Surname	Centre Number	Candidate Number
Other Names		0

WJEC CBAC

GCSE

4370/05

SOLUTIONS

MATHEMATICS – LINEAR
PAPER 1
HIGHER TIER

A.M. WEDNESDAY, 6 November 2013

2 hours

CALCULATORS ARE NOT TO BE USED FOR THIS PAPER

ADDITIONAL MATERIALS

A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

Take π as 3·14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded that assessment will take into account the quality of written communication (including mathematical communication) used in your answer to question **7**.

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1.	6	
2.	4	
3.	8	
4.	7	
5.	4	
6.	3	
7.	7	
8.	5	
9.	4	
10.	5	
11.	3	
12.	11	
13.	4	
14.	2	
15.	6	
16.	9	
17.	5	
18.	7	
Total	100	

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Examiner only

1. Given that $f = -3$, $g = 2$ and $h = 5$, find the value of the following expressions.

(a) $\dfrac{f^2 - h}{g}$ [2]

$= \dfrac{(-3)^2 - 5}{2} = \dfrac{9-5}{2} = \dfrac{4}{2} = 2$

(b) $(2h)^3$ [2]

$[2(5)]^3$

$= 10^3$

$= 1000$

(c) $g - f + \dfrac{1}{h}$ [2]

$2 - (-3) + \dfrac{1}{5}$

$= 2 + 3 + \dfrac{1}{5}$

$= 5\dfrac{1}{5}$

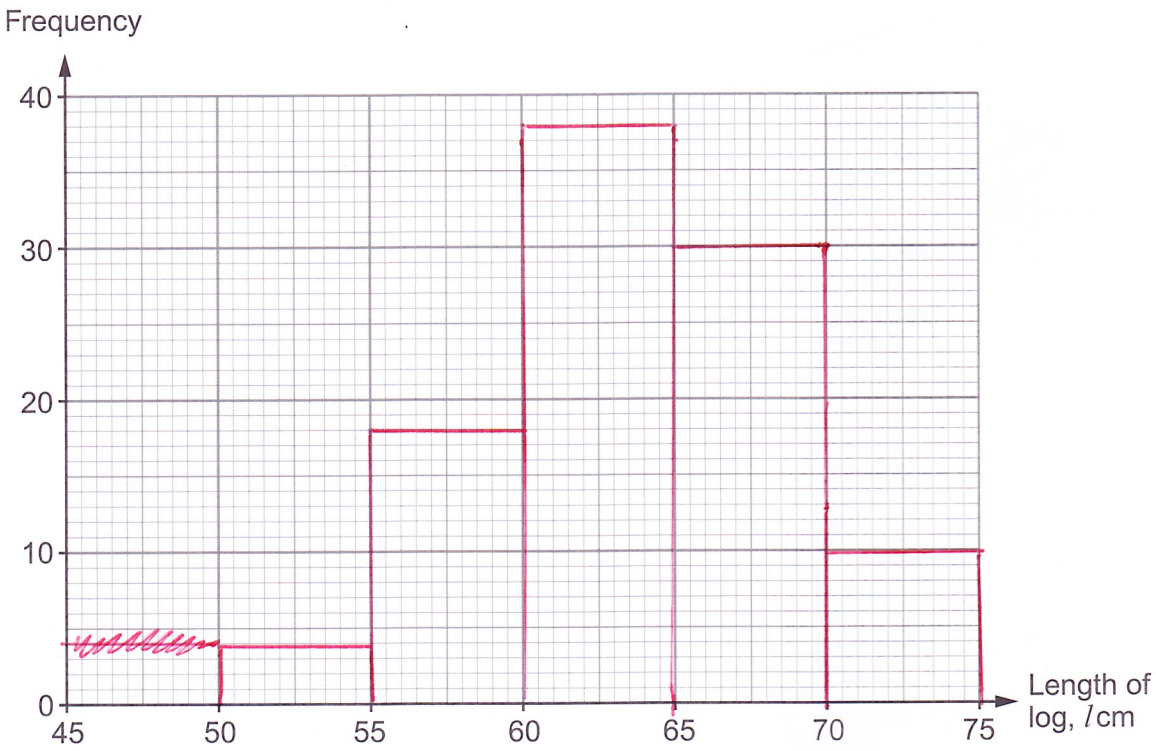
Examine only

2. Tom collected 100 logs and measured their lengths in centimetres.

The table below shows a grouped frequency distribution of his results.

