Surname		Centre Number	Candidate Number
Other Names			0

WJEC CBAC

GCSE

185/10

SOLUTIONS.

MATHEMATICS
HIGHER TIER
PAPER 2

A.M. THURSDAY, 17 November 2011

2 hours

ADDITIONAL MATERIALS

A calculator will be required for this paper.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

Take π as 3·14 or use the π button on your calculator.

INFORMATION FOR CANDIDATES

You should give details of your method of solution especially when a calculator is used.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1	5	
2	9	
3	9	
4	6	
5	6	
6	8	
7	6	
8	8	
9	4	
10	11	
11	3	
12	5	
13	7	
14	6	
15	7	
TOTAL MARK		

1. The distance-time graph shows a 140 km journey from Meadow Green to Tarn Close.

Distance in km
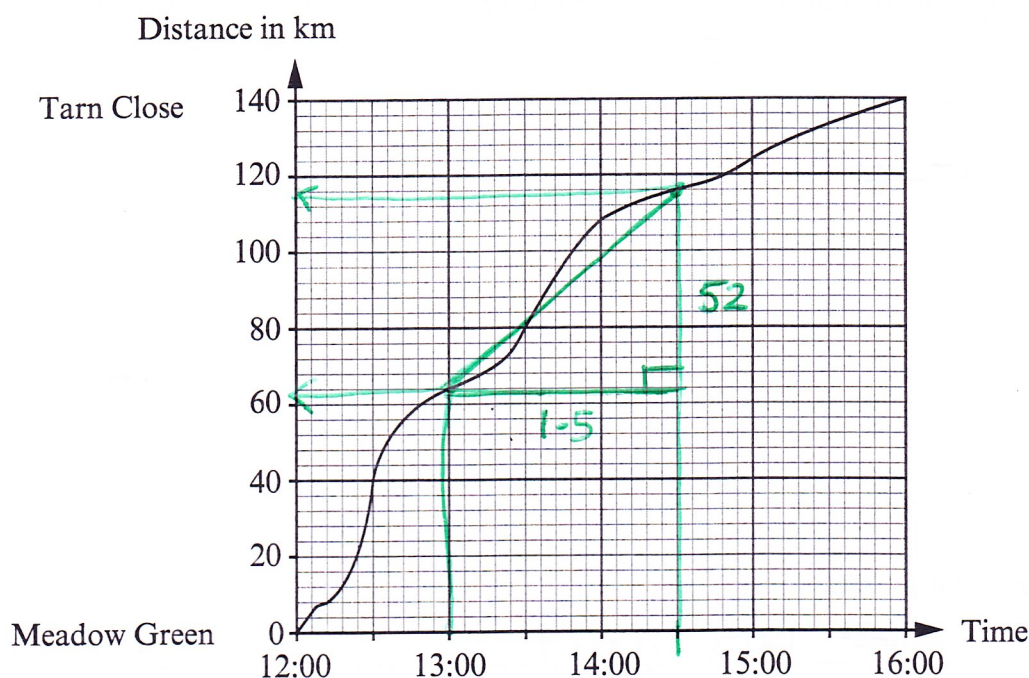
(a) (i) What is the distance travelled between 13:00 and 14:30?

116 - 64

= 52 km

(ii) Find the average speed for this part of the journey.

Use the triangles = $\frac{52}{1.5}$ = $\frac{104}{3}$ = $34\frac{2}{3}$ km/h
drawn

[4]

(b) Explain how you can tell from the graph that it took less time to travel the first 70 km of the journey than it took to travel the final 70 km of the journey.

Steeper 1st 70km ∴ faster

[1]

(185-10)

Turn over.

2. *(a)* The *n*th term of a sequence is $n^2 - 6$.
Write down the first three terms of the sequence.

$1^2 - 6 = -5$

$2^2 - 6 = -2$

$3^2 - 6 = 3$ $-5, -2, 3$

[2]

(b) Factorise $x^2 - 5x$.

$x(x-5)$

[1]

(c) Factorise $240y - 360$.

$120(2y - 3)$

[2]

(d) Solve $4(3x - 11) = 40$.

