Parameters with 2nd Derivatives : SOLUTIONS

1)

$$x = t^{-1} \qquad y = 3t^2 + 2$$

$$\frac{dx}{dt} = -t^{-2} = -\frac{1}{t^2} \qquad \frac{dy}{dt} = 6t$$

$$\frac{dt}{dx} = -t^2$$

$$\frac{dy}{dx} = 6t \times (-t^2) = -6t^3$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[-6t^3\right]$$

$$= -18t^2 \times \frac{dt}{dx}$$

$$= -18t^2 \times (-t^2)$$

$$= 18t^4$$

2)

$$x = t + 3 \qquad y = t^3 - 6t$$

$$\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = 3t^2 - 6$$

$$\frac{dy}{dx} = (3t^2 - 6) \times 1$$

$$= 3t^2 - 6$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[3t^2 - 6\right]$$

$$= 6t \times \frac{dt}{dx}$$

$$= 6t \times 1$$

$$= 6t$$

3) $x = 3 - 2t^2$ $\qquad$ $y = t^{-1}$

$$\frac{dx}{dt} = -4t \qquad \frac{dy}{dt} = \frac{-1}{t^2}$$

$$\frac{dt}{dx} = -\frac{1}{4t}$$

$$\frac{dy}{dx} = -\frac{1}{t^2} \times \left(\frac{-1}{4t}\right)$$

$$= \frac{1}{4t^3}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{1}{4}t^{-3}\right]$$

$$= \frac{-3}{4}t^{-4} \times \frac{dt}{dx}$$

$$= \frac{-3}{4t^4} \times \left(\frac{-1}{4t}\right)$$

$$= \frac{3}{16t^5}$$

4) $x = t^2 + 2t$ $\qquad$ $y = t^3 - 3t$

$$\frac{dx}{dt} = 2t + 2 \qquad \frac{dy}{dt} = 3t^2 - 3$$

$$\frac{dt}{dx} = \frac{1}{2t+2}$$

$$\frac{dy}{dx} = \frac{(3t^2-3)}{(2t+2)}$$

$$= \frac{3(t^2-1)}{2(t+1)}$$

$$= \frac{3(t-1)(t+1)}{2(t+1)}$$

$$= \frac{3(t-1)}{2}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{3(t-1)}{2}\right]$$

$$= \frac{d}{dx}\left[\frac{3t}{2} - \frac{3}{2}\right]$$

$$= \frac{3}{2} \times \frac{dt}{dx}$$

$$= \frac{3}{2} \times \frac{1}{2t+2} = \frac{3}{2(2t+2)}$$

$$= \frac{3}{4(t+1)}$$

5. $x = at^3 \qquad y = at^4$

$$\frac{dx}{dt} = 3at^2 \qquad \frac{dy}{dt} = 4at^3$$

$$\frac{dt}{dx} = \frac{1}{3at^2}$$

$$\frac{dy}{dx} = 4at^3 \times \frac{1}{3at^2}$$

$$= \frac{4t}{3}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{4t}{3}\right]$$

$$= \frac{4}{3} \times \frac{dt}{dx}$$

$$= \frac{4}{3} \times \frac{1}{3at^2}$$

$$= \frac{4}{9at^2}$$

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$$x = 2t \qquad y = e^t$$

$$\frac{dx}{dt} = 2 \qquad \frac{dy}{dt} = e^t$$

$$\frac{dt}{dx} = \frac{1}{2}$$

$$\frac{dy}{dx} = e^t \times \frac{1}{2}$$

$$= \frac{e^t}{2}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{e^t}{2}\right]$$

$$= \frac{1}{2}e^t \times \frac{dt}{dx}$$

$$= \frac{1}{2}e^t \times \frac{1}{2}$$

$$= \frac{e^t}{4}$$

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$$x = \ln t \qquad y = t^2$$

$$\frac{dx}{dt} = \frac{1}{t} \qquad \frac{dy}{dt} = 2t$$

$$\frac{dt}{dx} = t$$

$$\frac{dy}{dx} = 2t \times t$$

$$= 2t^2$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[2t^2\right]$$

$$= 4t \times \frac{dt}{dx}$$

$$= 4t \times t$$

$$= 4t^2$$

(8)