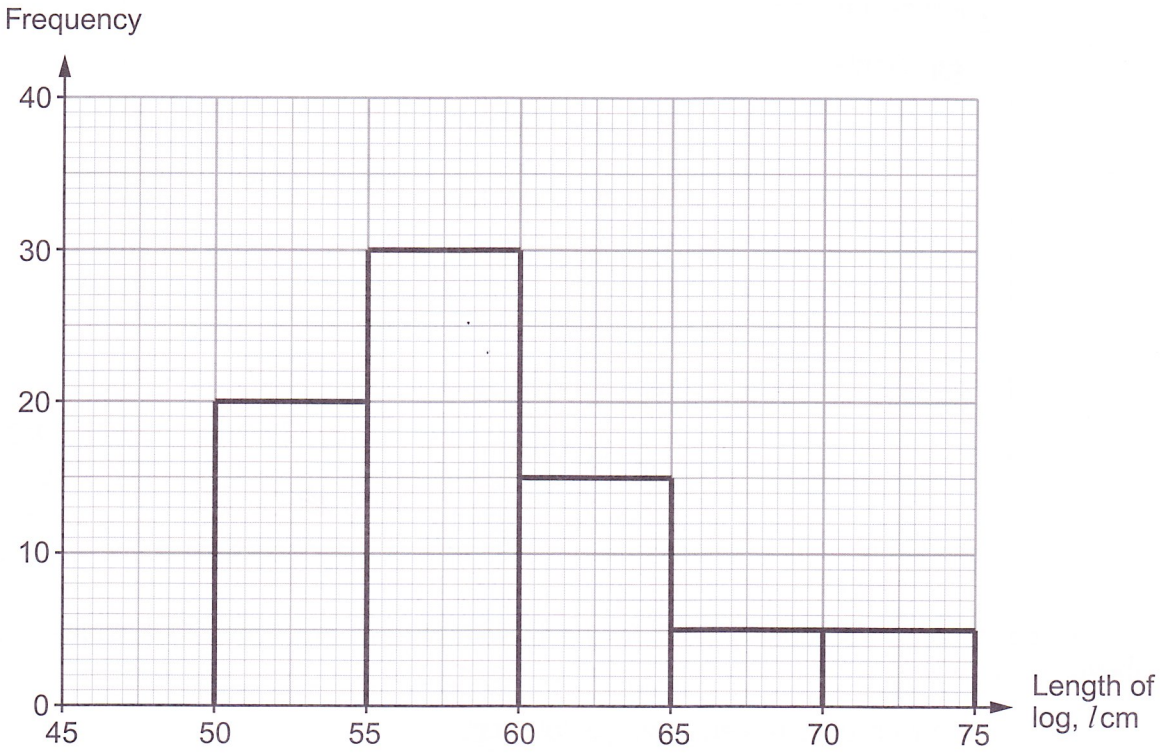
Length of log, l cm	$50 < l \leqslant 55$	$55 < l \leqslant 60$	$60 < l \leqslant 65$	$65 < l \leqslant 70$	$70 < l \leqslant 75$
Frequency	4	18	38	30	10

(a) On the graph paper below, draw a grouped frequency diagram to show this data. [2]



Examiner only

(b) Billy also collected and measured the lengths of some logs.
The grouped frequency diagram of his results is shown below.



(i) How many logs did Billy collect and measure? [1]

20 + 30 + 15 + 5 + 5

= 75

(ii) Was it Tom or Billy who collected the longer logs, on average? [1]

Tom

Explain how the grouped frequency diagrams help you to decide.

Taller bars are to the right for Tom.
This is where the lengths are greater values.

Turn over.

Examiner only

3.

Pasta with cheese and asparagus sauce

Serves **4** people

Ingredients:

4 ounces Butter → 115g

8 ounces Asparagus → 2 x 115 = 230g

12 ounces Pasta → 3 x 115 = 345g

1 Onion

2 tablespoons Stock → 2 x 15 = 30ml

$\frac{2}{3}$ cup Cream → $\frac{2}{3}$ of 240 = 240 ÷ 3 x 2 = 160 mL

3 ounces Cheese → 115 ÷ 4 x 3 = 86.25 g

The recipe in Tamara's cookery book for pasta with cheese and asparagus sauce is shown above.

Information to convert units is also given, as follows:

- 1 cup is approximately 240 ml
- 4 ounces is approximately 115 g
- 1 tablespoon is 15 ml

Examiner only

(a) Complete the recipe for serving **8 people** using **ml** and **g**. [4]

3 ounces = 115 ÷ 4 × 3 = 28·75

4 | 115·30

28·75
× 3
86·25

Pasta with cheese and asparagus sauce

Serves **8** people

Double all the quantities from 1st table

Ingredients:

230 g Butter

460 g Asparagus

690 g Pasta

2 Onions

60 ml Stock

320 ml Cream

173 g Cheese

(b) Tamara has a $\frac{1}{2}$ litre carton of cream.
She has large quantities of all the other ingredients.
Calculate the largest number of portions of pasta with cheese and asparagus sauce that Tamara can make using as much of the cream as possible. [4]

