

# Parameters with 2nd Derivatives : 2 : Answers

Q/ Solution

$$x = \ln t \quad y = 4t^4 - 3t^2$$

a)  $\frac{dx}{dt} = \frac{1}{t} \quad \frac{dy}{dt} = 16t^3 - 6t$

$$\frac{dt}{dx} = t$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{dt} \times \frac{dt}{dx} \\ &= (16t^3 - 6t) \times t \\ &= 16t^4 - 6t^2 \\ &= 2t^2(8t^2 - 3)\end{aligned}$$

b)  $\frac{d^2y}{dx^2} = \frac{d}{dx} [16t^4 - 6t^2]$

$$\begin{aligned}&= 64t^3 \times \frac{dt}{dx} - 12t \times \frac{dt}{dx} \\ &= 64t^3 \times t - 12t \times t \\ &= 64t^4 - 12t^2 \\ &= 4t^2(16t^2 - 3)\end{aligned}$$

c) Now  $\frac{d^2y}{dx^2} = 1$

$$4t^2(16t^2 - 3) = 1$$

$$64t^4 - 12t^2 - 1 = 0$$

$$(16t^2 + 1)(4t^2 - 1) = 0$$

either  $16t^2 + 1 = 0$  or  $4t^2 - 1 = 0$   
 $t^2 = \frac{1}{4}$

NO SOLNS

$t = \pm \frac{1}{2}$  but  $t > 0$   
 $\therefore t = \frac{1}{2}$ .

$$Q/ \quad x = e^t \quad y = \sin t$$

$$a) \quad \frac{dx}{dt} = e^t \quad \frac{dy}{dt} = \text{Cost}$$

$$\frac{dt}{dx} = \frac{1}{e^t}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \text{Cost} \times \frac{1}{e^t}$$

$$= \frac{\text{Cost}}{e^t}$$

$$b) \quad \frac{d^2y}{dx^2} = \frac{d}{dx} \left[ e^{-t} \text{Cost} \right]$$

\* probably easier  
to use  
product rule

$$u = e^{-t} \quad v = \text{Cost}$$

$$\frac{du}{dx} = -e^{-t} \times \frac{dt}{dx} \quad \frac{dv}{dx} = -\sin t \left( \frac{dt}{dx} \right)$$

$$= -e^{-t} \times e^{-t} \quad = -e^{-t} \sin t$$

$$= -e^{-2t}$$

$$\frac{d^2y}{dx^2} = -e^{-2t} \sin t + \text{Cost}(-e^{-2t})$$

$$= -e^{-2t}(\sin t + \text{Cost})$$

$$c) \quad x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y$$

$$= -x^2 e^{-2t} (\sin t + \text{Cost}) + x e^{-t} \text{Cost} + \sin t$$

$$= -x^2 e^{-2t} \sin t - x^2 e^{-2t} \text{Cost} + x e^{-t} \text{Cost} + \sin t$$

$$x = e^t$$

$$= -\frac{e^{2t}}{e^{2t}} \sin t - \frac{e^{2t}}{e^{2t}} \text{Cost} + \frac{e^t}{e^t} \text{Cost} + \sin t$$

$$= -\sin t - \text{Cost} + \text{Cost} + \sin t$$

$$= 0$$