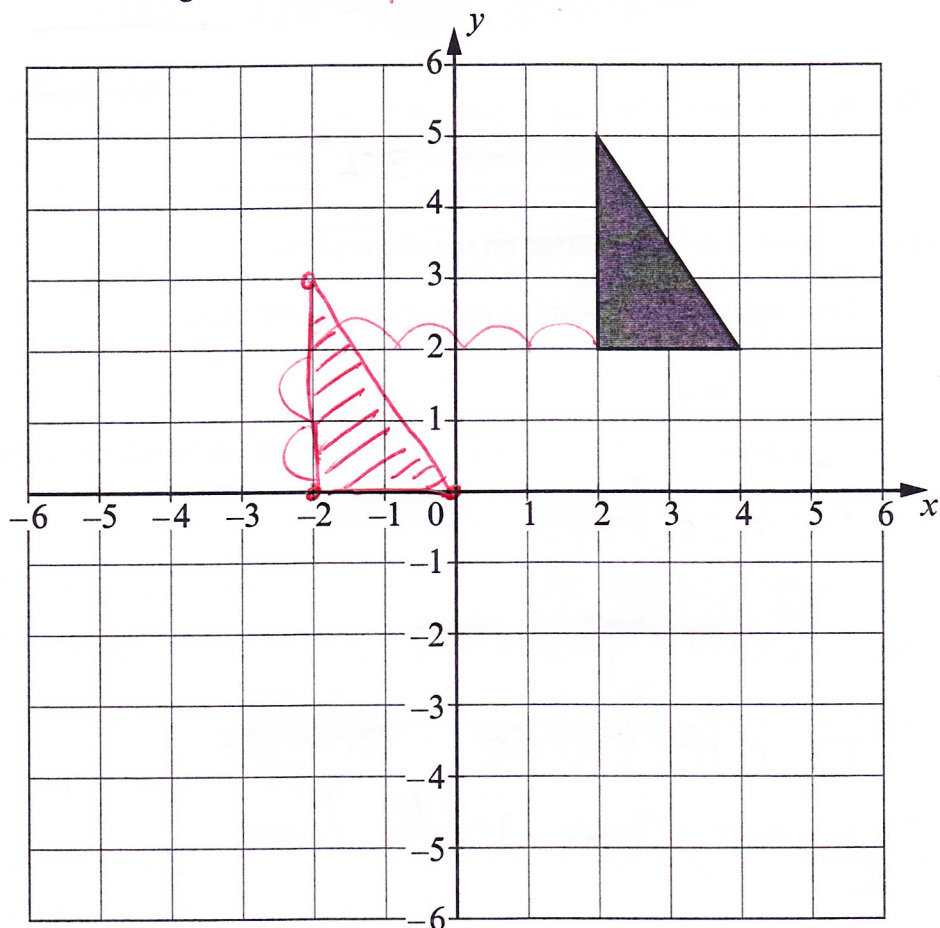
4. (a) Reflect the triangle in the line $x = -1$.

[2]



- (b) Translate the triangle shown 4 to the left and 2 down.

