

Surname		Centre Number	Candidate Number
Other Names			0



MATHEMATICS – LINEAR
PAPER 1
HIGHER TIER

A.M. WEDNESDAY, 5 November 2014
2 hours

CALCULATORS ARE
NOT TO BE USED
FOR THIS PAPER

ADDITIONAL MATERIALS

A ruler, a protractor and a pair of compasses may be required.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Write your name, centre number and candidate number in the spaces at the top of this page.
Answer all the questions in the spaces provided.
Take π as 3.14.

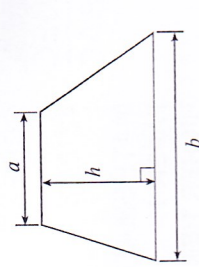
INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.
Unless stated, diagrams are not drawn to scale.
Scale drawing solutions will not be acceptable where you are asked to calculate.
The number of marks is given in brackets at the end of each question or part-question.
You are reminded that assessment will take into account the quality of written communication (including mathematical communication) used in your answer to question 4(c).

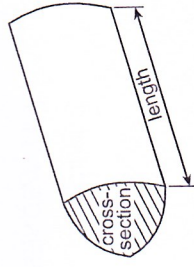
For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1.	4	
2.	11	
3.	6	
4.	9	
5.	7	
6.	4	
7.	7	
8.	8	
9.	11	
10.	10	
11.	3	
12.	7	
13.	5	
14.	5	
15.	3	
Total	100	

Formula List

$$\text{Area of trapezium} = \frac{1}{2}(a+b)h$$

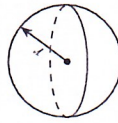


$$\text{Volume of prism} = \text{area of cross-section} \times \text{length}$$



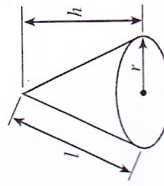
$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Surface area of sphere} = 4\pi r^2$$



$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Curved surface area of cone} = \pi r l$$

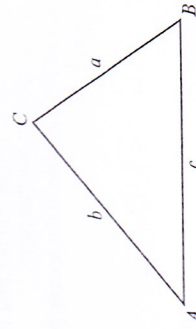


In any triangle ABC

$$\text{Sine rule } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Cosine rule } a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of triangle} = \frac{1}{2}ab \sin C$$



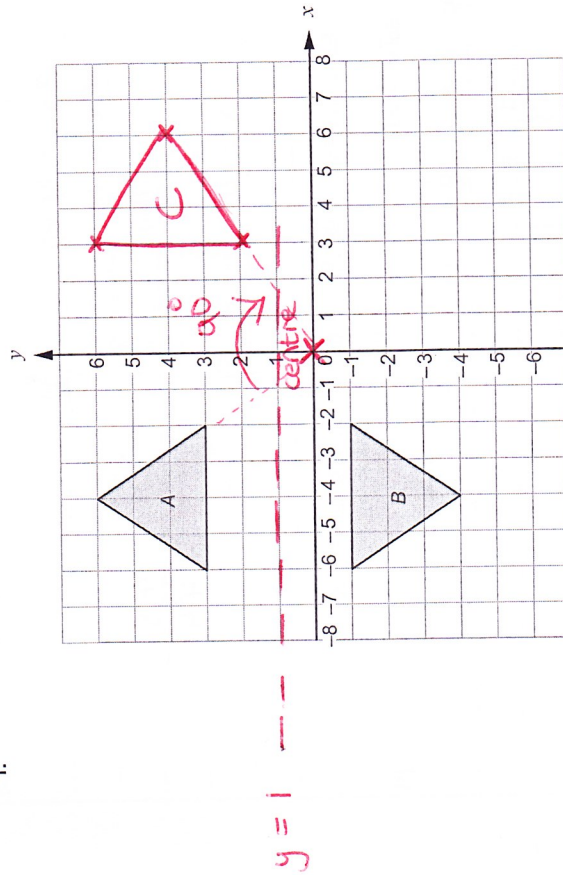
The Quadratic Equation

$$\text{The solutions of } ax^2 + bx + c = 0$$

where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

1.



The diagram shows the transformation of triangle A to triangle B.

(a) Fully describe a possible transformation of triangle A to triangle B.

[2]

Reflection in the line $y=1$

(b) Rotate triangle A through 90° clockwise about the origin.
Label your answer C.

[2]

Examiner only

2. (a) Solve $\frac{3x}{4} = 36$.