4 people need 160 ml cream

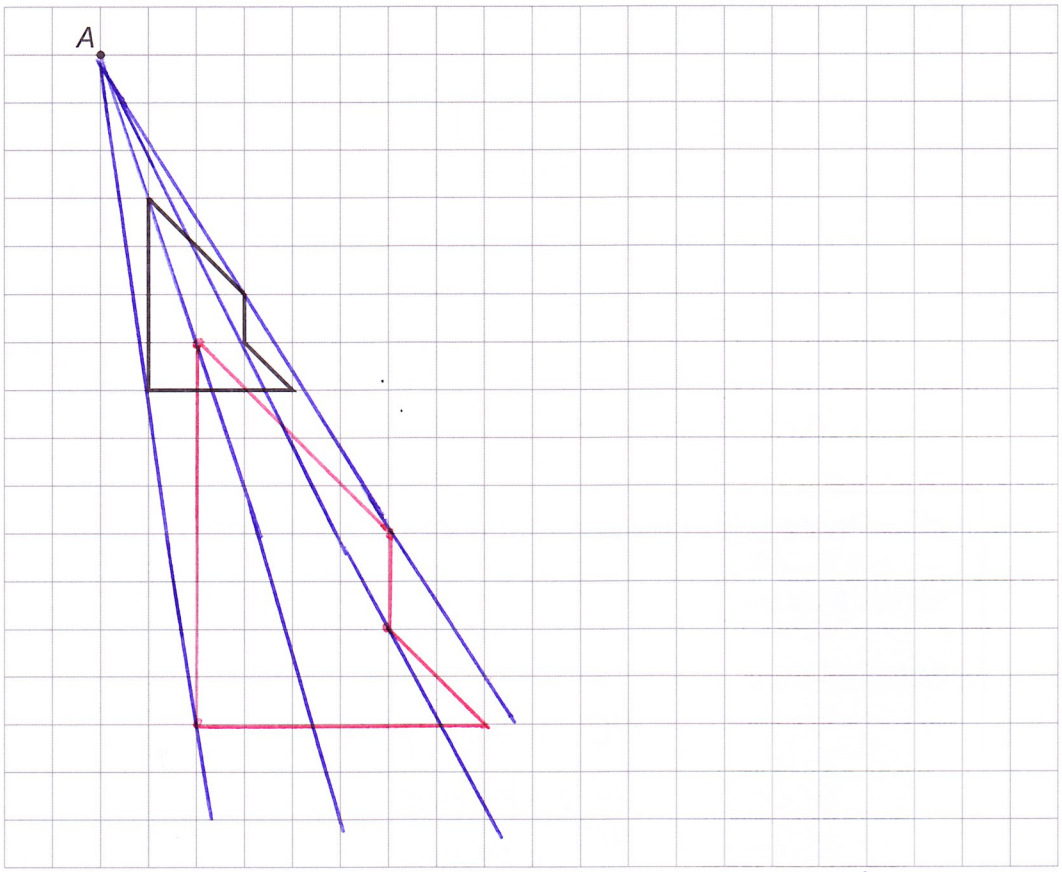
1 person needs $\frac{160}{4}$ = 40 ml cream.

So how many 40ml in ½ litre?

½ litre = 500ml so $\frac{500}{40} = \frac{50}{4} = 12.5$

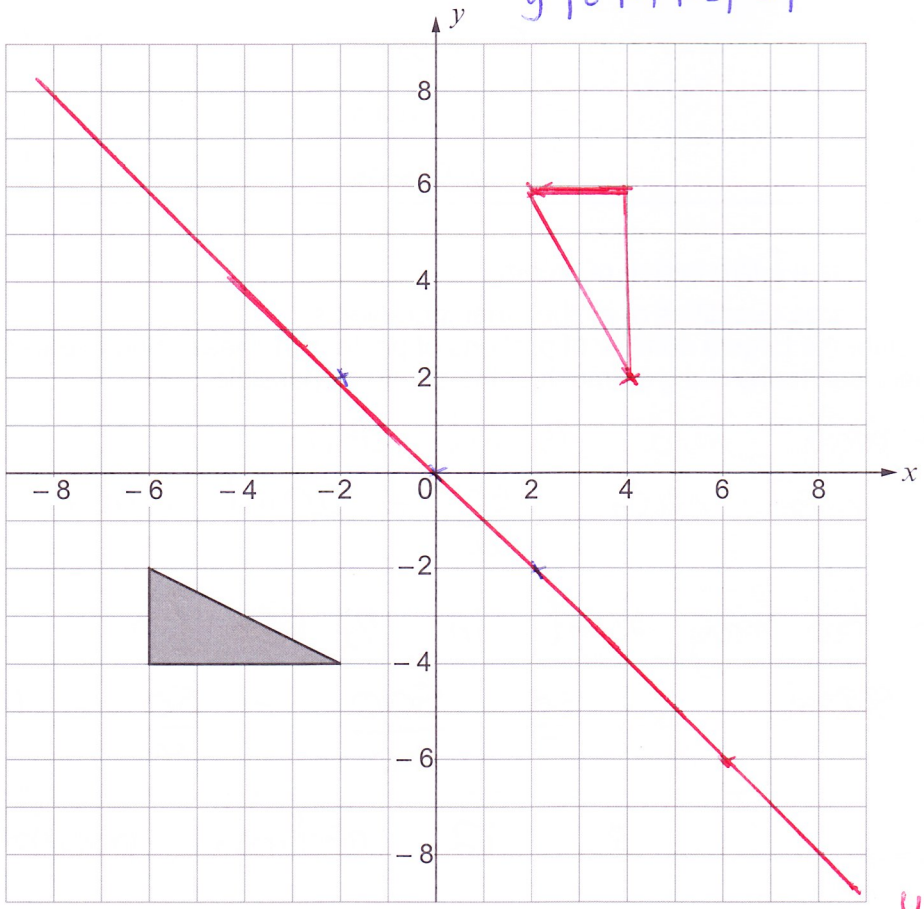
So 12 portions possible.

4. *(a)* Enlarge the shape shown on the grid by a scale factor of 2, using A as the centre of enlargement. [3]



(b) Reflect the triangle in the line $y = -x$. [2]