$x = \sin t$ $\qquad$ $y = \cos t$

$\frac{dx}{dt} = \cos t$ $\qquad$ $\frac{dy}{dt} = -\sin t$

$\frac{dt}{dx} = \frac{1}{\cos t}$

$$\frac{dy}{dx} = -\sin t \times \frac{1}{\cos t}$$

$$= -\tan t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[-\tan t\right]$$

$$= -\sec^2 t \times \frac{dt}{dx}$$

$$= -\sec^2 t \times \frac{1}{\cos t}$$

$$= -\frac{1}{\cos^2 t} \times \frac{1}{\cos t}$$

$$= -\frac{1}{\cos^3 t}$$

$$= -\sec^3 t$$

(9)

$x = \tan t$ $\qquad$ $y = \sin t$

$\frac{dx}{dt} = \sec^2 t$ $\qquad$ $\frac{dy}{dt} = \cos t$

$\frac{dt}{dx} = \frac{1}{\sec^2 t}$

$= \cos^2 t$

$$\frac{dy}{dx} = \cos t \times \cos^2 t$$

$$= \cos^3 t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\cos^3 t\right]$$

$$= 3\cos^2 t \times (-\sin t) \times \frac{dt}{dx}$$

$$= 3\cos^2 t \times (-\sin t) \times \cos^2 t$$

$$= -3\sin t\cos^4 t$$

(10) $x = \ln t$ $\qquad y = 3t^4 - 2t$

$$\frac{dx}{dt} = \frac{1}{t} \qquad \frac{dy}{dt} = 12t^3 - 2$$

$$= 12t^3 - 2$$

$$\frac{dt}{dx} = t$$

$$\frac{dy}{dx} = (12t^3 - 2) \times t$$

$$= 2t(6t^3 - 1) \quad \text{or} \quad 12t^4 - 2t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[12t^4 - 2t\right]$$

$$= 48t^3 \times \frac{dt}{dx} - 2 \times \frac{dt}{dx}$$

$$= 48t^3 \times t - 2t$$

$$= 48t^4 - 2t$$

$$= 2t(24t^3 - 1)$$

(11) $x = t - 2$ $\qquad y = t^3 + 3$

$$\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = 3t^2$$

$$\frac{dt}{dx} = 1$$

$$\frac{dy}{dx} = 3t^2 \times 1 = 3t^2$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[3t^2\right]$$

$$= 6t \times \frac{dt}{dx}$$

$$= 6t$$

when $t = 1$

$$\frac{d^2y}{dx^2} = 6(1)$$

$$= 6$$

(12) $x = t + 1$ $\qquad$ $y = 2t^2 - 5$

$\frac{dx}{dt} = 1$ $\qquad$ $\frac{dy}{dt} = 4t$

$\frac{dt}{dx} = 1$

$$\frac{dy}{dx} = 4t \times 1$$
$$= 4t$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}[4t]$$
$$= 4 \times \frac{dt}{dx}$$
$$= 4$$

when $t = 3$

$$\frac{d^2y}{dx^2} = 4$$

(13) $x = a\sin\theta$ $\qquad$ $y = 3a\cos\theta$

$\frac{dx}{d\theta} = a\cos\theta$ $\qquad$ $\frac{dy}{d\theta} = 3a\sin\theta$

$\frac{d\theta}{dx} = \frac{1}{a\cos\theta}$

$$\frac{dy}{dx} = 3a\sin\theta \times \frac{1}{a\cos\theta}$$
$$= 3\tan\theta$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[3\tan\theta\right]$$
$$= 3\sec^2\theta \times \frac{d\theta}{dx}$$
$$= 3\sec^2\theta \times \frac{1}{a\cos\theta}$$
$$= \frac{3}{a\cos^3\theta}$$

when $\theta = \pi/3$

$$\frac{d^2y}{dx^2} = \frac{3}{a\left(\cos^3 \pi/3\right)}$$

$$= \frac{3}{a\left(1/2\right)^3}$$

$$= \frac{3}{1/8\, a}$$

$$= \frac{3}{a} \times 8$$

$$= \frac{24}{a}$$