[2]

$$3x = 36 \times 4$$

$$3x = 144$$

$$x = \frac{144}{3}$$

$$x = 48$$

$$\begin{array}{r} 48 \\ 3 \overline{) 144} \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

- (b) Solve $\frac{9}{x} = 18$.

[1]

$$9 = 18x$$

$$\frac{9}{18} = x$$

$$\frac{1}{2} = x$$

- (c) Solve $5x - 12 = 3(x + 6)$.

[3]

$$5x - 12 = 3x + 18$$

$$5x - 3x = 18 + 12$$

$$2x = 30$$

$$x = \frac{30}{2}$$

$$x = 15$$

Examiner only

- (d) Solve the inequality $9x + 5 < 77$.

[2]

$$9x < 77 - 5$$

$$9x < 72$$

$$x < \frac{72}{9}$$

$$x < 8$$

- (e) Write down the smallest whole number that satisfies the inequality $4x > 45$.

[2]

$$x > \frac{45}{4}$$

$$x > 11.25 \quad \text{or} \quad x > 11\frac{1}{4}$$

Smallest whole number is 12

- (f) Simplify $10x \times 5x \times 2x$.

[1]

$$10 \times 5 \times 2 \times x \times x \times x$$

$$= 100x^3$$

3. (a) The product of three consecutive prime numbers is 385.
Find the **sum** of these three prime numbers. [3]

$$3 \times 5 \times 7 = 105 \quad \text{Too small}$$

NOTICE 9 is not a prime number

$$5 \times 7 \times 11 = 35 \times 11 = 385$$

$$\therefore \text{Sum} = 5 + 7 + 11$$

$$= 23$$

Sum of these three prime numbers = **23**

- (b) Express the highest common factor of 24 and 40 as a product of prime numbers in index form. [3]

HCF = highest number that divides exactly into 24 and 40

$$\text{HCF} = 8 \quad \frac{24}{8} = 3$$

$$\frac{40}{8} = 5$$

$$\therefore 8 = 2 \times 2 \times 2 = 2^3$$

4. Sophie is going on holiday.

- (a) Sophie travels a distance of 35 miles in 2 hours 30 minutes through busy traffic.

Calculate Sophie's average speed, in miles per hour.

$$\text{Time} = 2.5 \text{ hrs}$$

$$S = \frac{D}{T}$$

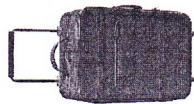
$$S = \frac{35}{2.5}$$

$$S = 14$$

$$S = \frac{70}{5} = 14 \text{ mph}$$

$$S = 14 \text{ mph}$$

- (b) Sophie's luggage weighs 22lb.



Approximately how much does her luggage weigh in kg?

$$1 \text{ kg} \approx 2.2 \text{ lb} \leftarrow \text{Learn this}$$

$$10 \text{ kg} \approx 22 \text{ lb}$$

- (c) You will be assessed on the quality of your written communication in this part of the question.

Sophie is due to fly from Glasgow to San Francisco. She arrives at Glasgow airport on Wednesday at 13:40. She has to wait 4 hours 25 minutes for her flight.

It takes 13 hours to fly between Glasgow and San Francisco. She knows that the time in Glasgow is 8 hours ahead of the time in San Francisco. For example, when it is 10:00 a.m. in Glasgow it is 2:00 a.m. in San Francisco.

On which day and at what time does Sophie expect to arrive in San Francisco airport? [5]

Arrival at Glasgow 13:40

wait 4:25

ADD 17:65

= 18:05

← only 60 minutes in 1 hour

Flight Time 13:00

Time difference -8:00

5:00 hrs

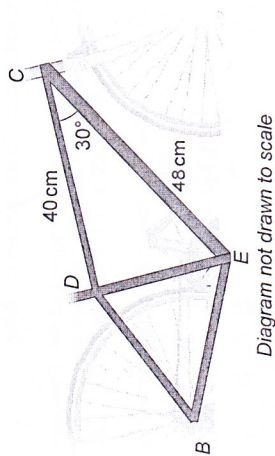
Arrive San Francisco 18:05

+ 5:00

23:05

Day Wednesday Time 23:05

5. (a) Abhu has an old bike that has been damaged. The wheels are missing but he still has the frame. He has made a sketch of the damaged frame as shown below.