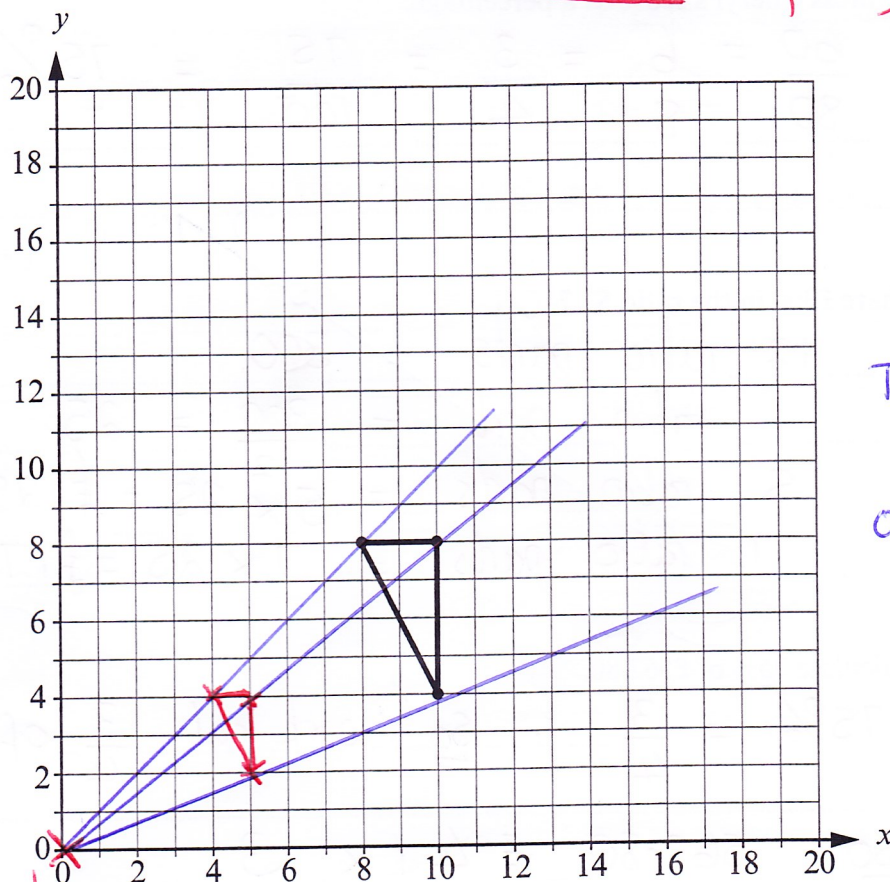
[1]



- (c) Enlarge the triangle using centre $(0, 0)$ by a scale factor of $\frac{1}{2}$.

← All sides $\div 2$
and distance
from $(0, 0)$
 $\div 2$

[2]

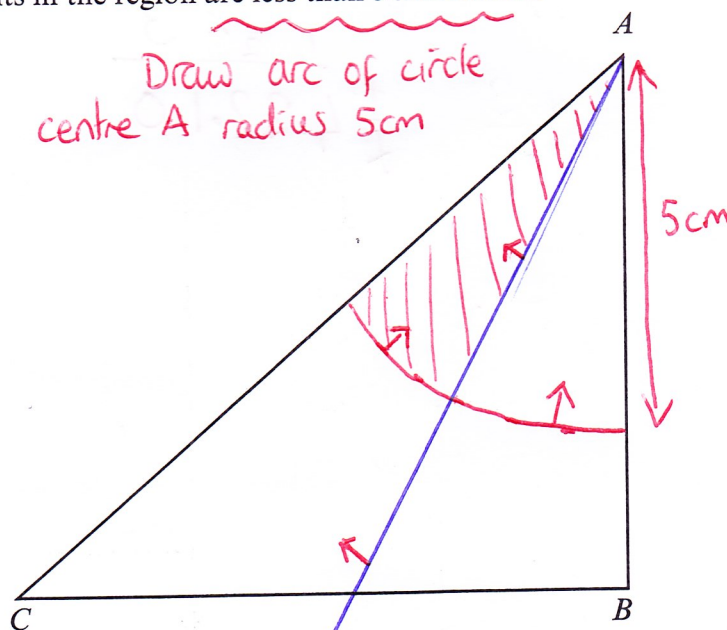


The projector
beam is
only a
check !!

- (d) The diagram below shows a triangle ABC . Find and **shade** the region inside the triangle which satisfies **both** of the following conditions.

- (i) All points in the region are nearer to AC than to AB .
(ii) All points in the region are less than 5 cm from A .

Draw arc of circle
centre A radius 5 cm



← Draw a line that cuts
the angle between AC
and AB in half
Use a protractor !!

[3]

Turn over.