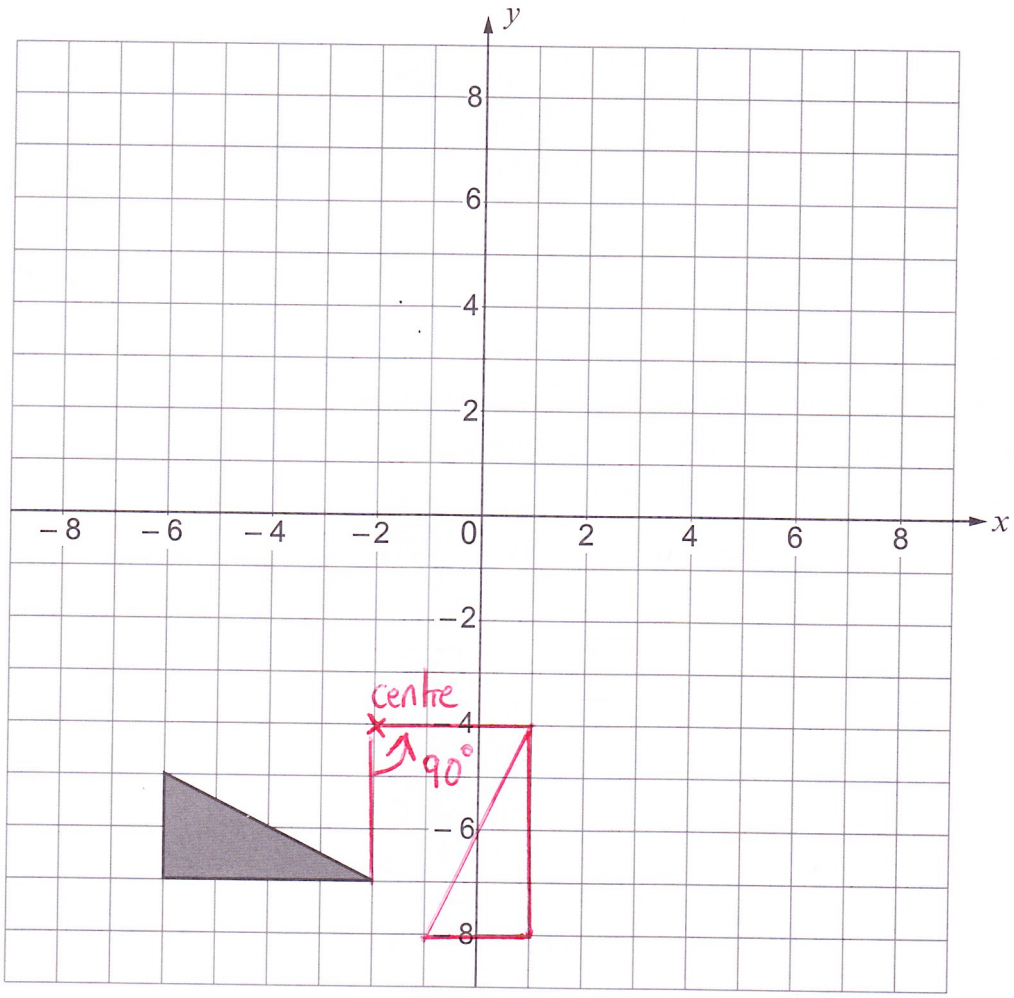
x	0	1	2	-2
y	0	-1	-2	2



$y = -x$

Examiner only

(c) Rotate the triangle shown on the grid below through 90° anticlockwise about the point (−2, −4). [2]



Examiner only

5. *(a)* Expand $y(y^5 + 3)$. [2]

$= y^6 + 3y$

(b) Factorise $4x^3 - 2x$. [2]

$2x(2x^2 - 1)$

6. Manilo won some money.
He gave each of his close friends $\frac{1}{24}$ of the money he won.
He kept the remaining $\frac{2}{3}$ of the money for himself.
How many close friends does Manilo have? [3]

If $\frac{2}{3}$ remained he gave $\frac{1}{3}$ away

So $\frac{1}{3} = \frac{\square}{24}$

Now $\frac{1}{3} = \frac{8}{24}$

$\therefore$ Must have 8 close friends

Examiner only

7. *You will be assessed on the quality of your written communication in this question.*

Dafydd works with his section manager in a department store.

He has a salary of £17 000 per annum and he usually receives a bonus every year.

Dafydd has to make a choice about which bonus to take from those offered by the store this year.

He can either have

- the smaller share when £2500 is shared in the ratio 2 : 3 with his section manager
  or
- a sum of money equal to 6 % of his salary.

Which of these two bonus offers should Dafydd accept?
You must show your working and give a reason for your choice. [7]

RATIO OPTION

5 ratio parts = 2500

1 ratio part = $\frac{2500}{5}$ = 500

2 ratio parts = 500 × 2 = £1000

∴ Bonus £1000

% OPTION

1% of £17 000 = $\frac{17000}{100}$ = £170

6% = 6 × 170 = 170 × 6 = £1020

4 × 6

1020

∴ Best bonus is 6% of salary, it is £20 more.