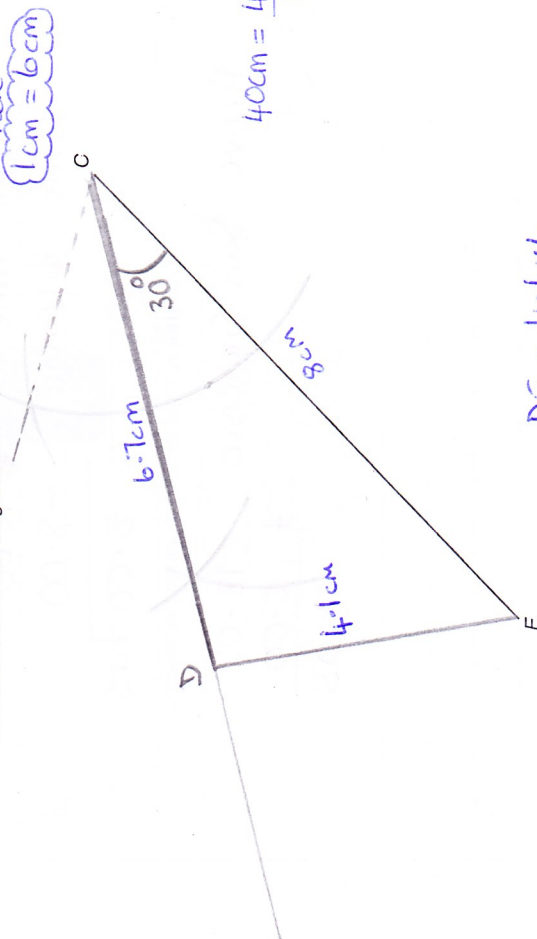


Abhu needs to know the actual length DE so that he can buy a new part.

Using a pair of compasses and a ruler only, construct a scale drawing of triangle DCE .
 Use a scale of 1 cm to represent 4 cm. *change scale for the A5 paper used*
 You must show all your construction lines.

Hence find the actual length of DE .

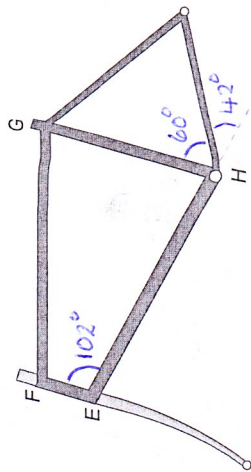
[5]



$$DE = 4.1 \times 6 = 24.6 \text{ cm}$$

Actual length of $DE = 24.6 \text{ cm}$

- (b) Abhu's friend Val has a bike frame for sale. The diagram below is a scale drawing of Val's bike frame.

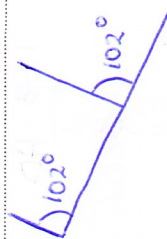


Abhu believes that FE is parallel to GH .
 Use your protractor to show that Abhu is correct.
 You must give reasons for your answer.

[2]

Corresponding angles are equal if you have parallel lines

In the diagram we have



∴ FE is parallel to GH

6. The diagram shows a rectangle ABCD.

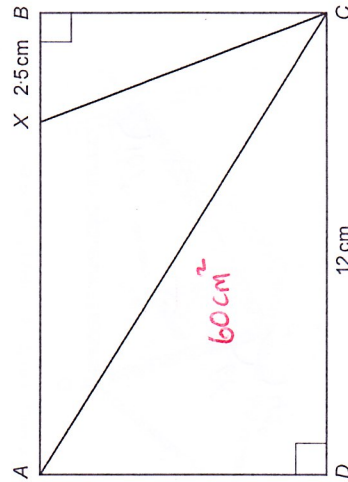


Diagram not drawn to scale

Given that $XB = 2.5\text{ cm}$, $DC = 12\text{ cm}$ and the area of triangle ADC is 60 cm^2 , calculate the area of triangle XBC.