Angle bisector

5. (a) Cheryl scored 60 marks out of 80 in a test.
Express Cheryl's score as a percentage.

$$\frac{60}{80} = \frac{6}{8} = \frac{3}{4} = \frac{75}{100} = 75\%$$

[2]

- (b) Share £300 in the ratio 5 : 7.

$$12 \text{ ratio parts} = 300$$

$$1 \text{ ratio part} = \frac{300}{12} = 25$$

$$5 \text{ ratio parts} = 5 \times 25 = \pounds 125$$

$$7 \text{ ratio parts} = 7 \times 25 = \pounds 175$$

[3]

- (c) Calculate 75% of £562.80.

$$75\% = \frac{3}{4}$$

So work out $\frac{3}{4}$ of £562.80

$$\text{So } 562.80 \div 4 \times 3$$

$$= 140.70$$

$$4 \overline{) 562.80}$$

$$\text{So } 140.70$$

$$\begin{array}{r} 140.70 \\ \times 3 \\ \hline \pounds 422.10 \end{array}$$

[2]

6. (a) Write down the n th term of the sequence 6, 10, 14, 18, 22, ...

$$4n + 2$$

starts $4n$ because of $+4$ above

[2]

- (b) Solve $3x + 4 = 8 - 7x$.

$$3x + 7x = 8 - 4$$

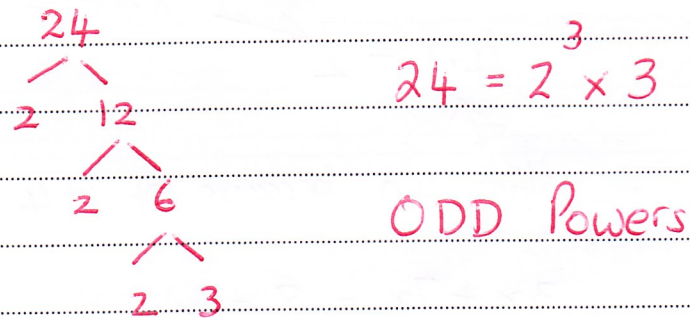
$$10x = 4$$

$$x = \frac{4}{10}$$

$$x = \frac{2}{5}$$

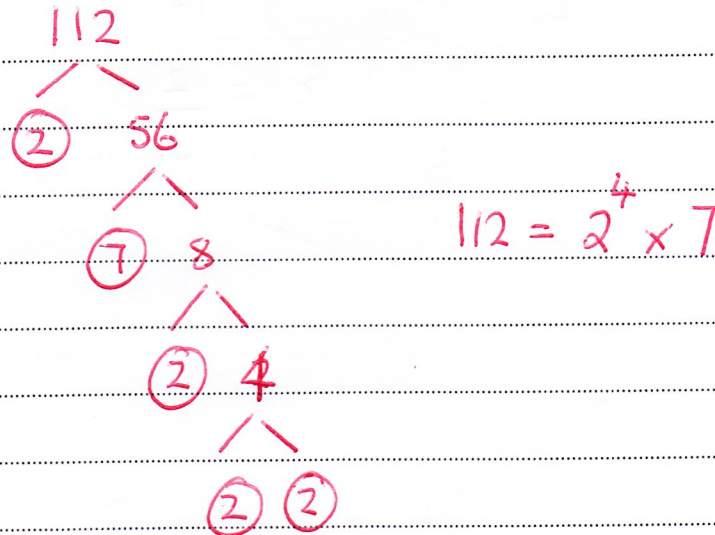
[2]

7. (a) Explain how you know that 24 is **not** a square number.



[1]

- (b) Express 112 as a product of prime numbers in index form.



[3]

- (c) Evaluate $\frac{1}{5} \div \frac{1}{3}$.

$$\begin{aligned}
 & \frac{1}{5} \times \frac{3}{1} \\
 & = \frac{3}{5}
 \end{aligned}$$

[1]

(d) Evaluate $2\frac{1}{3} \times 3\frac{3}{7}$. Simplify your answer.

$$= \frac{\cancel{1}^1 \times \cancel{24}^8}{\cancel{3}_1 \times \cancel{7}_1}$$

$$= \frac{8}{1}$$

$$= 8$$

Now cancel 7's
cancel 3's

[2]

8. The length of a table top is 2050 mm, measured to the nearest 10 mm.

(a) Write down the **least** and **greatest** possible values of the length of the table top.

