Rates of Change : 1 : Answers

1) a) $\frac{dr}{dt} = 2$ cm/s

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$= 2\pi r \times 2$

$= 4\pi r$ cm^2/s

$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

b) $r = 10$ cm

$\frac{dA}{dt} = 4\pi \times 10$

$= 40\pi$ cm^2/s

2) $\frac{dr}{dt} = 2$cm/s

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$= 4\pi r^2 \times 2$

$= 8\pi r^2$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

a) $r = 3$cm

$\frac{dV}{dt} = 8\pi(3^2)$

$= 72\pi$ cm^3/s

b) $r = 4$cm

$\frac{dV}{dt} = 8\pi(4^2)$

$= 128\pi$ cm^3/s

3) $\frac{dV}{dt} = 54$ cm^3/s

$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$

$= \frac{1}{4\pi r^2} \times 54$

$= \frac{27}{2\pi r^2}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{dr}{dV} = \frac{1}{4\pi r^2}$

when $V = 36\pi$

$36\pi = \frac{4}{3}\pi r^3$

$27 = r^3$

$3 = r$

$r = 3$ $\frac{dr}{dt} = \frac{27}{2\pi(3^2)} = \frac{3}{2\pi}$ cm/s

4) $\frac{dV}{dt} = 6\,cm^3/s$

$\frac{dA}{dt} = \frac{dA}{dV} \times \frac{dV}{dt}$

$= \frac{2}{r} \times 6$

$= \frac{12}{r}$

$r = 4cm$

$\frac{dA}{dt} = \frac{12}{4}$

$= 3\,cm^2/sec$

Sphere

$A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$

$\frac{dr}{dA} = \frac{1}{8\pi r}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{dr}{dV} = \frac{1}{4\pi r^2}$

$\therefore \frac{dA}{dV} = \frac{dA}{dr} \times \frac{dr}{dV}$

$= 8\pi r \times \frac{1}{4\pi r^2}$

$= \frac{8\pi r}{4\pi r^2}$

$= \frac{2}{r}$

5) $A = (2t+3)^5$

$\frac{dA}{dt} = 5(2t+3)^4 \times 2$

$\frac{dA}{dx} = \frac{dA}{dt} \times \frac{dt}{dx}$

$x = 2t^2 + 6t$

$\frac{dx}{dt} = 4t+6$

$\frac{dt}{dx} = \frac{1}{4t+6}$

$= 10(2t+3)^4 \times \frac{1}{4t+6}$

$= 10(2t+3)^4 \times \frac{1}{2(2t+3)}$

$= 5(2t+3)^3$

6) $T = 5p^2 + \frac{3}{p}$

$\frac{dT}{dp} = 10p - \frac{3}{p^2}$

$\frac{dT}{dq} = \frac{dT}{dp} \times \frac{dp}{dq}$

$= \left(10p - \frac{3}{p^2}\right) \times 4$ when $p = 2$

$= 4\left(20 - \frac{3}{4}\right)$

$= 4\left(77/4\right)$

$= 77$

7) a) $\frac{dr}{dt} = 1 \text{ mm/s} = 0.1 \text{ cm/s}$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$= 16\pi r \times 0.1$

NOTICE must change to cm

$= 1.6\pi r \text{ cm}^2/\text{s}$

Cylinder

$A = \pi r^2 + \pi r^2 + 2\pi rh$

$A = 2\pi r^2 + 2\pi r(3r)$

$A = 2\pi r^2 + 6\pi r^2$

$A = 8\pi r^2$

$\frac{dA}{dr} = 16\pi r$

b) $\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$

$= 9\pi r^2 \times 0.1$

$= 0.9\pi r^2 \text{ cm}^3/\text{s}$

$V = \pi r^2 h$

$V = \pi r^2 (3r)$

$V = 3\pi r^3$

$\frac{dv}{dr} = 9\pi r^2$

8) $\frac{dA}{dt} = 10\ cm^2/s$

Cube

$A = 6l^2$ $\quad V = l^3$

$\frac{dA}{dl} = 12l$ $\quad \frac{dV}{dl} = 3l^2$

$\frac{dl}{dA} = \frac{1}{12l}$ $\quad \frac{dl}{dV} = \frac{1}{3l^2}$

$$\frac{dV}{dA} = \frac{dV}{dl} \times \frac{dl}{dA}$$

$$= 3l^2 \times \frac{1}{12l}$$

$$= \frac{l}{4}$$

$$\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt}$$

$$= \frac{l}{4} \times 10$$

$$= \frac{5l}{2}\ cm^3/s$$

when $l = 12cm$

$$\frac{dV}{dt} = \frac{5 \times 12}{2}$$

$$= 30\ cm^3/s$$

9) $\frac{dA}{dt} = 5\ cm^2/s$

Circle

$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$\frac{dr}{dA} = \frac{1}{2\pi r}$