$\triangle ABC$

$$\text{Area} = 60\text{ cm}^2$$

$$A = \frac{bh}{2}$$

$$60 = \frac{12 \times AD}{2}$$

$$120 = 12 \times BC$$

$$\times 2 \quad 240 = 5BC$$

$$\times 2 \quad 480 = 10BC$$

$$\frac{480}{10} = BC$$

$$48\text{ cm} = BC$$

7. (a) Solve the following equation.

$$\frac{20+3x}{4} + \frac{5-x}{5} = \frac{13}{4}$$

$$\text{LCM} = 4 \times 5 = 20$$

$$\frac{5(20+3x)}{20} + \frac{4(5-x)}{20} = \frac{5(13)}{20}$$

$\times 20$

$$5(20+3x) + 4(5-x) = 65$$

$$100 + 15x + 20 - 4x = 65$$

$$11x = 65 - 100 - 20$$

$$11x = -55$$

$$x = \frac{-55}{11}$$

$$x = -5$$

- (b) Factorise $x^2 - 10x + 16$ and hence solve the equation $x^2 - 10x + 16 = 0$.

$$\begin{array}{r} x \quad + \\ +16 \quad | \quad -10 \\ (-8)(-2) \end{array}$$

$$(x-8)(x-2) = 0$$

either or

$$x-8=0$$

$$x=8$$

$$x-2=0$$

$$x=2$$

8. The table shows some of the values of $y = 3x^2 + 2$ for values of x from -2 to 2 .

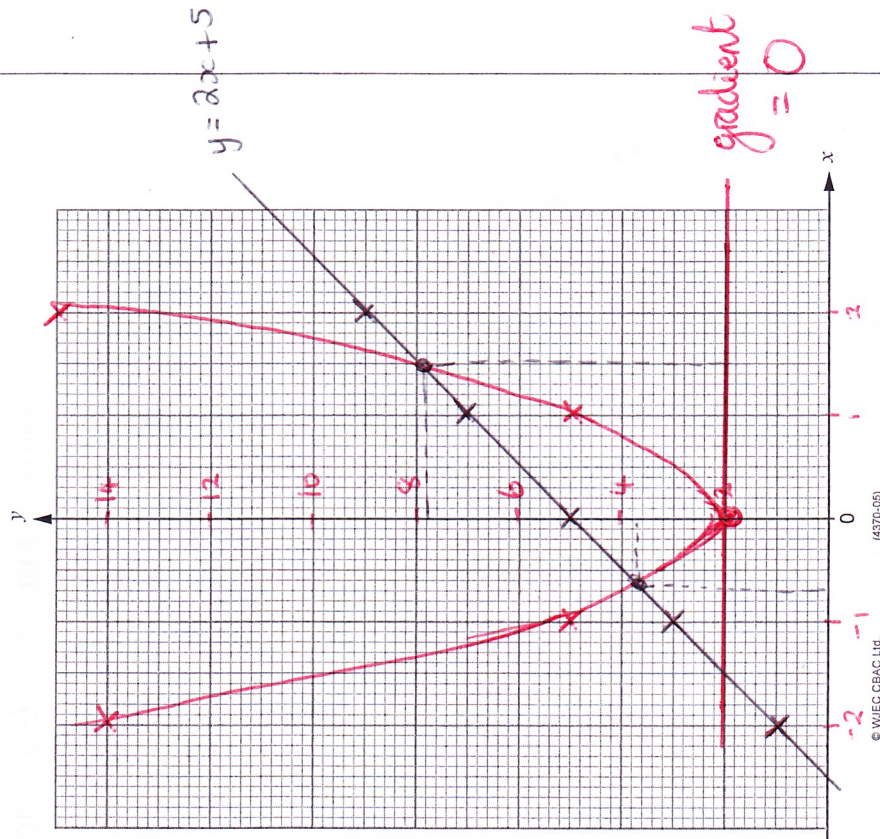
- (a) Complete the table by finding the values of y when $x = -2$ and $x = 2$.

x	-2	-1	0	1	2
y	14	5	2	5	14

$$x = -2 \quad y = 3(-2)^2 + 2 = 3(4) + 2 = 12 + 2 = 14$$

$$x = 2 \quad y = 3(2)^2 + 2 = 3(4) + 2 = 12 + 2 = 14$$

- (b) Using the graph paper below, draw the graph of $y = 3x^2 + 2$ for values of x between -2 and 2 .



- (c) Write down the coordinates of the point on $y = 3x^2 + 2$ where the gradient of the curve is zero.

(0, 2)

- (d) When the line $y = 2x + 5$ is drawn between $x = -2$ and $x = 2$, it intersects the curve $y = 3x^2 + 2$ at two points.

Use your graph to find the coordinates of the points of intersection.

$$y = 2x + 5 \quad x = -2 \quad y = 1 \quad x = 2 \quad y = 9$$

Points of intersection at $(-0.7, 3.8)$ and $(1.5, 7.8)$

9. (a) A fair dice is thrown twice.



The score is noted each time the dice is thrown.
Calculate the probability of getting at least one six.