Least value 2045 mm

Greatest value 2055 mm

[2]

(b) The width of the table top is 1040 mm, measured to the nearest 10 mm.
Find the least possible perimeter of the table top.

$$\text{Least } P = 2045 + 2045 + 1035 + 1035$$

$$= 6160 \text{ mm}$$

[3]

9. (a) Factorise $x^2 - 2x - 8$.

$$(x-4)(x+2)$$

$\times$	$+$
-8	-2

$$(-4) \times (+2)$$

[2]

- (b) Expand and simplify $(2x + 1)(x - 5)$.

$$= 2x^2 - 10x + x - 5$$

$$= 2x^2 - 9x - 5$$

[2]

- (c) Solve $\frac{21-2x}{5} = 4 - x$.

$\times$ everything by 5

$$21 - 2x = 20 - 5x$$

$$-2x + 5x = 20 - 21$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

[3]

(d) Simplify $6x^7y^2 \times 4x^3y^5$.

$$6 \times 4 x^{7+3} y^{2+5}$$
$$= 24 x^{10} y^7$$

[2]

(e) Simplify $a^5 \times a^{-2}$.

$$a^{5+(-2)} = a^3$$

[1]

10. The heart rate of each of 60 people at a gym was recorded.
The following table shows a grouped frequency distribution of the results.

Heart rate, r (beats per minute)	$80 \leq r < 90$	$90 \leq r < 100$	$100 \leq r < 110$	$110 \leq r < 120$	$120 \leq r < 130$	$130 \leq r < 140$
Number of people	4	14	22	10	6	4

- (a) Complete the following cumulative frequency table.

Heart rate (less than)	80	90	100	110	120	130	140
Cumulative frequency	0	4	18	40	50	56	60

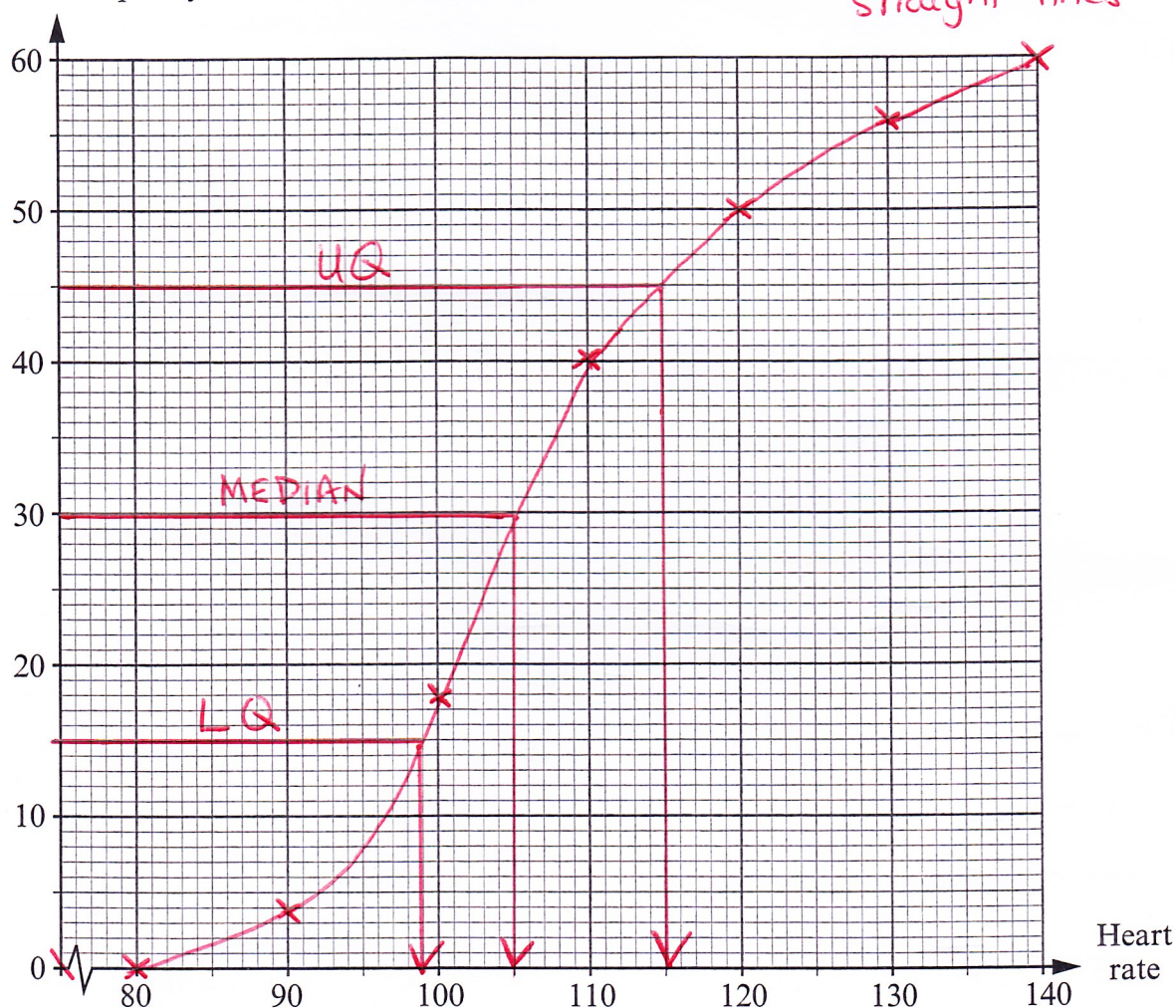
[1]

