Completing the Square : 6 : Answers

1) $x^2 + 8x + 2$

$= (x+4)^2 - 16 + 2$

$= (x+4)^2 - 14$ $\quad a = 4 \quad b = -14$

$\therefore$ Least value needed $= -14$ when $x = -4$

2) $x^2 + 4x + 9$

$= (x+2)^2 - 4 + 9$

$= (x+2)^2 + 5$ $\quad a = 2 \quad b = 5$

Min value $= 5$ when $x = -2$

$\therefore$ Maximum value of $\dfrac{1}{x^2+4x+9}$ is when bottom line is MIN

$= \dfrac{1}{5}$

3) $2x^2 + 4x + 5$

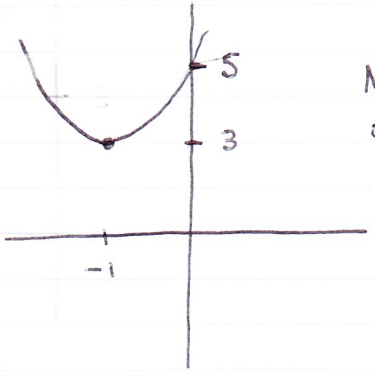
$= 2\left[x^2 + 2x + \frac{5}{2}\right]$

$= 2\left[(x+1)^2 - 1 + \frac{5}{2}\right]$

$= 2\left[(x+1)^2 + \frac{3}{2}\right]$

$= 2(x+1)^2 + 3$ $\quad a = 2 \quad b = 1 \quad c = 3$

Now for $y = 2x^2 + 4x + 5$



Min = 3 when $x = -1$

Now $2x^2 + 4x + 9$

$= (2x^2 + 4x + 5) + 4$

previous work

Min of this

$= (3) + 4$

$= 7$

$\therefore$ Greatest value needed

$= \dfrac{1}{7}$

4) $x^2 + 1.8x - 3.19$

$= (x + 0.9)^2 - 0.81 - 3.19$

$= (x + 0.9)^2 - 4$ $\quad p = 0.9$

$x^2 + 1.8x - 3.19 = 0$

$(x + 0.9)^2 - 4 = 0$

$(x + 0.9)^2 = 4$

$x + 0.9 = \pm 2$

either $x = 2 - 0.9$ $\quad$ or $\quad x = -2 - 0.9$

$x = 1.1$ $\quad x = -2.9$

5) $x^2 + 6x - 4$

$= (x + 3)^2 - 9 - 4$

$= (x + 3)^2 - 13$ $\quad a = 3 \quad b = -13$

Min value $= -13$ when $x = -3$

∴ Least value of $2x^2 + 12x - 8$

$= 2(-13)$ from above

$= -26$ when $x = -3$

NOTICE.

$2x^2 + 12x - 8$

$= 2(x^2 + 6x - 4)$

6) $3x^2 - 12x + 17$

$= 3\left[x^2 - 4x + \frac{17}{3}\right]$

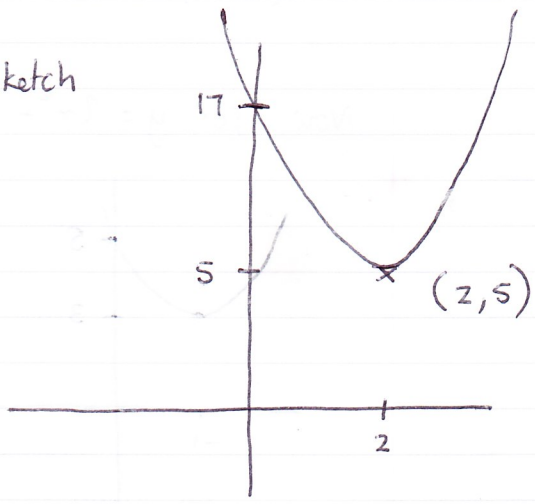
$= 3\left[(x-2)^2 - 4 + \frac{17}{3}\right]$

$= 3\left[(x-2)^2 + \frac{5}{3}\right]$

$= 3(x-2)^2 + 5$

$a = 3 \quad b = -2 \quad c = 5$

sketch



Min at $(2, 5)$

7) $x^2 - 5x + 8$

$= \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 8$

$= \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + \frac{32}{4}$

$= \left(x - \frac{5}{2}\right)^2 + \frac{7}{4}$

$a = -\frac{5}{2}$ $b = \frac{7}{4}$ Min $= \frac{7}{4}$ when $x = 5/2$

Now $-x^2 + 5x - 8$

$= -[x^2 - 5x + 8]$

previous part

Now Greatest value of $-[x^2 - 5x + 8]$

will be when the expression is Least negative.

ie as close to 0 as possible

$\therefore$ Greatest value is $-[7/4]$

$= -\frac{7}{4}$