Q/ Solution

$$x = \ln t \qquad y = 4t^4 - 3t^2$$

a) $$\frac{dx}{dt} = \frac{1}{t} \qquad \frac{dy}{dt} = 16t^3 - 6t$$

$$\frac{dt}{dx} = t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$
$$= (16t^3 - 6t) \times t$$
$$= 16t^4 - 6t^2$$
$$= 2t^2(8t^2 - 3)$$

b) $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[16t^4 - 6t^2\right]$$
$$= 64t^3 \times \frac{dt}{dx} - 12t \times \frac{dt}{dx}$$
$$= 64t^3 \times t - 12t \times t$$
$$= 64t^4 - 12t^2$$
$$= 4t^2(16t^2 - 3)$$

c) Now $$\frac{d^2y}{dx^2} = 1$$
$$4t^2(16t^2 - 3) = 1$$
$$64t^4 - 12t^2 - 1 = 0$$
$$(16t^2 + 1)(4t^2 - 1) = 0$$

either $16t^2 + 1 = 0$ or $4t^2 - 1 = 0$

NO SOLNS

$t^2 = 1/4$

$t = \pm \frac{1}{2}$ but $t > 0$

$\therefore \; t = \frac{1}{2}$.

Q/ $x = e^t$ $\qquad y = \sin t$

a) $\dfrac{dx}{dt} = e^t$ $\qquad \dfrac{dy}{dt} = \cos t$

$\dfrac{dt}{dx} = \dfrac{1}{e^t}$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \cos t \times \frac{1}{e^t}$$

$$= \frac{\cos t}{e^t}$$

b) $\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left[e^{-t}\cos t\right]$

* probably easier to use product rule

$u = e^{-t}$ $\qquad V = \cos t$

$\dfrac{du}{dx} = -e^{-t} \times \dfrac{dt}{dx}$ $\qquad \dfrac{dV}{dx} = -\sin t\left(\dfrac{dt}{dx}\right)$

$= -e^{-t} \times e^{-t}$ $\qquad = -e^{-t}\sin t$

$= -e^{-2t}$

$$\frac{d^2y}{dx^2} = -e^{-2t}\sin t + \cos t\left(-e^{-2t}\right)$$

$$= -e^{-2t}(\sin t + \cos t)$$

c) $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y$

$= -x^2e^{-2t}(\sin t + \cos t) + xe^{-t}\cos t + \sin t$

$= -x^2e^{-2t}\sin t - x^2e^{-2t}\cos t + xe^{-t}\cos t + \sin t$

$x = e^t$

$$= -\frac{e^{2t}}{e^{2t}}\sin t - \frac{e^{2t}}{e^{2t}}\cos t + \frac{e^t}{e^t}\cos t + \sin t$$

$$= -\sin t - \cos t + \cos t + \sin t$$

$$= 0$$