- (b) On the graph paper below, draw a cumulative frequency diagram to show this information.

Curve of
straight lines

[3]

Cumulative frequency



(c) Use your cumulative frequency diagram to find estimates for

(i) the median, = 30th result (1/2 of 60 results)
= 105 b.p.m.

(ii) the interquartile range.

Lower Quartile = 15th result (1/4 of 60)
= 99 b.p.m.

Upper Quartile = 45th result (3/4 of 60)
= 115 b.p.m. [3]

So IQR = 115 - 99 = 16 b.p.m.

11. (a) Factorise the expression $6x^2 - 11x - 10$ and hence solve the equation $6x^2 - 11x - 10 = 0$.

$$= 6x^2 - 15x + 4x - 10$$

$$= 3x(2x-5) + 2(2x-5)$$

$$= (2x-5)(3x+2) \quad (-15)(+4)$$

x	+
-60	-11

$$\therefore (2x-5)(3x+2) = 0$$

either $2x-5=0$ or $3x+2=0$

$$2x=5$$

$$3x=-2$$

$$x = \frac{5}{2}$$

$$x = -\frac{2}{3}$$

Find 2 numbers that
x to give (-60) and
add to give (-11)

[3]

(b) Factorise $4y^2 - 81$.

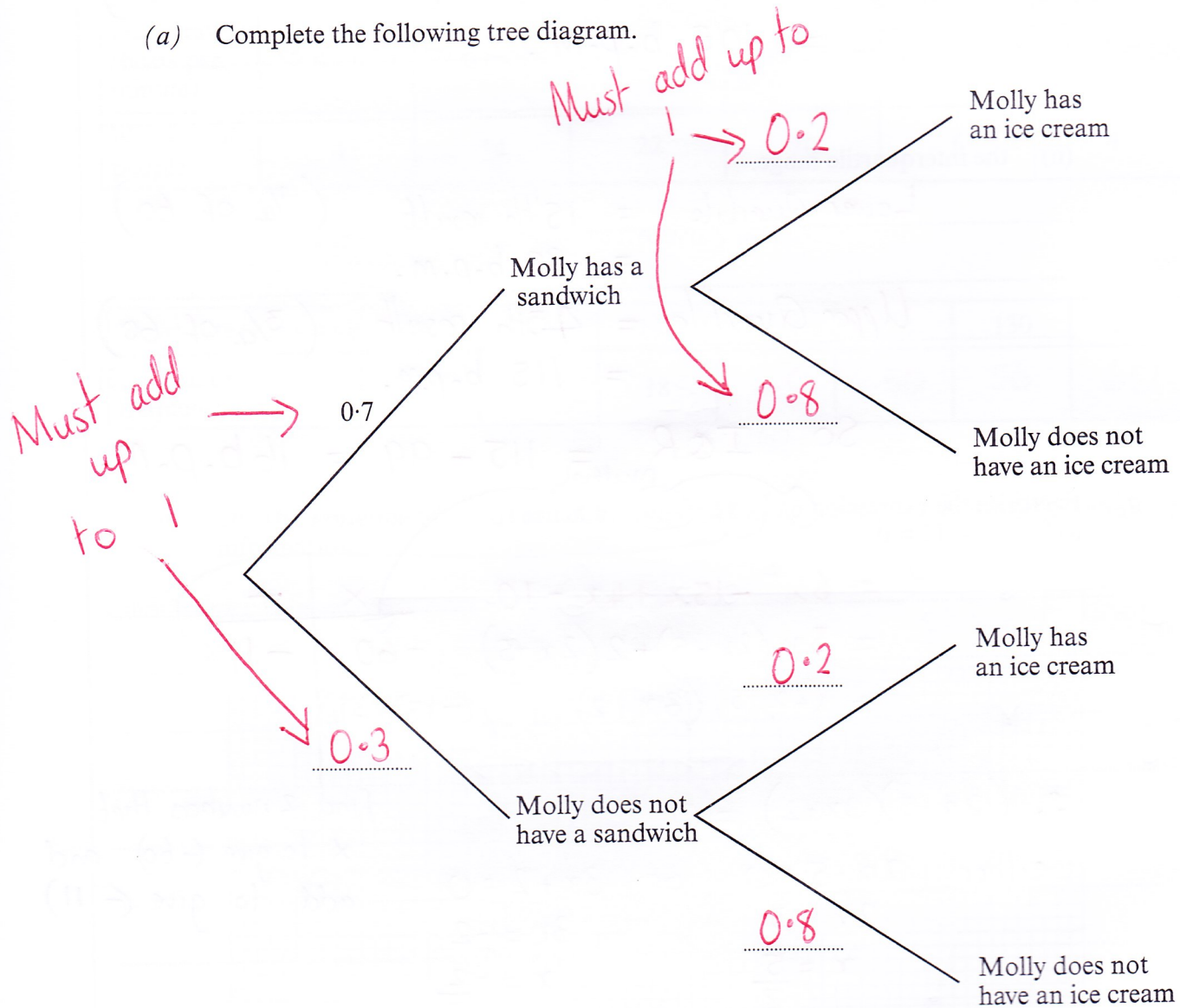
$$= (2y)^2 - 9^2$$

$$= (2y-9)(2y+9)$$

[2]

12. At lunchtime, the probability that Molly has a sandwich is 0.7. Whether or not Molly has a sandwich, the probability that Molly has an ice cream mid-afternoon is 0.2.

(a) Complete the following tree diagram.



[2]

- (b) Calculate the probability that Molly has a sandwich for lunch and has an ice cream mid-afternoon.

0.7×0.2
 $= 0.14$
 Do $7 \times 2 = 14$
 Then count
 2 decimals
 $\Rightarrow 0.14$

[2]

13. Vectors $\mathbf{OK}$, $\mathbf{OL}$ and $\mathbf{OM}$ are shown in the diagram below.

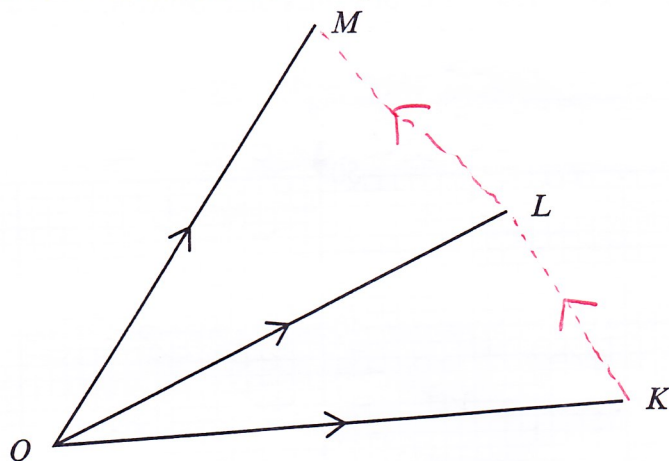


Diagram not drawn to scale

Given that $\mathbf{OK} = 2\mathbf{a} + \mathbf{b}$, $\mathbf{OL} = 5\mathbf{a} + 3\mathbf{b}$ and $\mathbf{OM} = 14\mathbf{a} + 9\mathbf{b}$,