$12x - 44 = 40$

$12x = 84$

$x = 7$

[3]

(e) Solve $\frac{45}{x} = 5$.

$45 = 5x$

$\frac{45}{5} = x$

$9 = x$

[1]

3. *(a)* Find $\dfrac{14\cdot5 \times 33\cdot4}{710\cdot7 - 35\cdot9}$ correct to two decimal places.

$\dfrac{484\cdot3}{674\cdot8}$

$= 0\cdot72$

[2]

(b) A trader sells a table for £45, excluding tax.
Given that tax is charged at a rate of 18%, calculate the price charged for the table.

$$45 \times 1.18$$
$$= £53.10$$

[3]

(c) Kim bought a scooter for £1600 on 1st January 2010.
Every year the value of the scooter depreciates by 8% of its value at the start of the year.
Find the value of the scooter on 1st January 2012.

$$1600 \times 0.92^2$$
$$= £1354.24$$

[4]

Turn over.

4. The exchange rate for buying Canadian dollars ($) at an exchange bureau is $1.64 for £1.
    Carys only has £700 to exchange into Canadian dollars.
    The lowest Canadian dollar notes the exchange bureau has are $50 notes.

    How many Canadian dollars can Carys buy and how much will this cost her?
    You must give the units of your answer.

    $700 \times 1.64 = \$1,148$

    $\therefore \$1,100$ taken (bought)

    $\therefore \dfrac{1100}{1.64} = £670.73$

[6]

5. *(a)* The lengths of 100 fish were recorded.
The results are summarised in the table below.

Length, l cm	Frequency
$0 < l \leqslant 8$	15
$8 < l \leqslant 16$	67
$16 < l \leqslant 24$	18

Find an estimate for the mean length of these fish.

$$\frac{(15 \times 4) + (67 \times 12) + (18 \times 20)}{100}$$

$$= \frac{60 + 804 + 360}{100}$$

$$= \frac{1224}{100}$$

$$= 12 \cdot 24 \text{ cm}.$$
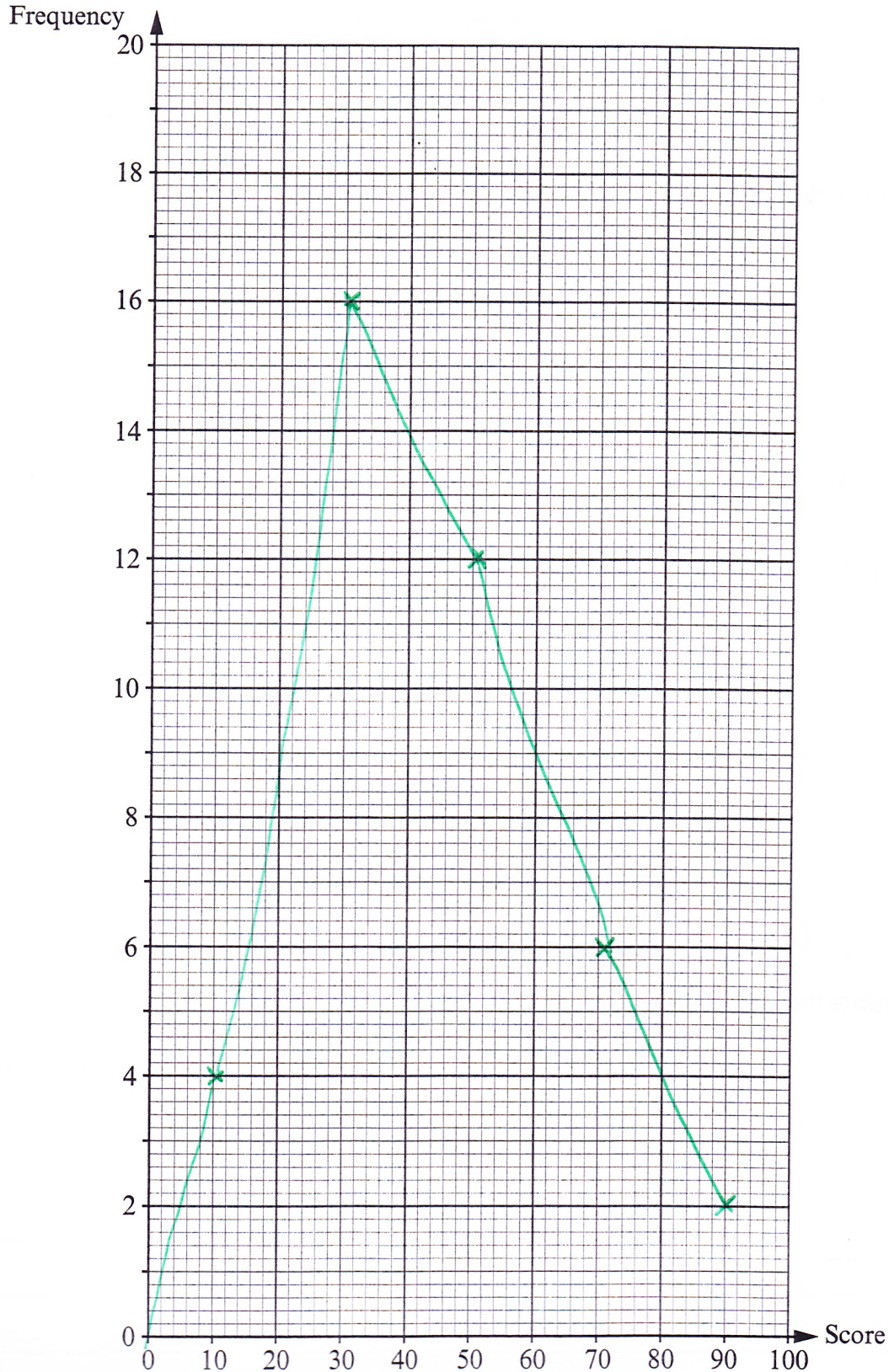
[4]

