Laws of Indices : 2 : SOLUTIONS

1) Expand and simplify where appropriate

a) $\left(\sqrt{x} + \sqrt[3]{x}\right)\left(\sqrt{x} - \sqrt[3]{x}\right)$

$= \left(x^{1/2} + x^{1/3}\right)\left(x^{1/2} - x^{1/3}\right)$

$= x^{1} - x^{5/6} + x^{5/6} - x^{2/3}$

$= x - \sqrt[3]{x^2}$

b) $\left(\sqrt[3]{x} - \sqrt[4]{x}\right)\left(\sqrt[4]{x} + \sqrt[3]{x}\right)$

$= \left(x^{1/3} - x^{1/4}\right)\left(x^{1/4} + x^{1/3}\right)$

$= x^{7/12} + x^{2/3} - x^{1/2} - x^{7/12}$

$= \sqrt[3]{x^2} - \sqrt{x}$

c) $\left(\sqrt{x} - \frac{1}{x^2}\right)\left(\frac{1}{x^2} + \sqrt{x}\right)$

$= \left(x^{1/2} - x^{-2}\right)\left(x^{-2} + x^{1/2}\right)$

$= x^{-3/2} + x^{1} - x^{-4} - x^{-3/2}$

$= x - \frac{1}{x^4}$

OR

$= \frac{x^5}{x^4} - \frac{1}{x^4}$

$= \frac{x^5 - 1}{x^4}$

2) a) $8^{3x-1} = 16^{x}$

$(2^3)^{3x-1} = (2^4)^{x}$

$2^{9x-3} = 2^{4x}$

$\therefore \quad 9x-3 = 4x$

$5x = 3$

$x = 3/5$

b) $9^{5-2x} = 27^{3x+1}$

$(3^2)^{5-2x} = (3^3)^{3x+1}$

$3^{10-4x} = 3^{9x+3}$

$\therefore \quad 10-4x = 9x+3$

$7 = 13x$

$\frac{7}{13} = x$

c) $\left(\frac{1}{27}\right)^{2x-1} = 3^{4x}$

$(3^{-3})^{2x-1} = 3^{4x}$

$3^{-6x+3} = 3^{4x}$

$\therefore \quad -6x+3 = 4x$

$3 = 10x$

$\frac{3}{10} = x$

d) $4^{3x} = \left(\frac{1}{32}\right)^{3-x}$

$4^{3x} = (2^{-5})^{3-x}$

$(2^2)^{3x} = (2^{-5})^{3-x}$

$2^{6x} = 2^{-15+5x}$

$\therefore \quad 6x = -15+5x$

$x = -15$

e) $27^{-x} = \left(\frac{1}{81}\right)^{4x+1}$

$(3^3)^{-x} = (3^{-4})^{4x+1}$

$3^{-3x} = 3^{-16x-4}$

$\therefore \quad -3x = -16x-4$

$13x = -4$

$x = \frac{-4}{13}$

f) $100^{5x} = \left(\frac{1}{1000}\right)^{6-3x}$

$(10^2)^{5x} = (10^{-3})^{6-3x}$

$10^{10x} = 10^{-18+9x}$

$\therefore \quad 10x = -18+9x$

$x = -18$