Laws of Indices : 1 : Solutions

(1) a) $9^{1/2} = \sqrt{9} = 3$

b) $8^{1/3} = \sqrt[3]{8} = 2$

c) $100^{-1/2} = \frac{1}{100^{1/2}} = \frac{1}{\sqrt{100}} = \frac{1}{10}$

d) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

e) $8^{2/3} = (8^{1/3})^2 = (\sqrt[3]{8})^2 = (2)^2 = 4$

f) $9^{3/2} = (9^{1/2})^3 = (\sqrt{9})^3 = (3)^3 = 27$

g) $27^{-2/3} = \frac{1}{27^{2/3}} = \frac{1}{(27^{1/3})^2} = \frac{1}{(\sqrt[3]{27})^2} = \frac{1}{(3)^2} = \frac{1}{9}$

h) $16^{-3/4} = \frac{1}{16^{3/4}} = \frac{1}{(16^{1/4})^3} = \frac{1}{(\sqrt[4]{16})^3} = \frac{1}{(2)^3} = \frac{1}{8}$

(2) a) $\frac{x^4 \times x^3}{x^9} = \frac{x^7}{x^9} = x^{-2}$

b) $\frac{x^3 \times x^{-7}}{x^{-5}} = \frac{x^{-4}}{x^{-5}} = x^1 = x$

c) $\frac{(x^{-2})^4 \times x^3}{(x^{-3})^{-4}} = \frac{x^{-8} \times x^3}{x^{12}} = \frac{x^{-5}}{x^{12}} = x^{-17}$

d) $(2x^3)^3 \times (3x^2)^2 = 8x^9 \times 9x^4 = 72x^{13}$

e) $x^{1/2} \times x^{1/5} = x^{7/10}$

$\frac{1}{2} + \frac{1}{5} = \frac{5}{10} + \frac{2}{10} = \frac{7}{10}$

f) $\frac{x^{2/3} \times x^{-3/4}}{x^{1/2}} = x^{-1/12} = x^{-7/12}$

$\frac{2}{3} + \left(\frac{-3}{4}\right) = \frac{2}{3} - \frac{3}{4} = \frac{8}{12} - \frac{9}{12} = \frac{-1}{12}$

$-\frac{1}{12} - \frac{1}{2} = -\frac{1}{12} - \frac{6}{12} = -\frac{7}{12}$

(3) a) $\sqrt{x} \times \frac{1}{x}$

$= x^{1/2} \times x^{-1}$

$= x^{-1/2}$

$= \frac{1}{\sqrt{x}}$

b) $\sqrt[3]{x} \times \sqrt{x}$

$= x^{1/3} \times x^{1/2}$

$= x^{5/6}$

$= \sqrt[6]{x^5}$

$\frac{1}{3} + \frac{1}{2}$

$= \frac{2}{6} + \frac{3}{6}$

$= \frac{5}{6}$

c) $\sqrt[3]{x^2} \times \frac{1}{x^2}$

$= x^{2/3} \times x^{-2}$

$= x^{-4/3}$

$= \frac{1}{x^{4/3}}$

$= \frac{1}{\sqrt[3]{x^4}}$

$\frac{2}{3} + (-2)$

$= \frac{2}{3} - \frac{6}{3}$

$= -\frac{4}{3}$

d) $\sqrt{x^3} \times \frac{1}{\sqrt{x}}$

$= x^{3/2} \times x^{-1/2}$

$= x^1$

$= x$

$\frac{3}{2} + (-\frac{1}{2})$

$= \frac{3}{2} - \frac{1}{2}$

$= \frac{2}{2}$

$= 1$

(4) a) $\sqrt{x}\left(x - \frac{1}{x}\right)$

$= x^{1/2}(x^1 - x^{-1})$

$= x^{3/2} - x^{-1/2}$

$= \sqrt{x^3} - \frac{1}{\sqrt{x}}$

b) $\sqrt[3]{x}\left(\sqrt{x} + \sqrt[3]{x^2}\right)$

$= x^{1/3}\left(x^{1/2} + x^{2/3}\right)$

$= x^{5/6} + x^{3/3}$

$= \sqrt[6]{x^5} + x$

(5) a) $2^{8x} = 8^{x+1}$

$2^{8x} = (2^3)^{x+1}$

$2^{8x} = 2^{3x+3}$

$\therefore \quad 8x = 3x + 3$

$5x = 3$

$x = \frac{3}{5}$

b) $3^{x-2} = 9^{4x}$

$3^{x-2} = (3^2)^{4x}$

$3^{x-2} = 3^{8x}$

$\therefore \quad x - 2 = 8x$

$-2 = 7x$

$-\frac{2}{7} = x$

c) $8^{x+3} = 16^{x-2}$

$(2^3)^{x+3} = (2^4)^{x-2}$

$2^{3x+9} = 2^{4x-8}$

$\therefore \quad 3x + 9 = 4x - 8$

$9 + 8 = 4x - 3x$

$17 = x$

d) $27^{2x+1} = 9^{x-2}$

$(3^3)^{2x+1} = (3^2)^{x-2}$

$3^{6x+3} = 3^{2x-4}$

$\therefore \quad 6x + 3 = 2x - 4$

$4x = -7$

$x = -\frac{7}{4}$

e) $\left(\frac{1}{100}\right)^{2x} = 10^{2x+1}$

$(10^{-2})^{2x} = 10^{2x+1}$

$10^{-4x} = 10^{2x+1}$

$\therefore \quad -4x = 2x + 1$

$-1 = 6x$

$-\frac{1}{6} = x$

f) $\left(\frac{1}{16}\right)^{x} = \left(\frac{1}{8}\right)^{3x-1}$

$(2^{-4})^{x} = (2^{-3})^{3x-1}$

$2^{-4x} = 2^{-9x+3}$

$\therefore \quad -4x = -9x + 3$

$5x = 3$

$x = \frac{3}{5}$