(a) express each of the following in terms of $\mathbf{a}$ and $\mathbf{b}$ in their simplest form,

(i) $\mathbf{KL}$

$$\vec{KL} = \vec{KO} + \vec{OL}$$

$$= -2\mathbf{a} - \mathbf{b} + 5\mathbf{a} + 3\mathbf{b}$$

(ii) $\mathbf{LM}$

$$\vec{LM} = \vec{LO} + \vec{OM}$$

$$= -5\mathbf{a} - 3\mathbf{b} + 14\mathbf{a} + 9\mathbf{b}$$

$$= 9\mathbf{a} + 6\mathbf{b}$$

[3]

(b) show that $\mathbf{KM} = p\mathbf{KL}$ where p is a constant,

$$\vec{KM} = \vec{KL} + \vec{LM}$$

$$= 3\mathbf{a} + 2\mathbf{b} + 9\mathbf{a} + 6\mathbf{b}$$

$$= 12\mathbf{a} + 8\mathbf{b}$$

$$= 4(3\mathbf{a} + 2\mathbf{b})$$

$$= 4\vec{KL}$$

[2]

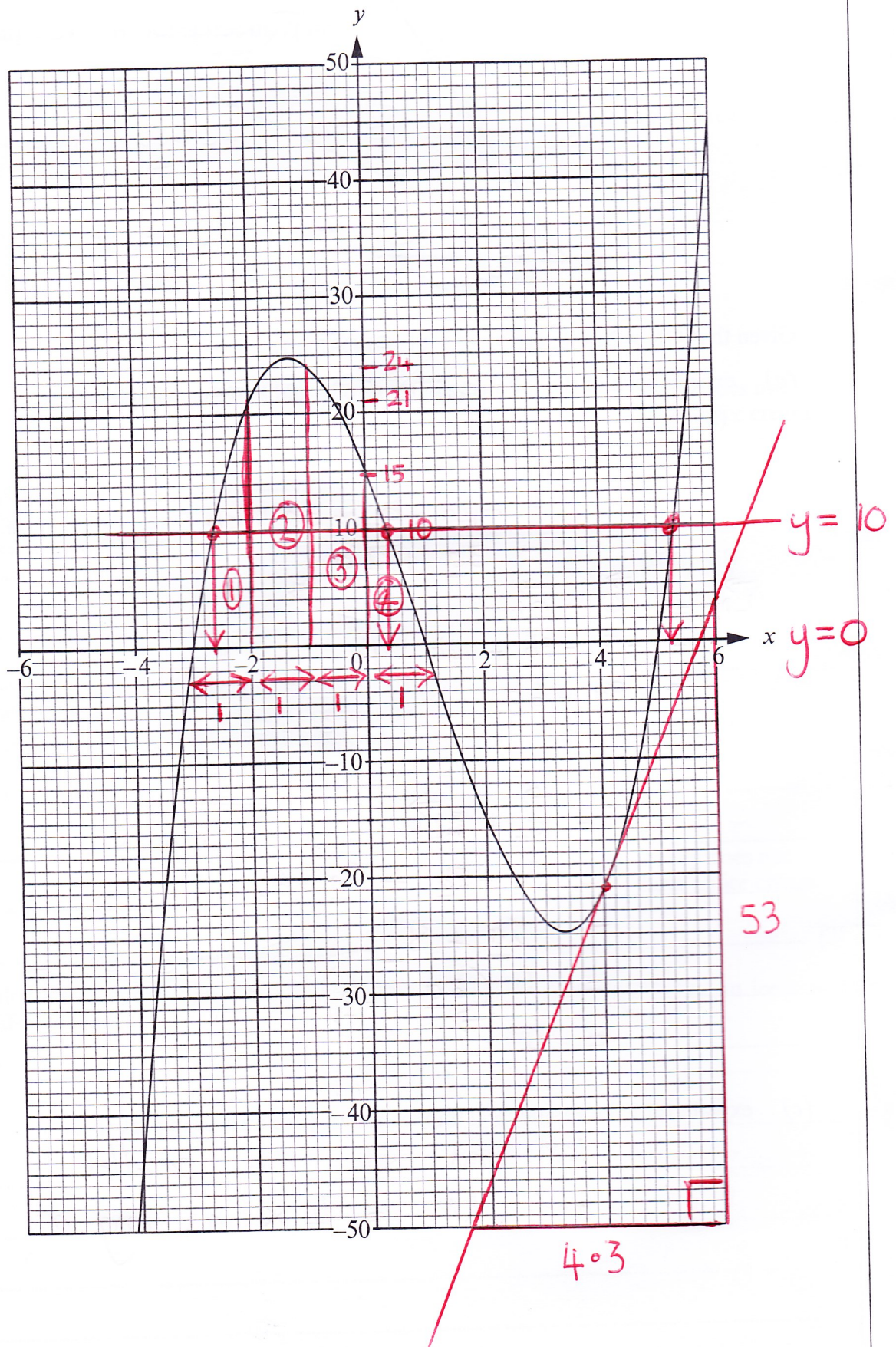
(c) explain fully the geometrical implication of your answer in part (b).

