Related Rates of Change

1) The radius of a circle is increasing at the rate of 2cm/s.

a) Find an expression in terms of r for the rate at which the area is increasing. (Ans: $4\pi r$ cm^2/sec)

b) Find the rate at which the area is increasing when $r = 10$cm. (Ans: $44\pi r$ cm^2/sec)

2) The radius of a sphere increases at a rate of 2cm/s.

Find the rate at which the volume is increasing when

a) $r = 3$cm (Ans: 72π cm^3/sec)

b) $r = 4$cm (Ans: 128π cm^3/sec)

3) Air is being pumped into a spherical balloon at a rate of 54 cm^3/s. Find the rate at which the radius is increasing when the volume is 36πcm^3.

(Ans: $\frac{3}{2\pi}$ cm/sec)

4) If the volume of a sphere increases at a rate of 6cm^3/s, find the rate of increase in the surface area of the sphere when the radius is 4cm.

(Ans: 3cm^2/sec)

5) If $A = (2t+3)^5$ and $x = 2t^2 + 6t$ find an expression for $\frac{dA}{dx}$ in terms of t.

(Ans: $5(2t+3)^3$)

6) If $T = 5p^2 + \frac{3}{p}$, find $\frac{dT}{dq}$ when $p = 2$, given that for that value of p, $\frac{dp}{dq} = 4$.

(Ans: 77)

7) A closed right-circular cylinder has base radius r cm and height $3r$ cm. If r is increased at a rate of 1mm/sec, find expressions in terms of r for the rate of increase of

a) the total external surface area (Ans: $1.6\pi r$ cm^2/s)

b) the volume of the cylinder (Ans: $0.9\pi r^2$ cm^3/s)

8) The surface area of a cube is increasing at the rate of $10\,cm^2/s$. Find the rate of increase of the volume of the cube when the edge is of length 12cm.

(Ans: $30\,cm^3/sec$)

9) Ink is dropped on to paper forming a circular stain which increases in area at a rate of $5\,cm^2/s$. Find the rate of change of the radius when the area is $30\,cm^2$.

(Ans: 0·2575 cm/sec)

* Negative rate of change here!

10) A container in the shape of a right-circular cone of height 20cm and radius 5cm is held vertex downward and filled with water which then drips out from the vertex at a rate of $5\,cm^3/s$. Find the rate of change of the height of water in the cone when it is half empty by volume.

(Ans: $-0·1011$ cm/s)

11) A container in the shape of a hollow cone of semi-vertical angle 45° is held with its vertex pointing downwards. Water drips into the container at a rate of $3\,cm^3$ per minute. Find the rate at which the depth of water in the cone is increasing when this depth is 2cm.

(Ans: $\frac{3}{4\pi}$ cm per min)

12) A circle has radius r cm, circumference C cm and area A cm^2.

a) Show that $\frac{dC}{dA} = \frac{1}{r}$

b) The area of a circle is increasing at $2\,cm^2/sec$.
Find the rate of increase of the circumference when the radius is 3cm.

(Ans: $\frac{2}{3}$ cm/sec)