Examiner only

8. Martha is laying out a new design for a flowerbed in her garden, as shown in the diagram below.

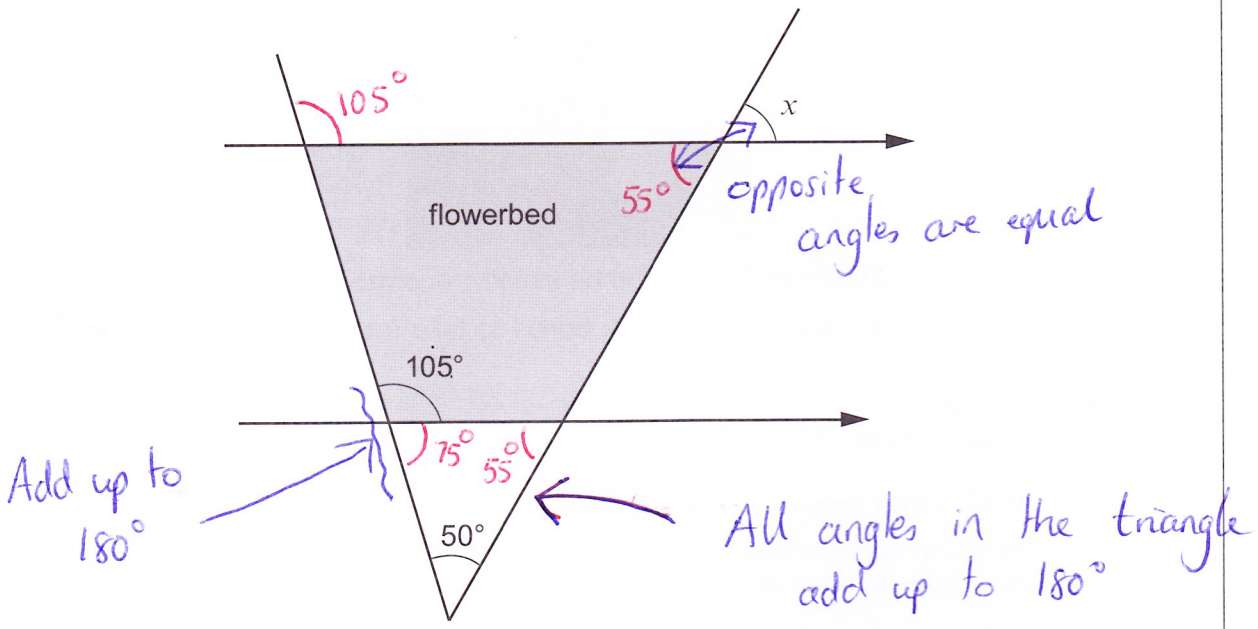


Diagram not drawn to scale

(a) Calculate the size of angle x. [2]

Use bottom triangle to help

$x =$ 55° °

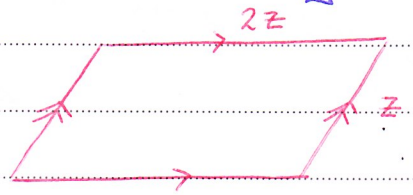
Examiner only

(b) Martha has another flowerbed in the shape of a parallelogram.

The longer sides measure twice the length of the shorter sides of the parallelogram.
The perimeter of this flowerbed is 24 metres.

Let the length of one of the shorter sides of the flowerbed be z metres.
Form an equation in terms of z.
Solve your equation to find the length of one of the shorter sides of the parallelogram. [3]

long side is twice as long as short side



Perimeter is distance all the way around the outside

$$2z + z + 2z + z = 24$$

$$6z = 24$$

$$z = \frac{24}{6}$$

$$z = 4\text{ m}$$

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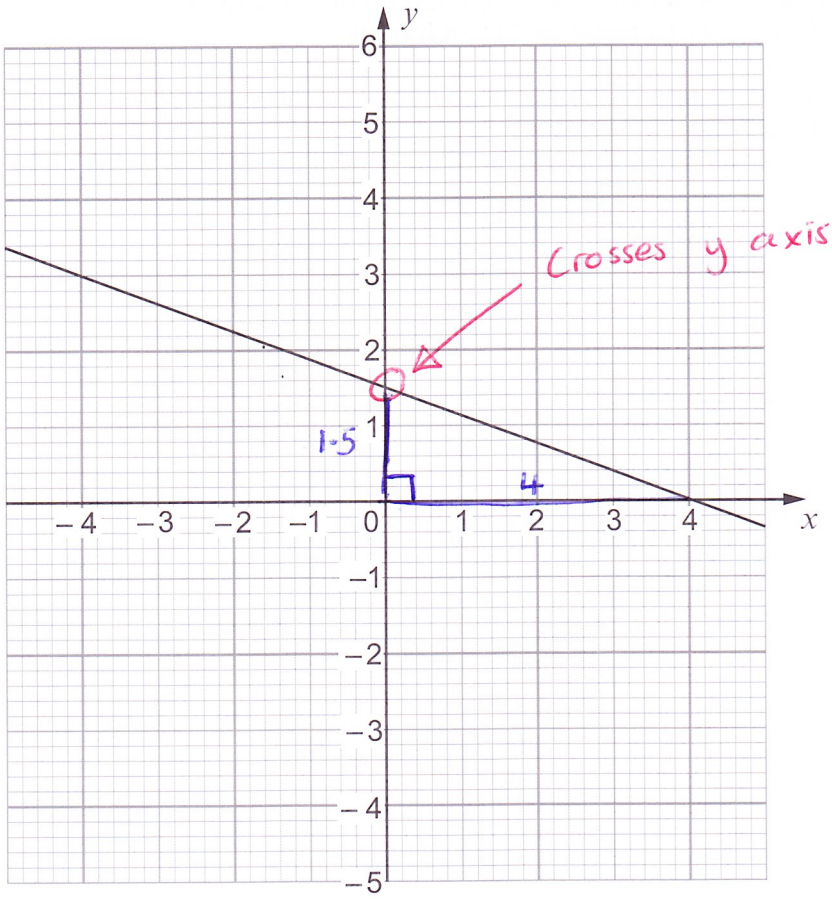
Exam only

9. The graph of a straight line is shown below.



(a) You are asked to match one of the equations given below with the straight line.
Put a ring around your choice of equation.
You must show your working **or** give an explanation for your choice of answer. [2]

$y = -4x + 1{\cdot}5$ $\quad$ $1{\cdot}5y = 4x$ $\quad$ $8y = 3x + 12$

$8y = -3x + 12$ $\quad$ $y = 4x + 1{\cdot}5$ $\quad$ $y = -\frac{1}{2}x + 1{\cdot}5$