$$\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt}$$

$$= \frac{1}{2\pi r} \times 5$$

$$= \frac{5}{2\pi r}$$

If area $= 30cm^2$

$30 = \pi r^2$

$\frac{30}{\pi} = r^2$

$\sqrt{\frac{30}{\pi}} = r$

3.09 [illegible] $cm = r$

$A = 30cm^2$

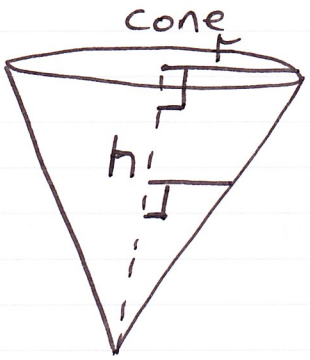
$$\frac{dr}{dt} = \frac{5}{2\pi(3.09)}$$

$$= 0.2575\ cm/s$$

10) $\frac{dV}{dt} = -5\ cm^3/s$

Now $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ (*)

First find $\frac{dh}{dV}$



Using similar triangles

$h = 4r$

$\frac{dh}{dr} = 4$

$\frac{dr}{dh} = \frac{1}{4}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2 (4r)$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{dr}{dV} = \frac{1}{4\pi r^2}$

$\frac{dh}{dV} = \frac{dh}{dr} \times \frac{dr}{dV}$

$= 4 \times \frac{1}{4\pi r^2}$

$\frac{dh}{dV} = \frac{1}{\pi r^2}$

Now return to (*)

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

$= \frac{1}{\pi r^2} \times (-5)$

$\frac{dh}{dt} = \frac{-5}{\pi r^2}$

Now find this value when V = ½ of full cone

Total volume = $\frac{1}{3}\pi \times 5^2 \times 20 = 523.6\ cm^3$

Half full = $261.8\ cm^3$

Now find r when $V = 261.8$ and $h = 4r$

$$V = \frac{1}{3}\pi r^2 h$$

$$3 \times 261.8 = \pi r^2 (4r)$$

$$\sqrt[3]{\frac{3 \times 261.8}{\pi \times 4}} = r$$

$$\sqrt[3]{62.5} = r$$

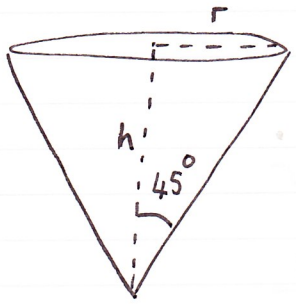
$$3.97 \text{ cm} = r$$

$\therefore$ $\frac{dh}{dt}$ when $\frac{1}{2}$ full

$$= -\frac{5}{\pi \times 3.97^2}$$

$$= -0.10098 \text{ cm/s}$$

11)



If angle is $45°$ as shown then $r = h$

Now $\frac{dV}{dt} = 3 \text{ cm}^3/\text{min}$

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} \qquad (*)$$

First find $\frac{dh}{dV}$

$h = r$

$\frac{dh}{dr} = 1$

$\frac{dr}{dh} = 1$

$V = \frac{1}{3}\pi r^2(r)$

$V = \frac{1}{3}\pi r^3$

$\frac{dV}{dr} = \pi r^2$

$\frac{dr}{dV} = \frac{1}{\pi r^2}$

$$\frac{dh}{dV} = \frac{dh}{dr} \times \frac{dr}{dV}$$

$$= 1 \times \frac{1}{\pi r^2}$$

$$\frac{dh}{dV} = \frac{1}{\pi r^2}$$

$\therefore$ (*) Gives

$$\frac{dh}{dt} = \frac{1}{\pi r^2} \times 3$$

Now if depth is 2cm

$h = r = 2\text{cm}$

$$\frac{dh}{dt} = \frac{3}{\pi \times 2^2}$$

$$= \frac{3}{4\pi} \text{ cm/min}$$

$$= 0.24 \text{ cm/min}$$

12) circle

a) $C = 2\pi r$ $\quad A = \pi r^2$

$\frac{dC}{dr} = 2\pi$ $\quad \frac{dA}{dr} = 2\pi r$

$\frac{dr}{dC} = \frac{1}{2\pi}$ $\quad \frac{dr}{dA} = \frac{1}{2\pi r}$

$$\frac{dC}{dA} = \frac{dC}{dr} \times \frac{dr}{dA}$$

$$= 2\pi \times \frac{1}{2\pi r}$$

$$= \frac{1}{r}$$

b) $\frac{dA}{dt} = 2\ cm^2/s$

find $\frac{dC}{dt}$

$$\frac{dC}{dt} = \frac{dC}{dA} \times \frac{dA}{dt}$$

$$= \frac{1}{r} \times 2$$

$r = 3cm$ $\quad \frac{dC}{dt} = \frac{1}{3} \times 2$

$$= \frac{2}{3}\ cm/s$$