(b) The points scored by 40 fishermen in a competition were recorded.
The table shows a grouped frequency distribution of the results.

Score	1 to 20	21 to 40	41 to 60	61 to 80	81 to 100
Frequency	4	16	12	6	2

On the graph paper below, draw a frequency polygon to show the data.

[2]

Turn over.

(185-10)

6. *(a)* Calculate the circumference of a circle with a radius of 7·2 cm, giving your answer to an appropriate degree of accuracy.

$C = \pi D$

$C = 3.14 \times 14.4$

$C = 45.22 \text{ cm}$

[3]

(b) Calculate the area of a semicircle with a diameter of 44·8 cm.

$A = \dfrac{\pi r^2}{2}$

$A = \dfrac{3.14 \times 22.4 \times 22.4}{2}$
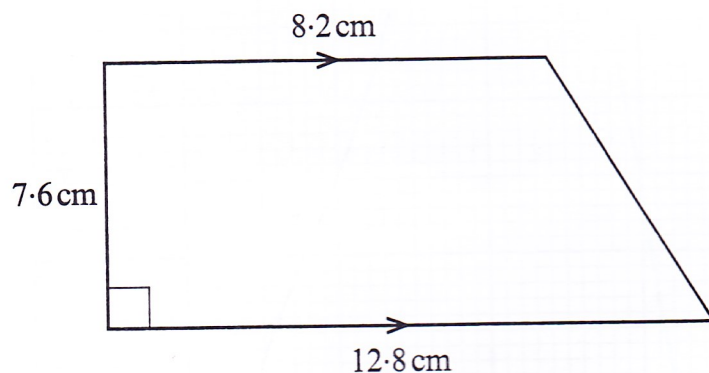
$A = 787.76 \text{ cm}^2$

[3]

(c)

8·2 cm

7·6 cm

12·8 cm

Diagram not drawn to scale

Calculate the area of the trapezium.