Crosses y axis at 1·5

Gradient $m = \frac{\text{height}}{\text{base}} = \frac{1{\cdot}5}{4} = \frac{3}{8}$ but \ so gradient $m = -\frac{3}{8}$ NOTICE

Equation is $y = -\frac{3}{8}x + 1{\cdot}5$

$\times 8$ $\quad 8y = -3x + 12$

Examiner only

(b) Find the coordinates of the midpoint of the straight line which joins (2, –4) and (–2, 6). [2]

(x_1, y_1) (x_2, y_2)

$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= M\left(\frac{2+(-2)}{2}, \frac{-4+6}{2}\right)$

$= M\left(\frac{0}{2}, \frac{2}{2}\right)$

(0 , 1)

10. *(a)* The nth term of a sequence is $3n^2 + 2n$.
Write down the first three terms of the sequence. [2]

$n=1 \quad 3(1^2) + 2(1) = 3 + 2 = 5$

$n=2 \quad 3(2^2) + 2(2) = 12 + 4 = 16$

$n=3 \quad 3(3^2) + 2(3) = 27 + 6 = 33$

$\therefore$ 5, 16, 33, ...

(b) The nth term of a sequence is $5n - n^2$.
Find the 10th term of the sequence. [1]

10th term $n = 10 \quad 5(10) - 10^2$

$= 50 - 100$

$= -50$

(c) Find the nth term of the sequence $-9, -6, -1, 6, 15, 26, \ldots$ [2]

$+3 \quad +5 \quad +7 \quad +9 \quad +11$

$+2 \quad +2 \quad +2 \quad +2$

2nd difference = +2 each time so a squared formul

$n^2 - 10$

If $n=1$

$n^2 = 1$ but we need -9

Examiner only

11. Harriet invests a sum of money into a savings account that pays compound interest at 3% per annum.
No further deposits or withdrawals are made.

A spreadsheet is used to calculate the total amount, £A, in Harriet's account.
It contains the formula

Amount invested
x years

$$A = 220 \times 1{\cdot}03^{x},$$

3% interest increase x 1·03

where x is the number of years since the investment was started.

(a) How much did Harriet initially invest in her savings account? [1]

£220

(b) Calculate the amount in Harriet's savings account after 1 year. [2]

$A = 220 \times 1{\cdot}03^{1}$

$A = 220 \times 1{\cdot}03$

OR

1% = £2·20

3% = 3 × 2·20 = £6·60

∴ 3% increase = 220 + 6·60

= £226·60

12. *(a)* Factorise $x^2 - 4x - 21$ and hence solve $x^2 - 4x - 21 = 0$. [3]

$= (x-7)(x+3)$

x	$+$
-21	-4
$(-7)(+3)$	

$\therefore$ Eqn. becomes $(x-7)(x+3) = 0$

either $x - 7 = 0$ or $x + 3 = 0$

$x = 7$ $\quad$ $x = -3$

Brackets needed

(b) Solve $\frac{(2x+3)}{3} + \frac{(4x+1)}{2} = \frac{43}{2}$. [4]

LCM = 6

$\frac{2(2x+3)}{6} + \frac{3(4x+1)}{6} = \frac{129}{6}$

$\times 6$ $\quad 2(2x+3) + 3(4x+1) = 129$

$4x + 6 + 12x + 3 = 129$

$16x + 9 = 129$

$16x = 129 - 9$

$16x = 120$

$x = \frac{120}{16}$

$x = \frac{30}{4} = \frac{15}{2}$

OR $\quad x = 7.5$

Examiner only

(c) Make e the subject of the following formula. [4]

$$\frac{d(2+e)}{(5-e)} = 3$$

Put bracket in

$\times (5-e)$

$d(2+e) = 3(5-e)$

$2d + de = 15 - 3e$

collect e's together

$de + 3e = 15 - 2d$

$e(d+3) = 15-2d$

common factor of e

$e = \frac{(15-2d)}{(d+3)}$

Examiner only

13. The diagram shows a cylinder.

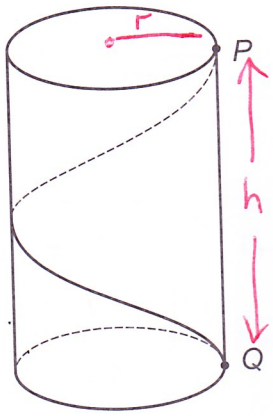


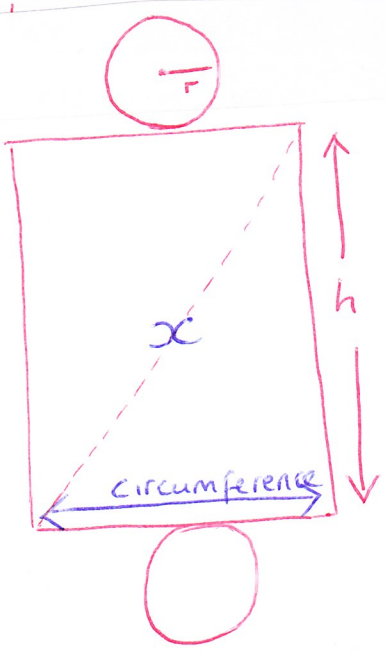
Diagram not drawn to scale

The cylinder has radius r cm and height h cm.

