

Completing the Square 2 : Answers

$$\begin{aligned} 1) \quad & x^2 + 2x + 5 \\ &= (x+1)^2 - 1 + 5 \\ &= (x+1)^2 + 4 \end{aligned}$$

$$\begin{aligned} 2) \quad & x^2 - 4x + 1 \\ &= (x-2)^2 - 4 + 1 \\ &= (x-2)^2 - 3 \end{aligned}$$

$$\begin{aligned} 3) \quad & x^2 - 8x - 3 \\ &= (x-4)^2 - 16 - 3 \\ &= (x-4)^2 - 19 \end{aligned}$$

$$\begin{aligned} 4) \quad & x^2 + \frac{4}{5}x + 1 \\ &= \left(x + \frac{2}{5}\right)^2 - \frac{4}{25} + \frac{25}{25} \\ &= \left(x + \frac{2}{5}\right)^2 + \frac{21}{25} \end{aligned}$$

$$\begin{aligned} 5) \quad & x^2 - \frac{3}{4}x + 2 \\ &= \left(x - \frac{3}{8}\right)^2 - \frac{9}{64} + \frac{128}{64} \\ &= \left(x - \frac{3}{8}\right)^2 + \frac{119}{64} \end{aligned}$$

$$\begin{aligned} 6) \quad & 2x^2 + 6x - 3 \\ &= 2\left[x^2 + 3x - \frac{3}{2}\right] \\ &= 2\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{6}{4}\right] \\ &= 2\left[\left(x + \frac{3}{2}\right)^2 - \frac{3}{4}\right] \\ &= 2\left(x + \frac{3}{2}\right)^2 - \frac{3}{2} \end{aligned}$$

$$\begin{aligned} 7) \quad & 2x^2 - 3x - 1 \\ &= 2\left[x^2 - \frac{3}{2}x - \frac{1}{2}\right] \\ &= 2\left[\left(x - \frac{3}{4}\right)^2 - \frac{9}{16} - \frac{8}{16}\right] \\ &= 2\left[\left(x - \frac{3}{4}\right)^2 - \frac{17}{16}\right] \\ &= 2\left(x - \frac{3}{4}\right)^2 - \frac{17}{8} \end{aligned}$$

$$\begin{aligned} 8) \quad & 3x^2 + 9x - 1 \\ &= 3\left[x^2 + 3x - \frac{1}{3}\right] \\ &= 3\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{1}{3}\right] \\ &= 3\left[\left(x + \frac{3}{2}\right)^2 - \frac{27}{12} - \frac{4}{12}\right] \\ &= 3\left[\left(x + \frac{3}{2}\right)^2 - \frac{31}{12}\right] \\ &= 3\left(x + \frac{3}{2}\right)^2 - \frac{31}{4} \end{aligned}$$

$$\begin{aligned} 9) \quad & 3x^2 - 4x + 1 \\ &= 3\left[x^2 - \frac{4}{3}x + \frac{1}{3}\right] \\ &= 3\left[\left(x - \frac{2}{3}\right)^2 - \frac{4}{9} + \frac{3}{9}\right] \\ &= 3\left[\left(x - \frac{2}{3}\right)^2 - \frac{1}{9}\right] \\ &= 3\left(x - \frac{2}{3}\right)^2 - \frac{1}{3} \end{aligned}$$

$$\begin{aligned} 10) \quad & 5x^2 - 5x + 1 \\ &= 5\left[x^2 - x + \frac{1}{5}\right] \\ &= 5\left[\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + \frac{1}{5}\right] \\ &= 5\left[\left(x - \frac{1}{2}\right)^2 - \frac{5}{20} + \frac{4}{20}\right] \\ &= 5\left[\left(x - \frac{1}{2}\right)^2 - \frac{1}{20}\right] \\ &= 5\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} \end{aligned}$$

$$\begin{aligned}
 11) \quad & 4x^2 + 3x - 2 \quad \swarrow \frac{2}{4} \\
 & = 4 \left[x^2 + \frac{3}{4}x - \frac{1}{2} \right] \\
 & = 4 \left[\left(x + \frac{3}{8} \right)^2 - \frac{9}{16} - \frac{8}{16} \right] \\
 & = 4 \left[\left(x + \frac{3}{8} \right)^2 - \frac{17}{16} \right] \\
 & = 4 \left(x + \frac{3}{8} \right)^2 - \frac{17}{4}
 \end{aligned}$$

$$\begin{aligned}
 12) \quad & -2x^2 + x - 1 \\
 & = -2 \left[x^2 - \frac{x}{2} + \frac{1}{2} \right] \\
 & = -2 \left[\left(x - \frac{1}{4} \right)^2 - \frac{1}{16} + \frac{8}{16} \right] \\
 & = -2 \left[\left(x - \frac{1}{4} \right)^2 + \frac{7}{16} \right] \\
 & = -2 \left(x - \frac{1}{4} \right)^2 - \frac{7}{8}
 \end{aligned}$$

$$\begin{aligned}
 13) \quad & -3x^2 - 2x + 5 \\
 & = -3 \left[x^2 + \frac{2}{3}x - \frac{5}{3} \right] \\
 & = -3 \left[\left(x + \frac{1}{3} \right)^2 - \frac{1}{9} - \frac{15}{9} \right] \\
 & = -3 \left[\left(x + \frac{1}{3} \right)^2 - \frac{16}{9} \right] \\
 & = -3 \left(x + \frac{1}{3} \right)^2 + \frac{16}{3}
 \end{aligned}$$

$$\begin{aligned}
 14) \quad & -x^2 + 3x - 1 \\
 & = - \left[x^2 - 3x + 1 \right] \\
 & = - \left[\left(x - \frac{3}{2} \right)^2 - \frac{9}{4} + \frac{4}{4} \right] \\
 & = - \left[\left(x - \frac{3}{2} \right)^2 - \frac{5}{4} \right] \\
 & = - \left(x - \frac{3}{2} \right)^2 + \frac{5}{4}
 \end{aligned}$$