$A = \dfrac{(8.2 + 12.8)\, 7.6}{2}$

$A = \dfrac{21 \times 7.6}{2}$

$A = 79.8 \text{ cm}^2$

[2]

7. *(a)* Make k the subject of the formula

$$6(3k - q) = dk + 7.$$

$$18k - 6q = dk + 7$$

$$18k - dk = 7 + 6q$$

$$k(18 - d) = 7 + 6q$$
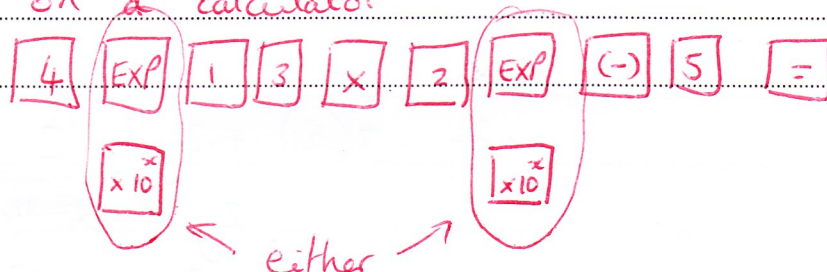
$$k = \frac{(7 + 6q)}{(18 - d)}$$

[4]

(b) (i) Write 76 million in standard form.

$$7.6 \times 10^7$$

(ii) Find, in standard form, the value of $(4 \times 10^{13}) \times (2 \times 10^{-5})$.

$$8 \times 10^8$$

Press on a calculator

$$4 \; \boxed{EXP} \; 1 \; 3 \; \boxed{\times} \; 2 \; \boxed{EXP} \; \boxed{(-)} \; 5 \; \boxed{=}$$

$$\boxed{\times 10^x}$$

$$\boxed{\times 10^x}$$

← either →

[2]

Turn over.

8. *(a)* A solution to the equation $2x^3 + x - 100 = 0$ lies between 3 and 4.
Use the method of trial and improvement to find this solution correct to 1 decimal place.

$x = 3.5$

$LHS = 2(3.5)^3 + 3.5 - 100$

$\quad = -10.75 \qquad$ Too small.

$x = 3.7 \qquad LHS = 2(3.7)^3 + 3.7 - 100 = 5.006$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ Too Big

$x = 3.6 \qquad LHS = 2(3.6)^3 + 3.6 - 100 = -3.088$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ Too Small

$x = 3.65 \qquad LHS = 2(3.65)^3 + 3.6 - 100 = +0.85$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ Too Big

$\therefore \quad x = 3.6$ to 1 d.p.

[4]

(b) Solve the following simultaneous equations by an algebraic (not graphical) method.
Show all your working.

$$7x + 2y = 2 \qquad \times \; 5$$
$$2x - 5y = 34 \qquad \times \; 2$$

$35x + 10y = 10$

$4x - 10y = 68$

ADD) $\quad 39x \qquad = 78$

$\qquad\quad x \qquad = 2$

$\therefore \quad 7(2) + 2y = 2$

$\qquad\qquad 2y = 2 - 14$

$\qquad\qquad 2y = -12$

$\qquad\qquad\quad y = -6$

$\therefore \quad x = 2 \;\Big\}$ Ans
$\qquad\quad y = -6 \;\Big\}$

[4]

9. Triangles *LMN* and *PQR* are similar.

L

P

3·6

N

3

R

5·4

2·5

M

Q
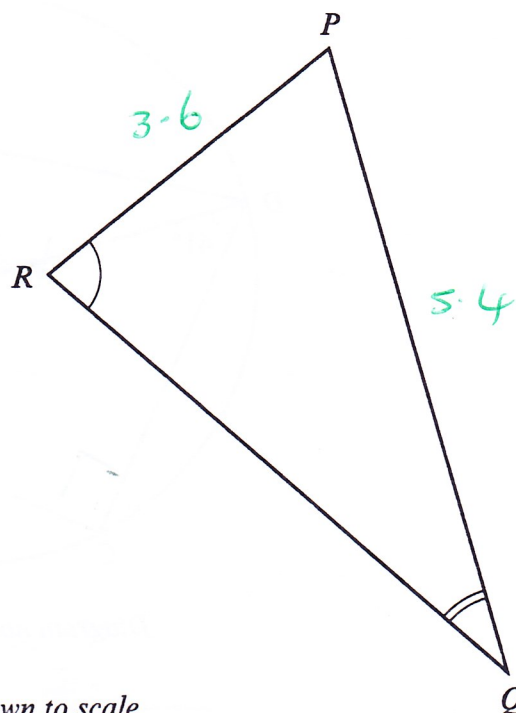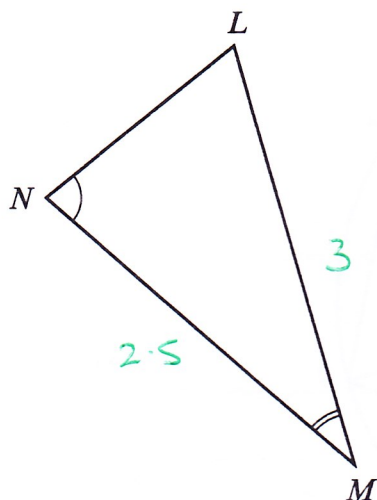
Diagrams not drawn to scale