The points P and Q are on the circumferences at opposite ends of the cylinder.
The point P is vertically above point Q.

By considering the net of the cylinder, find an expression for the shortest distance from P to Q when travelling **around** the cylinder.
Give your expression in terms of π, h and r. [4]

NET



Circumference Circle

$C = \pi D$

$C = \pi(2r)$

$C = 2\pi r$

Distance needed = dotted line

Use Pythagoras $x^2 = (\text{circumference})^2 + h^2$

$x^2 = (2\pi r)^2 + h^2$

$x^2 = 4\pi^2 r^2 + h^2$

$x = \sqrt{4\pi^2 r^2 + h^2}$

Examiner only

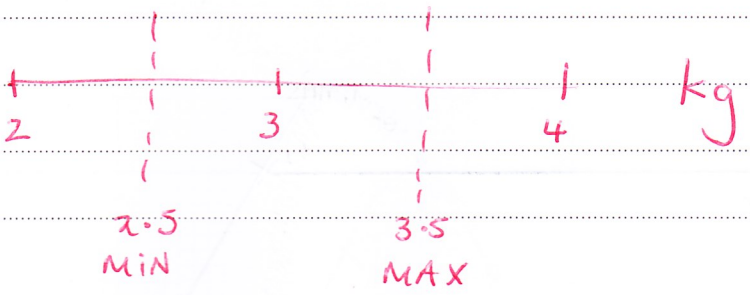
14. A bag of potatoes weighs 3 kg **to the nearest kilogram**.
A sack contains 5 bags of potatoes.

Complete the following sticker to attach to this sack of potatoes. [2]

This sack of 5 bags of potatoes weighs

at least ..12·5.. kg

Min weight of each bag = 2·5kg.



∴ For 5 bags minimum

= 5 x 2·5

= 12·5 kg.

Examiner only

15. The table shows some of the values of $y = 4x^3 - 12x^2$ for values of x from -1 to 3.

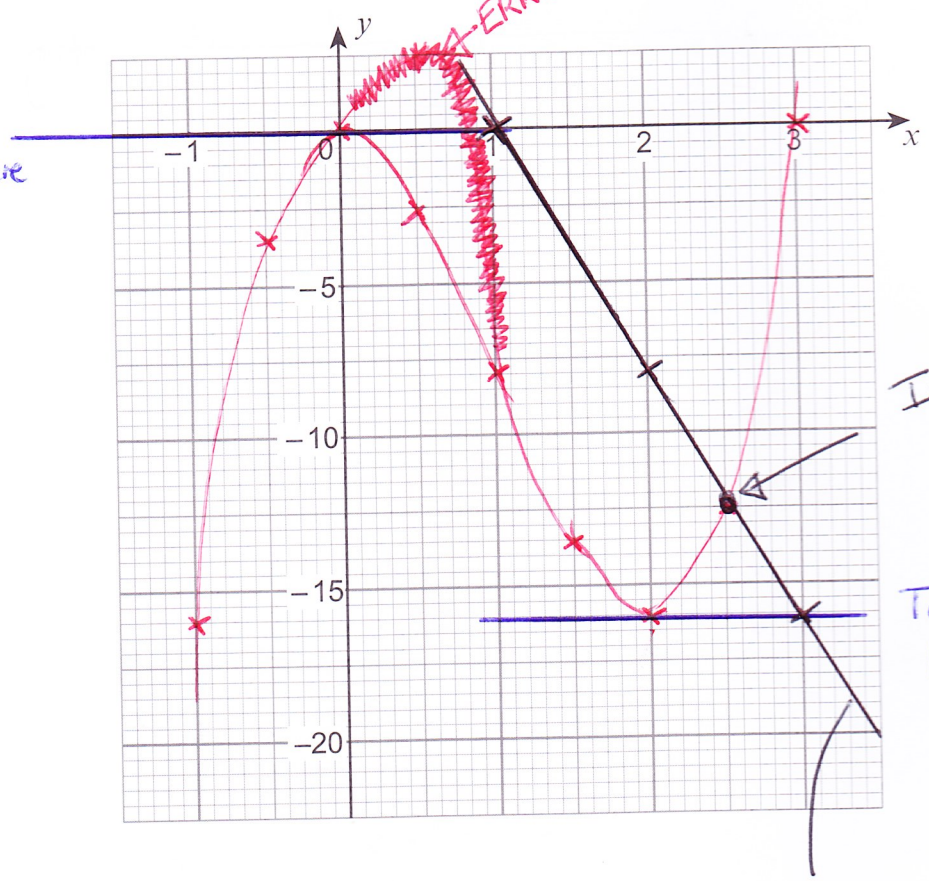
(a) Complete the table by finding the value of y when $x = -1$ and $x = 1$. [1]

x	-1	$-0{\cdot}5$	0	0·5	1	1·5	2	2·5	3
y	-16	$-3{\cdot}5$	0	$-2{\cdot}5$	-8	$-13{\cdot}5$	-16	$-12{\cdot}5$	0

$x = -1$ $y = 4(-1)^3 - 12(-1)^2$
$y = 4(-1) - 12(1)$
$y = -4 - 12$
$y = -16$

$x = 1$
$y = 4(1^3) - 12(1^2)$
$y = 4 - 12$
$y = -8$

(b) Using the graph paper below, draw the graph of $y = 4x^3 - 12x^2$ for values of x between -1 and 3. [2]