[3]

$$P(\text{no six}) = P(\text{not 6 and not 6})$$

$$= \frac{5 \times 5}{6 \times 6} = \frac{25}{36} \quad \leftarrow \text{must + to give 1}$$

$$\therefore P(\text{at least one 6}) = 1 - \frac{25}{36} = \frac{11}{36}$$

- (b) Carwyn has bought a 10-sided dice.



Carwyn was not sure if the dice was a fair dice.
He carried out an experiment.

He recorded the number of sixes he threw in every 10 throws of the dice.

Number of throws	10	10	10	10	10	10	10	10	10
Number of sixes	4	3	2	3	2	4	2	4	3

He decided to create a table to show the cumulative number of sixes thrown, to calculate the relative frequencies.

Number of throws	10	20	30	40	50	60	70	80	90	100
Number of sixes thrown	4	7	9	12	14	18	20	24	27	29
Relative frequency of throwing a six	$\frac{4}{10}$	$\frac{7}{20}$	$\frac{9}{30}$	$\frac{12}{40}$	$\frac{14}{50}$	$\frac{18}{60}$	$\frac{20}{70}$	$\frac{24}{80}$	$\frac{27}{90}$	$\frac{29}{100}$
	0.4	0.35	0.3	0.3	0.28	0.3	0.29	0.3	0.3	0.29

$$\frac{29}{100} \div \frac{3}{10} = \frac{29}{30}$$

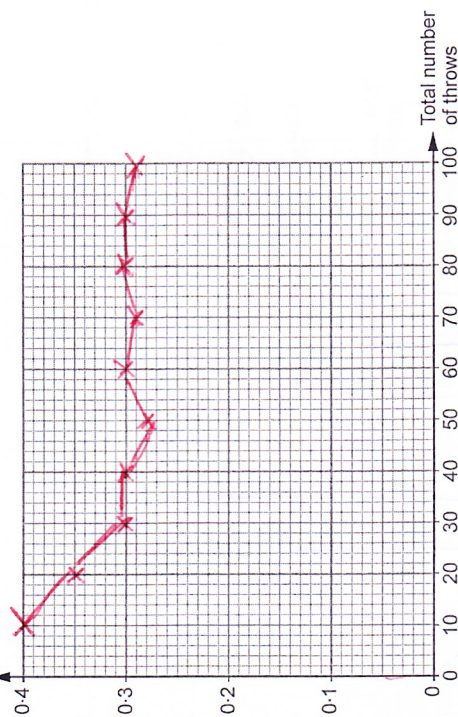
- (i) Complete Carwyn's table opposite.

[3]

- (ii) Draw a graph to illustrate the relative frequency of throwing a six on Carwyn's 10-sided dice.

[2]

Relative frequency of throwing a six



- (iii) Using the above results, write down the best estimate for the probability of not obtaining a six on Carwyn's dice, giving a reason for your answer.

[2]

$$1 - 0.3$$

$$= 0.7$$

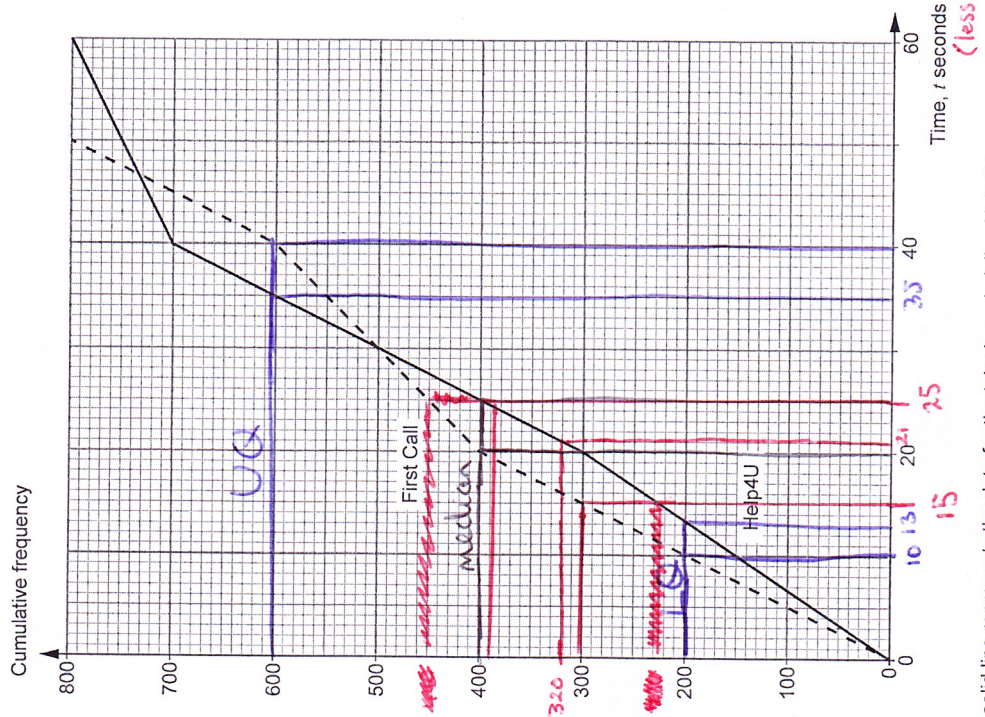
- (iv) Carwyn says that the dice is not a fair dice.
Explain why Carwyn's statement could be true.