LM = 3 cm, *MN* = 2·5 cm, *PR* = 3·6 cm and *PQ* = 5·4 cm.
Showing all your working, find the length of

(a) *RQ*,

Scale factor SF = $\frac{5·4}{3}$ = $\frac{54}{30}$ = $\frac{18}{10}$ = 1·8

so RQ = 2·5 × 1·8 = 4·5 cm

[2]

(b) *LN*.

$\frac{3·6}{1·8}$ = 2 cm.

[2]

10. The points *A, B, C* and *D* all lie on the circumference of a circle with centre *O*.
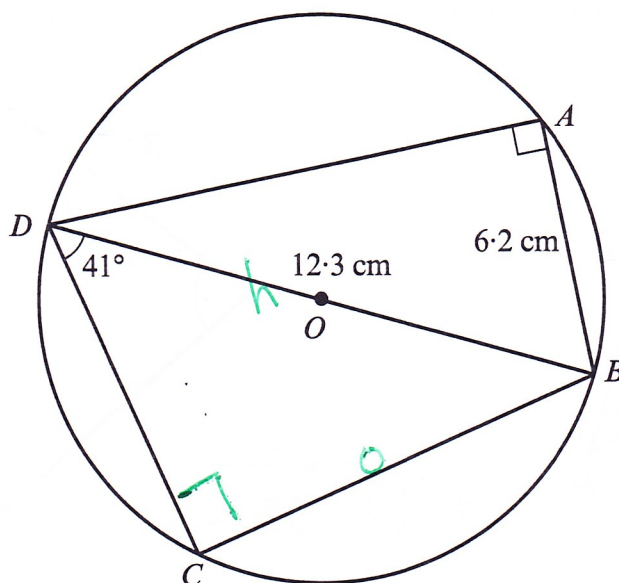

Diagram not drawn to scale

Given that $BD = 12·3$ cm, $AB = 6·2$ cm, $B\widehat{D}C = 41°$ and $D\widehat{A}B = 90°$, calculate

(a) the length *AD*,

$$12·3^2 = 6·2^2 + AD^2$$
$$151·29 = 38·44 + AD^2$$
$$112·85 = AD^2$$
$$10·62 \text{ cm} = AD$$

[3]

(b) the area of triangle *BCD*.

$\sin 41 = \dfrac{O}{12.3}$ $\cos 41 = \dfrac{a}{12.3}$

$12.3 \sin 41 = O$ $12.3 \cos 41 = a$

$8.07 \text{ cm} = BC$ $9.28 \text{ cm} = CD$

$\text{so } A = \dfrac{bh}{2}$

$= \dfrac{8.07 \times 9.28}{2}$

$= 37.44 \text{ cm}^2$

[8]

11. There are 100 pupils in year 8.
The time taken by each pupil to answer a question was recorded.
The following grouped frequency distribution was obtained.