A-ERROR

Tangent is Horizontal here so gradient = 0

Intersect here

Tangent is horizontal here so gradient = 0

$y = 8 - 8x$
OR $y = -8x + 8$

Examiner only

(c) Write down the coordinates of the points on $y = 4x^3 - 12x^2$ where the gradient is zero. [1]

(0 , 0) and (2 , −16)

(d) When the line $y = 8 - 8x$ is drawn between $x = 1$ and $x = 3$, it intersects the curve $y = 4x^3 - 12x^2$ at one point.
Use your graph to find the coordinates of this point of intersection. [2]

$y = 8 - 8x$

x	0	1	2	3
y	8	0	−8	−16

This is a straight line graph same as $y = -8x + 8$

Intersect at (2·5, −12·5)

Examiner only

16. *(a)* Express $0\dot{3}4\dot{2}\dot{7}$ as a fraction. [2]

Let $x = 0.34\dot{2}\dot{7}$

$100x = 34.\dot{2}\dot{7}$ — (1)

$10000x = 3427.\dot{2}\dot{7}$ — (2)

subtract (1) from (2) so decimals vanish

$99900x = 3427 - 34$

$x = \frac{3393}{99900}$

(b) Write down any three values of x for which $x^{\frac{3}{2}}$ is rational. [2]

$x^{3/2} = \sqrt{x^3}$

If $x = 1$ $x^{3/2} = \sqrt{1^3} = \sqrt{1} = 1 = \frac{1}{1}$ Rational

$x =$

(c) Give an example of an irrational number

(i) whose square is rational, [1]

$\sqrt{2}$ because $\sqrt{2} \times \sqrt{2} = 2 = \frac{2}{1}$ (rational)

(ii) whose square is irrational. [1]

π

(d) Evaluate $\left(\sqrt{32} + \sqrt{2}\right)^2$. [3]

$= (\sqrt{32} + \sqrt{2})(\sqrt{32} + \sqrt{2})$

$= 32 + \sqrt{2}\sqrt{32} + \sqrt{32}\sqrt{2} + 2$

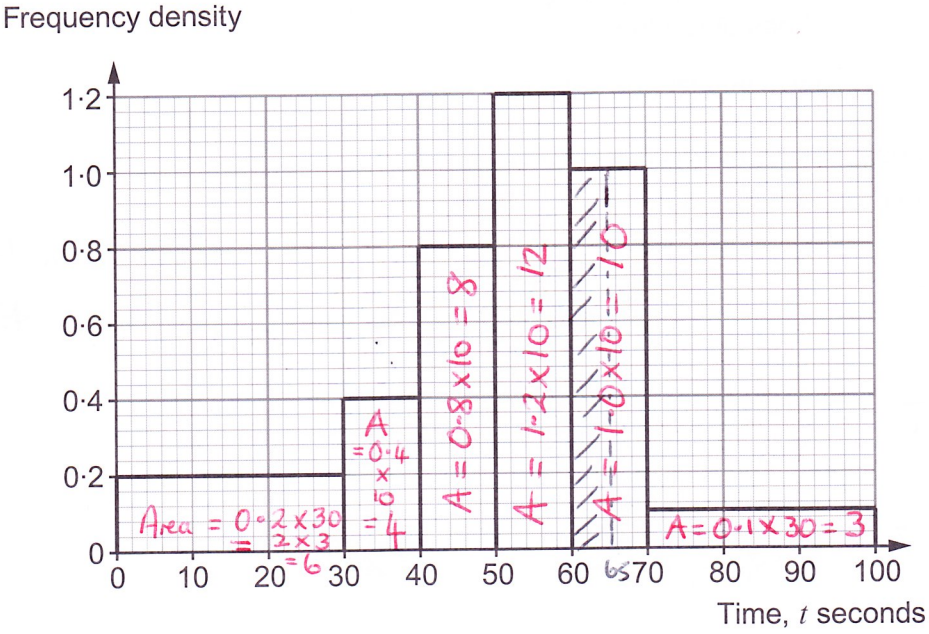
$= 34 + \sqrt{64} + \sqrt{64}$

$= 34 + 8 + 8$

$= 50$

Examiner only

17. The histogram shows the times taken by people in a group to climb a set of stairs.



(a) Calculate the number of people in the group. [3]

$6 + 4 + 8 + 12 + 10 + 3$

$= 43$

(b) Calculate an estimate for the number of people who climbed the stairs in less than 65 seconds. [2]

$6 + 4 + 8 + 12 + 5 = 35$

↑ area is half of full bar

Turn over.

Examiner only

18. Rhodri has four pairs of shoes.
The colours of the pairs of shoes are red, purple, black and white.
The shoes are kept in a trunk in a dark room.
Rhodri selects two shoes at random.

Calculate the probability that Rhodri selects

(a) two shoes, neither of which is purple, [3]

P(NOT Purple and NOT Purple)

$= \frac{6}{8} \times \frac{5}{7}$ ← only 5 remain that are not purple
← only 7 shoes in total left

$= \frac{30}{56}$

$= \frac{15}{28}$

(b) a matching pair of shoes. [4]

P(R and R or P and P or B and B or W and W)

$= \left(\frac{2}{8} \times \frac{1}{7}\right) + \left(\frac{2}{8} \times \frac{1}{7}\right) + \left(\frac{2}{8} \times \frac{1}{7}\right) + \left(\frac{2}{8} \times \frac{1}{7}\right)$

$= \frac{2}{56} + \frac{2}{56} + \frac{2}{56} + \frac{2}{56}$

$= \frac{8}{56}$

$= \frac{1}{7}$

END OF PAPER