Discriminants and Roots of Equations : Answers

1) $kx^2 - 4x + k - 3 = 0$

$a = k \quad b = -4 \quad c = k-3$

$b^2 - 4ac = 0$ for equal roots

$(-4)^2 - 4(k)(k-3) = 0$

$16 - 4k^2 + 12k = 0$

$0 = 4k^2 - 12k - 16$

$0 = k^2 - 3k - 4$

$0 = (k-4)(k+1)$

either $k - 4 = 0$, $k = 4$ or $k + 1 = 0$, $k = -1$

2) $kx^2 - 4x + (k-3) = 0 \quad a = k \quad b = -4 \quad c = k-3$

$b^2 - 4ac \geqslant 0$ for real roots

$(-4)^2 - 4(k)(k-3) \geqslant 0$

$16 - 4k^2 + 12k \geqslant 0$

$4 - k^2 + 3k \geqslant 0$

$0 \geqslant k^2 - 3k - 4$

$\therefore \quad k^2 - 3k - 4 \leqslant 0$ as required

Now from algebra

$(k-4)(k+1) \leqslant 0$

either $(+) \times (-) = (-)$

or $(-) \times (+) = (-)$

either $k - 4 \geqslant 0$ and $k + 1 \leqslant 0$

$k \geqslant 4 \quad k \leqslant -1$

IMPOSSIBLE

or $k - 4 \leqslant 0$ and $k + 1 \geqslant 0$

$k \leqslant 4 \quad k \geqslant -1$

$-1 \leqslant k \leqslant 4$

3) $x^2 + (2k+1)x + (k^2+k+1) = 0$

$a = 1 \quad b = 2k+1 \quad c = k^2+k+1$

Now $b^2 - 4ac$

$= (2k+1)^2 - 4(1)(k^2+k+1)$

$= \cancel{4k^2} + \cancel{4k} + 1 - \cancel{4k^2} - \cancel{4k} - 4$

$= -3$

$\therefore$ because $b^2 - 4ac < 0$ there are no real roots for any k value.

4) $3x^2 + 2x - k = 0$

$a = 3 \quad b = 2 \quad c = -k$

$b^2 - 4ac > 0$ for 2 distinct real roots

$(2)^2 - 4(3)(-k) > 0$

$4 + 12k > 0$

$12k > -4$

$k > \frac{-4}{12}$

$k > -\frac{1}{3}$

5) $3x^2 - 6x + m = 0$

$a = 3 \quad b = -6 \quad c = m$

$b^2 - 4ac < 0$ for no real roots

$(-6)^2 - 4(3)(m) < 0$

$36 - 12m < 0$

$36 < 12m$

$3 < m$

or

$m > 3$

6) $(3k-2)x^2 + 8x + k = 0$

$a = 3k-2 \qquad b = 8 \qquad c = k$

$b^2 - 4ac < 0$ for no real roots

$(8)^2 - 4(3k-2)(k) < 0$

$64 - 12k^2 + 8k < 0$

$16 - 3k^2 + 2k < 0$

$0 < 3k^2 - 2k - 16$

So $3k^2 - 2k - 16 > 0$ as required

From algebra $(3k - 8)(k+2) > 0$

either $(+) \times (+) = (+)$

or $(-) \times (-) = (+)$

either $3k - 8 > 0$ and $k + 2 > 0$

$k > \frac{8}{3}$ $\qquad$ $k > -2$

$k > \frac{8}{3}$

or $3k - 8 < 0$ and $k + 2 < 0$

$k < 8/3$ $\qquad$ $k < -2$

$k < -2$

7) $(k+1)x^2 + 2kx + (k-1) = 0$

$a = k+1 \qquad b = 2k \qquad c = k-1$

$b^2 - 4ac$

$= (2k)^2 - 4(k+1)(k-1)$

$= 4k^2 - 4(k^2 - 1)$

$= 4k^2 - 4k^2 + 4$

$= 4$

$(k+1)(k-1)$
$= (k^2 - 1)$

ie $b^2 - 4ac > 0$

$\therefore$ 2 distinct real roots

8) $kx^2 + 3x - 5 = 0$

$a = k \quad b = 3 \quad c = -5$

$b^2 - 4ac < 0$ for no real roots

$(3)^2 - 4(k)(-5) < 0$

$9 + 20k < 0$

$20k < -9$

$k < -\frac{9}{20}$

9) $2x^2 + kx + 18 = 0$

$a = 2 \quad b = k \quad c = 18$

$b^2 - 4ac < 0$ for no real roots

$(k)^2 - 4(2)(18) < 0$

$k^2 - 144 < 0$

$(k-12)(k+12) < 0$

either $(+) \times (-) = (-)$

or $(-) \times (+) = (-)$

either $k - 12 > 0$ and $k + 12 < 0$

$k > 12$ $\quad$ $k < -12$

IMPOSSIBLE

or $k - 12 < 0$ and $k + 12 > 0$

$k < 12$ $\quad$ $k > -12$

$-12 < k < 12$

10) $2x^2 + (3k-1)x + (3k^2 - 1) = 0$

$a = 2 \quad b = 3k - 1 \quad c = 3k^2 - 1$

$b^2 - 4ac > 0$ for two distinct real roots

$(3k-1)^2 - 4(2)(3k^2 - 1) > 0$

$9k^2 - 6k + 1 - 24k^2 + 8 > 0$

$-15k^2 - 6k + 9 > 0$

$-5k^2 - 2k + 3 > 0$

$0 > 5k^2 + 2k - 3$

So $5k^2 + 2k - 3 < 0$

$(5k - 3)(k + 1) < 0$

either $(+) \times (-) = (-)$

or $(-) \times (+) = (-)$

continue

either $5k - 3 > 0$ and $k + 1 < 0$

$k > \frac{3}{5}$ $\quad$ $k < -1$

IMPOSSIBLE

or $5k - 3 < 0$ and $k + 1 > 0$

$k < \frac{3}{5}$ $\quad$ $k > -1$

$-1 < k < \frac{3}{5}$