Time, t seconds	$0 < t \leqslant 10$	$10 < t \leqslant 20$	$20 < t \leqslant 30$	$30 < t \leqslant 40$	$40 < t \leqslant 60$
Number of pupils	10	16	24	42	8

freq density

$$\frac{10}{10} = 1 \qquad \frac{16}{10} = 1.6 \qquad \frac{24}{10} = 2.4 \qquad \frac{42}{10} = 4.2 \qquad \frac{8}{20} = 0.4$$

(a) Find an estimate for the median of this distribution.
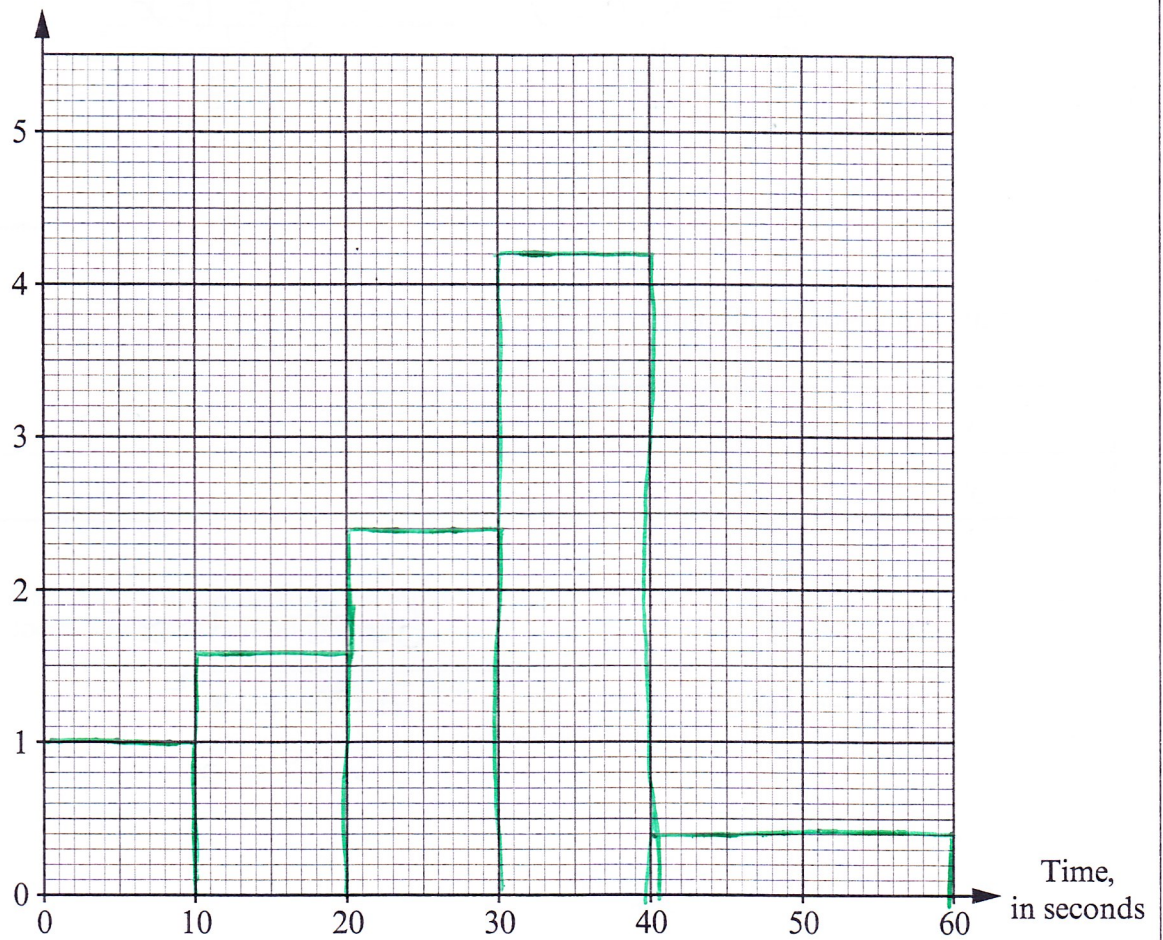
50th result.   30 secs.

[1]

(b) Draw a histogram to illustrate the distribution on the graph paper opposite.

Use the frequency density figures above

[2]

Time taken to answer in seconds

Frequency density

Time, in seconds

Turn over.

