Completing the Square : 7 : Answers

1) $4x^2 - 8x + 7$

$= 4\left[x^2 - 2x + \frac{7}{4}\right]$

$= 4\left[(x-1)^2 - 1 + \frac{7}{4}\right]$

$= 4\left[(x-1)^2 + \frac{3}{4}\right]$

$= 4(x-1)^2 + 3$ $\quad a = 4 \quad b = -1 \quad c = 3$

Min value $= +3$ when $x = 1$

∴ Greatest value of $\frac{1}{4x^2 - 8x + 7}$ occurs when bottom line of fraction is minimum

ie Greatest value $= \frac{1}{3}$

2) $2x^2 + 12x - 7$

$= 2\left[x^2 + 6x - \frac{7}{2}\right]$

$= 2\left[(x+3)^2 - 9 - \frac{7}{2}\right]$

$= 2\left[(x+3)^2 - \frac{25}{2}\right]$

$= 2(x+3)^2 - 25$ $\quad a = 2 \quad b = 3 \quad c = -25$

Min value $= -25$ when $x = -3$

Now

$6x^2 + 36x - 17$

$= 3(2x^2 + 12x - 7) + 4$

This is from the previous part

∴ Least value as needed

$= 3(-25) + 4$

$= -75 + 4$

$= -71$

3) $x^2 - 1.4x - 8.51$

$= (x - 0.7)^2 - 0.49 - 8.51$

$= (x - 0.7)^2 - 9$ $\quad p = -0.7$

NOW $x^2 - 1.4x - 8.51 = 0$

$(x - 0.7)^2 - 9 = 0$

$(x - 0.7)^2 = 9$

$x - 0.7 = \pm 3$

$x = 3 + 0.7$ or $-3 + 0.7$

$x = 3.7$ $\quad x = -2.3$

4) $-x^2 + 6x - 7$

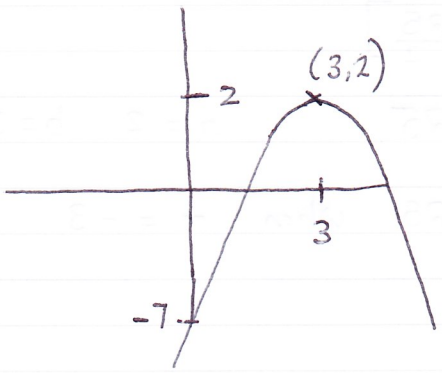
$= -[x^2 - 6x + 7]$

$= -[(x-3)^2 - 9 + 7]$

$= -[(x-3)^2 - 2]$

$= -(x-3)^2 + 2$ $\quad a = -3 \quad b = 2$

Sketch $y = -x^2 + 6x - 7$



MAX point (3,2)

5) $3x^2 - 6x + 5$

$= 3\left[x^2 - 2x + \frac{5}{3}\right]$

$= 3\left[(x-1)^2 - 1 + \frac{5}{3}\right]$

$= 3\left[(x-1)^2 + \frac{2}{3}\right]$

$= 3(x-1)^2 + 2$ $\quad a = 3 \quad b = -1 \quad c = 2$

Min value $= +2$ when $x = 1$

Now $3x^2 - 6x + 11$

$= (3x^2 - 6x + 5) + 6$

from above

$\therefore$ Min value of $3x^2 - 6x + 11$

$= (2) + 6$

$= 8$

$\therefore$ Greatest value of $\dfrac{1}{3x^2 - 6x + 11}$

$= \dfrac{1}{8}$

6) $3x^2 - 12x + 29$

$= 3\left[x^2 - 4x + \frac{29}{3}\right]$

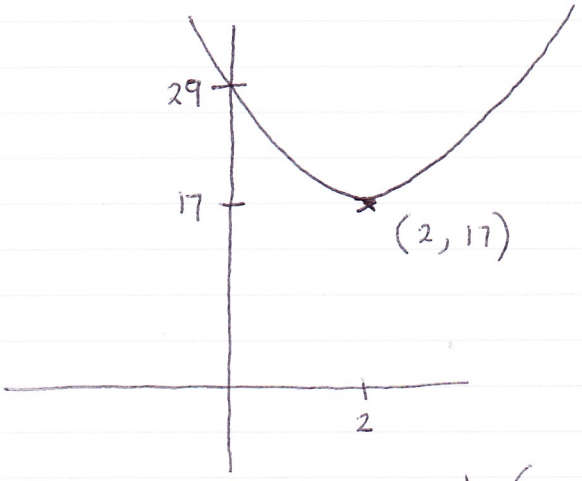
$= 3\left[(x-2)^2 - 4 + \frac{29}{3}\right]$

$= 3\left[(x-2)^2 + \frac{17}{3}\right]$

$= 3(x-2)^2 + 17$

$a = 3 \quad b = -2 \quad c = 17$

sketch $y = 3x^2 - 12x + 29$



Min point $(2, 17)$

7) $x^2 + 8x + 5$

$= (x+4)^2 - 16 + 5$

$= (x+4)^2 - 11$ $\quad a = 4 \quad b = -11$

MIN VALUE $= -11$ when $x = -4$

Now $3x^2 + 24x + 15$

$= 3(x^2 + 8x + 5)$

previous section

$\therefore$ Min or least value of $3x^2 + 24x + 15$

$= 3(-11)$

$= -33$ when $x = -4$