

SURDS: 5: SOLUTIONS

$$1) \frac{(4\sqrt{5} - \sqrt{3})}{(\sqrt{5} + \sqrt{3})} \times \frac{(\sqrt{5} - \sqrt{3})}{(\sqrt{5} - \sqrt{3})}$$

$$= \frac{20 - 4\sqrt{15} - \sqrt{15} + 3}{5 - \sqrt{15} + \sqrt{15} - 3}$$

$$= \frac{23 - 5\sqrt{15}}{2}$$

$$2) \frac{(3\sqrt{7} + 5\sqrt{3})}{(\sqrt{7} + \sqrt{3})} \times \frac{(\sqrt{7} - \sqrt{3})}{(\sqrt{7} - \sqrt{3})}$$

$$= \frac{21 - 3\sqrt{21} + 5\sqrt{21} - 15}{7 - \sqrt{21} + \sqrt{21} - 3}$$

$$= \frac{6 + 2\sqrt{21}}{4}$$

$$= \frac{3 + \sqrt{21}}{2}$$

$$3) \frac{1}{(a + \sqrt{7})} \times \frac{(a - \sqrt{7})}{(a - \sqrt{7})}$$

$$= \frac{a - \sqrt{7}}{a^2 - a\sqrt{7} + a\sqrt{7} - 7}$$

$$= \frac{a - \sqrt{7}}{a^2 - 7}$$

$$4) \frac{1}{(2 - \sqrt{a})} \times \frac{(2 + \sqrt{a})}{(2 + \sqrt{a})}$$

$$= \frac{2 + \sqrt{a}}{4 + 2\sqrt{a} - 2\sqrt{a} - a}$$

$$= \frac{2 + \sqrt{a}}{4 - a}$$

$$5) \frac{a}{(3 + \sqrt{b})} \times \frac{(3 - \sqrt{b})}{(3 - \sqrt{b})}$$

$$= \frac{3a - a\sqrt{b}}{9 - 3\sqrt{b} + 3\sqrt{b} - b}$$

$$= \frac{3a - a\sqrt{b}}{9 - b}$$

$$6) \frac{(p + 2\sqrt{3})}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}$$

$$= \frac{p\sqrt{3} + p + 6 + 2\sqrt{3}}{3 + \sqrt{3} - \sqrt{3} - 1}$$

$$= \frac{\sqrt{3}p + p + 2\sqrt{3} + 6}{2}$$

$$= \frac{(p + 2)\sqrt{3} + p + 6}{2}$$

$$\begin{aligned}
 7) \quad & \sqrt{192} + \frac{2\sqrt{6q^2}}{\sqrt{2}} - \sqrt{27} \\
 &= 8\sqrt{3} + \frac{2\sqrt{6}q}{\sqrt{2}} - 3\sqrt{3} \\
 &= 8\sqrt{3} + \underbrace{\sqrt{2}}_{\frac{2}{\sqrt{2}}} \underbrace{\sqrt{3}\sqrt{2}}_{\sqrt{6}} q - 3\sqrt{3} \\
 &= 8\sqrt{3} + 2q\sqrt{3} - 3\sqrt{3} \\
 &= 5\sqrt{3} + 2q\sqrt{3} \\
 &= (5 + 2q)\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 8) \quad & (\sqrt{x} + 2)^2 - (\sqrt{x} - 2)^2 \\
 &= x + 4 + 4\sqrt{x} - [x + 4 - 4\sqrt{x}] \\
 &= x + 4 + 4\sqrt{x} - x - 4 + 4\sqrt{x} \\
 &= 8\sqrt{x}
 \end{aligned}$$

$$\begin{aligned}
 9) \quad & (\sqrt{y} - 3)^2 - (\sqrt{y} + 3)^2 \\
 &= y + 9 - 6\sqrt{y} - [y + 9 + 6\sqrt{y}] \\
 &= y + 9 - 6\sqrt{y} - y - 9 - 6\sqrt{y} \\
 &= -12\sqrt{y}
 \end{aligned}$$

$$\begin{aligned}
 10) \quad & \frac{24\sqrt{t}}{(\sqrt{t} + 3)^2 - (\sqrt{t} - 3)^2} \\
 &= \frac{24\sqrt{t}}{t + 9 + 6\sqrt{t} - [t + 9 - 6\sqrt{t}]} \\
 &= \frac{24\sqrt{t}}{t + 9 + 6\sqrt{t} - t - 9 + 6\sqrt{t}} \\
 &= \frac{24\sqrt{t}}{12\sqrt{t}} \\
 &= 2
 \end{aligned}$$