12. Given that y is proportional to x^2, and that $y = 4$ when $x = 0.5$,

(a)   find an expression for y in terms of x,

$$y = kx^2$$
$$4 = k(0.5^2)$$
$$4 = \frac{k}{4}$$
$$16 = k$$

$$\therefore y = 16x^2$$

[3]

(b)   use the expression you found in (a) to complete the following table.

x	0.5	3	20
y	4	144	6400

$$y = 16 \times 9$$
$$y = 144$$

$$6400 = 16x^2$$
$$400 = x^2$$
$$20 = x$$

[2]

13. A solid cuboid has length $(x + 5)$ cm, width $(x + 4)$ cm and height 6 cm.
The surface area of the cuboid is 205 cm^2.

(a) Show that x satisfies the equation $2x^2 + 42x - 57 = 0$.

$$2(x+5)(x+4) + 2(x+5)6 + 2(x+4)6 = 205$$
$$2x^2 + 18x + 40 + 12x + 60 + 12x + 48 = 205$$
$$2x^2 + 42x - 57 = 0$$

[3]

(b) Use the formula method to solve the equation $2x^2 + 42x - 57 = 0$, giving solutions correct to two decimal places.

$$x = \frac{-42 \pm \sqrt{42^2 - 4(2)(-57)}}{4}$$

$$x = \frac{-42 \pm \sqrt{1764 + 456}}{4}$$

$$x = \frac{-42 \pm \sqrt{2220}}{4}$$

$$x = \frac{-42 \pm 47.117}{4}$$

$$x = \frac{-8\,7.117}{4} \quad \text{oR} \quad \frac{5.117}{4}$$

$$= -21.78 \qquad 1.28$$

[3]

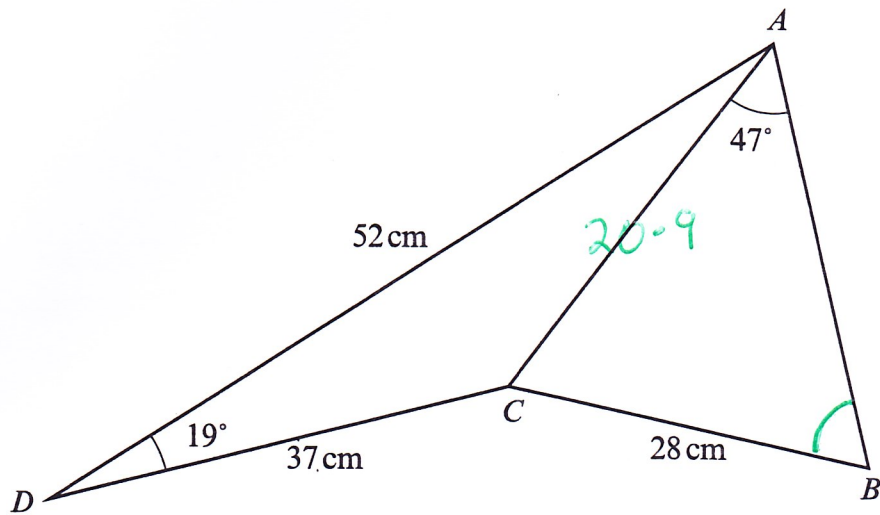(c) Hence write down the dimensions of the cuboid.

$$\therefore \quad x + 5 = 6.28 \text{ cm}$$
$$x + 4 = 5.28 \text{ cm}$$
$$\text{and} \quad 6 \text{ cm}.$$

[1]

Turn over.

15.

Diagram not drawn to scale

Calculate the size of $A\hat{B}C$.

$\triangle ACD$ Cos rule.

$$B \quad d^2 = a^2 + c^2 - 2ac \cos D$$

$$d^2 = 37^2 + 52^2 - 2(37)(52)\cos 19°$$

$$d^2 = 1369 + 2704 - 3848 \cos 19$$

$$d^2 = 4073 - 3638 \cdot 36$$

$$d^2 = 434 \cdot 64$$

$$d = 20 \cdot 85 \text{ cm}$$

$$\frac{\sin B}{20 \cdot 9} = \frac{\sin 47}{28}$$

$$\sin B = \frac{20 \cdot 9 \sin 47}{28}$$

$$= 0 \cdot 5459$$

$$\therefore A\hat{B}C = 33 \cdot 1°$$

[7]