$\mathbf{KM}$ and $\mathbf{KL}$ are parallel

and $\mathbf{KM} = 4 \times$ as long as $\mathbf{KL}$

[2]

14. The graph of $y = x^3 - 3x^2 - 13x + 15$, for values of x between $x = -4$ and $x = 6$, has been drawn below.



- (a) Use the graph to solve $x^3 - 3x^2 - 13x + 15 = 0$.

Plot curve and $y=0$ & x axis

Intersect at

$$x = -3 \quad x = 1 \quad x = 5$$

[1]

- (b) Using the graph, estimate the gradient of the curve $y = x^3 - 3x^2 - 13x + 15$ when $x = 4$.

Draw a tangent. Gradient = $\frac{\text{height}}{\text{base}} = \frac{53}{4.3} = 12.3$

But you could use any sized triangle!!! Remember + because

- (c) By drawing an appropriate line on the graph, solve the equation

$$x^3 - 3x^2 - 13x + 5 = 0.$$

$$x^3 - 3x^2 - 13x + 5 + 10 = 10 \quad \text{Add 10 to both sides}$$

Draw curve given and $y = 10$

Intersect at $x = -2.6 \quad x = 0.4 \quad x = 5.2$

[3]

- (d) Use the trapezium rule with 4 strips to estimate the area of the region enclosed by the curve $y = x^3 - 3x^2 - 13x + 15$ and the x -axis between $x = -3$ and $x = 1$.

$$\text{Area} = \textcircled{1} + \textcircled{2} + \textcircled{3} + \textcircled{4}$$

$$= \frac{bh}{2} + \frac{(a+b)h}{2} + \frac{(a+b)h}{2} + \frac{bh}{2}$$

$$= \frac{1(21)}{2} + \frac{(21+24)1}{2} + \frac{(24+15)1}{2} + \frac{1(15)}{2}$$

$$= 10.5 + 22.5 + 19.5 + 7.5$$

$$= 60 \text{ units}^2$$

[4]

15. (a) Express $0.\dot{6}5\dot{2}$ as a fraction.

$$\text{Let } x = 0.\dot{6}5\dot{2}$$

$$10x = 6.\dot{5}2 \quad \text{--- (1)}$$

$$1000x = 652.\dot{5}2 \quad \text{--- (2)}$$

$\therefore$ SUBTRACT (1) from (2)

$$990x = 652.\dot{5}2 - 6.\dot{5}2$$

$$990x = 646$$

$$x = 646/990$$

[2]

- (b) Given that $p = \sqrt{5}$, $q = \sqrt{13}$ and $r = \sqrt{325}$, simplify pqr .

$$pqr = \sqrt{5}\sqrt{13}\sqrt{325}$$

$$= \sqrt{5}\sqrt{13} 5\sqrt{13}$$

$$= 5\sqrt{5} \times 13$$

$$= 65\sqrt{5}$$

[2]

$$\begin{aligned} \sqrt{325} &= \sqrt{25 \times 13} \\ &= \sqrt{25} \sqrt{13} \\ &= 5\sqrt{13} \end{aligned}$$

$$\sqrt{13} \times \sqrt{13} = 13$$