[1]

You would expect a 10 sided die to have $P(6) = 0.1$.

10. Help4U and First Call are two different telephone help lines. Information was recorded for the first 800 callers of the day for each helpline.

The cumulative frequency diagram illustrates the time it took for these callers to get through to each of these helplines.



The solid line represents the data for the telephone helpline Help4U.
The dotted line represents the data for the telephone helpline First Call.

- (a) How many of the 800 callers to First Call took less than 15 seconds to get through? [1]

300

- (b) Help4U were set a target to answer 40% of all calls in less than 25 seconds. They actually met the target with these 800 calls. By how many seconds did they beat their target?

25 secs $\Rightarrow$ 390 calls less than 25 secs [3]

~~% answered in time = $\frac{390}{800} \times 100$~~ ~~HARD !!~~

~~40% of 800 = $80 \times 4 = 320$~~

$\therefore$ 320 calls = 21 secs

$\therefore$ Beat target by 4 secs.

- (c) Which of the two companies had the lower median time and by how many seconds? [2]

First call 400th = median = 20 secs

Help4U 400th = median = 25 secs

First Call lower median by 5 secs

- (d) Calculate the interquartile range of the times for each company. [3]

First Call

Lower Quartile = 200th = 10 secs

Upper Quartile = 600th = 40 secs

IQR = 40 - 10 = 30 secs

Help4U

LQ = 13 secs

UQ = 35 secs

IQR = 35 - 13 = 22 secs

- (e) One of these companies was given an award for the speed of answering the telephone. Which company do you think won the award? You **must** give a reason for your answer. [1]

First Call Lower median

by 5 secs.

11. The diagram shows a circle with centre O. The three points A, B and C lie on the circumference of the circle. The tangent XAY touches the circle at A.

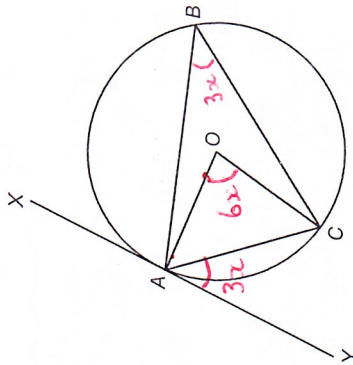


Diagram not drawn to scale

Given that $\angle AOC = 6x^\circ$, find an expression for $\angle ABC$ in terms of x . You must give a reason for each stage of your working. Express your answer in its simplest form.

[3]

$\angle ABC = 3x$ because angle at circumference is half the angle to the centre

$\angle YAC = 3x$ because of alternate segment theorem ($\angle YAC = \angle ABC$)

12. Write each of the following in its simplest form.

[1]

$$\begin{aligned} (a) \sqrt{50} &= \sqrt{25 \times 2} \\ &= \sqrt{25} \sqrt{2} = 5\sqrt{2} \end{aligned}$$

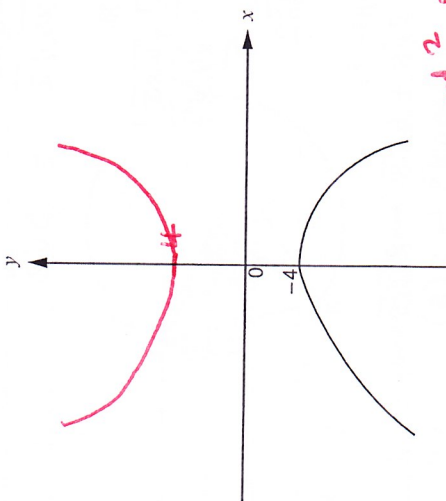
[3]

$$\begin{aligned} (b) \sqrt{40} \times \sqrt{20} &= \sqrt{8 \times 5} \sqrt{4 \times 5} \\ &= \sqrt{8} \sqrt{5} \sqrt{4} \sqrt{5} \\ &= \sqrt{32} \times 5 \\ &= 5\sqrt{32} \end{aligned}$$

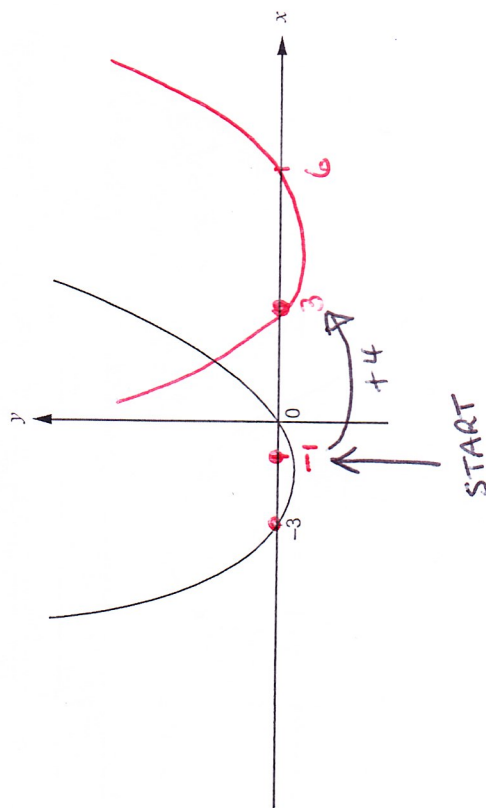
[3]

$$\begin{aligned} (c) (3 + 2\sqrt{5})(2 - \sqrt{5}) &= 6 - 3\sqrt{5} + 4\sqrt{5} - 2 \times 5 \\ &= 6 - 3\sqrt{5} + 4\sqrt{5} - 10 \\ &= \sqrt{5} - 4 \end{aligned}$$

13. (a) The diagram shows a sketch of $y = f(x)$.
On the same diagram, sketch the curve $y = -f(x)$.
Mark clearly the coordinates of any point where this curve crosses an axis. [2]



- (b) The diagram shows a sketch of $y = g(x+2)$.
On the same diagram, sketch the curve $y = g(x-4)$.
Mark clearly the coordinates of the points where this curve crosses the x-axis. [3]



14. Express the following as a single fraction in its simplest form. [5]

$$\begin{aligned}
 LCM &= (2x-1)(3x+1) \\
 &= \frac{(3x+1)(4x+3)}{(2x-1)(3x+1)} - \frac{(2x-1)(6x-5)}{(2x-1)(3x+1)} \\
 &= \frac{(3x+1)(4x+3) - (2x-1)(6x-5)}{(2x-1)(3x+1)} \\
 &= \frac{12x^2 + 9x + 4x + 3 - [12x^2 - 10x - 6x + 5]}{(2x-1)(3x+1)} \\
 &= \frac{12x^2 + 13x + 3 - [12x^2 - 16x + 5]}{(2x-1)(3x+1)} \\
 &= \frac{12x^2 + 13x + 3 - 12x^2 + 16x - 5}{(2x-1)(3x+1)} \\
 &= \frac{29x - 2}{(2x-1)(3x+1)}
 \end{aligned}$$

15.

A	$\sin 46^\circ$
---	-----------------

E	$\sin 44^\circ$
---	-----------------

B	$\sin 146^\circ$
---	------------------

F	$\sin 316^\circ$
---	------------------

C	$\sin 134^\circ$
---	------------------

G	$\sin 224^\circ$
---	------------------

D	$\sin 314^\circ$
---	------------------

H	$\sin 136^\circ$
---	------------------

Sallie and Bethan are playing a game by matching cards of equal value.

Sallie matches card A with card D.

Bethan matches card E with card H.

Who is correct?

You must sketch a graph to explain your answer.

[3]

From sketch

Bethan is correct as both

$\sin 44^\circ$ and $\sin 136^\circ$ give the same
y axis reading

For sallie one y values is +ve and one
is -ve so they are different values

Space for sketch:

