

Simultaneous Equations : 2 : Answers

1) $y = 10x^2 - 5x - 2 \quad -\textcircled{1}$
 $y = 2x - 3 \quad -\textcircled{2}$

sub $\textcircled{2}$ into $\textcircled{1}$

$$2x - 3 = 10x^2 - 5x - 2$$

$$0 = 10x^2 - 7x + 1$$

$$0 = (5x - 1)(2x - 1)$$

either or

$$5x - 1 = 0$$

$$x = \frac{1}{5}$$

\Rightarrow

$$y = 2\left(\frac{1}{5}\right) - 3$$

$$y = \frac{2}{5} - \frac{15}{5}$$

$$y = -\frac{13}{5}$$

$$2x - 1 = 0$$

$$x = \frac{1}{2}$$

$$y = 2\left(\frac{1}{2}\right) - 3$$

$$y = 1 - 3$$

$$y = -2$$

$$\begin{cases} x = \frac{1}{5} \\ y = -\frac{13}{5} \end{cases} \text{ Ans}$$

$$\begin{cases} x = \frac{1}{2} \\ y = -2 \end{cases} \text{ Ans}$$

2) $y - 3 = 2x \quad -\textcircled{1}$
 $y + 7 = x^2 - x \quad -\textcircled{2}$

~~2~~ $\textcircled{1} \Rightarrow y = 2x + 3 \quad (*)$

Sub into $\textcircled{2} \Rightarrow 2x + 3 + 7 = x^2 - x$

$$0 = x^2 - 3x - 10$$

$$0 = (x - 5)(x + 2)$$

either

$$x - 5 = 0$$

$$x = 5$$

from (*) \downarrow

$$y = 2(5) + 3$$

$$y = 13$$

$$x + 2 = 0$$

$$x = -2$$

\downarrow

$$y = 2(-2) + 3$$

$$y = -4 + 3$$

$$y = -1$$

$$\begin{cases} x = 5 \\ y = 13 \end{cases} \text{ Ans}$$

$$\begin{cases} x = -2 \\ y = -1 \end{cases} \text{ Ans}$$

$$3) \quad \begin{array}{l} x^2 + y^2 = 25 \\ 2y + x = 10 \end{array} \quad \begin{array}{l} -\textcircled{1} \\ -\textcircled{2} \end{array}$$

$$\textcircled{2} \Rightarrow x = 10 - 2y \quad (*)$$

$$\text{sub into } \textcircled{1} \Rightarrow (10 - 2y)^2 + y^2 = 25$$

$$100 - 40y + 4y^2 + y^2 = 25$$

$$5y^2 - 40y + 75 = 0$$

$$y^2 - 8y + 15 = 0$$

$$(y-5)(y-3) = 0$$

$$\text{either } y-5=0 \quad \text{or} \quad y-3=0$$

$$\begin{array}{l} y=5 \\ (*) \end{array} \quad \begin{array}{l} y=3 \\ \downarrow \end{array}$$

$$x = 10 - 2(5)$$

$$x = 0$$

$$x = 10 - 2(3)$$

$$x = 4$$

$$\begin{array}{l} y=5 \\ y=0 \end{array} \quad \left. \begin{array}{l} \text{ANS} \\ \text{ANS} \end{array} \right\}$$

$$\begin{array}{l} x=4 \\ x=4 \\ y=3 \end{array} \quad \left. \begin{array}{l} \text{ANS} \\ \text{ANS} \end{array} \right\}$$

$$4) \quad \begin{array}{l} y = x \\ y^2 = 4x \end{array} \quad \begin{array}{l} -\textcircled{1} \\ -\textcircled{2} \end{array}$$

$$\textcircled{2} \Rightarrow x^2 = 4x$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$\text{either} \quad \begin{array}{l} x=0 \\ x=4 \end{array} \quad \text{or} \quad \begin{array}{l} x-4=0 \\ x=4 \end{array}$$

$$\begin{array}{l} \textcircled{1} \quad \downarrow \\ y=0 \end{array} \quad \begin{array}{l} \textcircled{2} \quad \downarrow \\ y=4 \end{array}$$

$$\begin{array}{l} x=0 \\ y=0 \end{array} \quad \left. \begin{array}{l} \text{ANS} \\ \text{ANS} \end{array} \right\}$$

$$\begin{array}{l} x=4 \\ y=4 \end{array} \quad \left. \begin{array}{l} \text{ANS} \\ \text{ANS} \end{array} \right\}$$

$$5) \quad \begin{aligned} xy &= 64 \quad -\textcircled{1} \\ 4x - y &= 60 \quad -\textcircled{2} \end{aligned}$$

$$\textcircled{2} \Rightarrow 4x - 60 = y \quad (*)$$

sub into $\textcircled{1}$

$$\begin{aligned} x(4x - 60) &= 64 \\ 4x^2 - 60x &= 64 \\ 4x^2 - 60x - 64 &= 0 \\ x^2 - 15x - 16 &= 0 \\ (x - 16)(x + 1) &= 0 \end{aligned}$$

either

$$\begin{aligned} x - 16 &= 0 & x + 1 &= 0 \\ x &= 16 & x &= -1 \end{aligned}$$

$$(*) \quad \begin{array}{c} \downarrow \\ \end{array} \quad \begin{array}{c} \downarrow \\ \end{array}$$

$$\begin{aligned} 4(16) - 60 &= y & 4(-1) - 60 &= y \\ 64 - 60 &= y & -4 - 60 &= y \\ 4 &= y & -64 &= y \end{aligned}$$

$$\begin{cases} x = 16 \\ y = 4 \end{cases} \text{ANS}$$

$$\begin{cases} x = -1 \\ y = -64 \end{cases} \text{ANS}$$

$$6) \quad \begin{aligned} xc - 3y &= 8 \quad -\textcircled{1} \\ xc^2 - 3y^2 &= 22 \quad -\textcircled{2} \end{aligned}$$

$$\textcircled{1} \Rightarrow xc = 8 + 3y \quad (*)$$

$$\textcircled{2} \Rightarrow (8 + 3y)^2 - 3y^2 = 22$$

$$\begin{aligned} 64 + 48y + 9y^2 - 3y^2 &= 22 \\ 6y^2 + 48y + 42 &= 0 \\ y^2 + 8y + 7 &= 0 \\ (y + 7)(y + 1) &= 0 \end{aligned}$$

either

$$\begin{aligned} y + 7 &= 0 & y + 1 &= 0 \\ y &= -7 & y &= -1 \end{aligned}$$

$$(*) \quad \begin{array}{c} \downarrow \\ \end{array} \quad \begin{array}{c} \downarrow \\ \end{array}$$

$$\begin{aligned} xc &= 8 + 3(-7) & xc &= 8 + 3(-1) \\ xc &= 8 - 21 & xc &= 8 - 3 \\ xc &= -13 & xc &= 5 \end{aligned}$$

$$\begin{cases} x = -13 \\ y = -7 \end{cases} \text{ANS}$$

$$\begin{cases} x = 5 \\ y = -1 \end{cases} \text{ANS}$$

$$7) \begin{array}{l} 2x - 5y = 3 \\ 2xy + y^2 = 9 \end{array} \quad \begin{array}{l} -(1) \\ -(2) \end{array}$$

$$(1) \Rightarrow 2x = 3 + 5y \quad (*)$$

$$(2) \Rightarrow (3 + 5y)y + y^2 = 9$$

$$3y + 5y^2 + y^2 = 9$$

$$6y^2 + 3y - 9 = 0$$

$$2y^2 + y - 3 = 0$$

$$(2y + 3)(y - 1) = 0$$

either $2y + 3 = 0$ or $y - 1 = 0$

$$y = -\frac{3}{2}$$

$$y = 1$$

(*)

$$2x = 3 + 5\left(-\frac{3}{2}\right)$$

↓

$$2x = 3 + 5(1)$$

$$2x = \frac{6}{2} - \frac{15}{2}$$

$$2x = 8$$

$$x = 4$$

$$2x = -\frac{9}{2}$$

$$x = -\frac{9}{4}$$

$$\begin{cases} x = -\frac{9}{4} \\ y = -\frac{3}{2} \end{cases} \text{ANS}$$

$$\begin{cases} x = 4 \\ y = -\frac{3}{2} \end{cases} \text{ANS.}$$

$$8) \begin{array}{l} 2x - 3y = 3 \\ 2x^2 + 3y^2 = 21 \end{array} \quad \begin{array}{l} -(1) \\ -(2) \end{array}$$

$$(1) \Rightarrow 2x = 3 + 3y$$

$$x = \frac{(3+3y)}{2} \quad (*)$$

$$(2) \Rightarrow 2 \left(\frac{(3+3y)^2}{4} \right) + 3y^2 = 21$$

$$\frac{(3+3y)^2}{4} + 3y^2 = 21$$

$$(3+3y)^2 + 6y^2 = 42$$

$$9 + 18y + 9y^2 + 6y^2 = 42$$

$$15y^2 + 18y - 33 = 0$$

$$5y^2 + 6y - 11 = 0$$

$$(5y + 11)(y - 1) = 0$$

either $5y + 11 = 0$ or $y - 1 = 0$

$$y = -\frac{11}{5}$$

(*) ↓

$$x = \frac{3 + 3\left(-\frac{11}{5}\right)}{2}$$

$$x = \frac{9}{5}$$

$$\begin{cases} x = \frac{9}{5} \\ y = -\frac{11}{5} \end{cases} \text{ANS}$$

$$\begin{cases} x = 3 \\ y = 1 \end{cases} \text{ANS}$$

$$x = \frac{3 + 3(1)}{2}$$

$$x = 3$$

$$9) \begin{array}{l} 2x+3y=8 \\ x^2+y^2=5 \end{array} \quad \begin{array}{l} -\textcircled{1} \\ -\textcircled{2} \end{array}$$

$$\textcircled{1} \Rightarrow 2x = 8 - 3y \\ x = \frac{(8-3y)}{2} \quad (*)$$

$$\textcircled{2} \Rightarrow \frac{(8-3y)^2}{4} + y^2 = 5$$

$$\times 4 \quad (8-3y)^2 + 4y^2 = 20 \\ 64 - 48y + 9y^2 + 4y^2 = 20 \\ 13y^2 - 48y + 44 = 0 \\ (13y - 22)(y - 2) = 0$$

$$\text{either } y = \frac{22}{13} \quad \text{or} \quad y = 2$$

$$(*) \quad \downarrow \quad x = \frac{8 - 3(\frac{22}{13})}{2}$$

$$x = \frac{\cancel{104} - \cancel{66}}{\cancel{13} \quad 2}$$

$$\downarrow \quad x = \frac{8 - 3(2)}{2}$$

$$x = 1$$

$$\left. \begin{array}{l} x = 1 \\ y = 2 \end{array} \right\} \text{ANS}$$

$$x = \frac{38}{26} \\ x = \frac{19}{13}$$

$$x = \frac{19}{13} \\ y = \frac{22}{13} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{ANS.}$$

$$10) \begin{array}{l} 3x - 2y = 4 \\ x^2 - y^2 = 3 \end{array} \quad \begin{array}{l} -\textcircled{1} \\ -\textcircled{2} \end{array}$$

$$\textcircled{1} \Rightarrow 3x = 4 + 2y \\ x = \frac{(4+2y)}{3} \quad (*)$$

$$\textcircled{2} \Rightarrow \frac{(4+2y)^2}{9} - y^2 = 3$$

$$\times 9 \quad (4+2y)^2 - 9y^2 = 27$$

$$16 + 16y + 4y^2 - 9y^2 = 27$$

$$0 = 5y^2 - 16y + 11$$

$$0 = (5y - 11)(y - 1)$$

$$\text{either } y = \frac{11}{5} \quad \text{or} \quad y = 1$$

$$x = \frac{4+2(\frac{11}{5})}{3}$$

$$x = \frac{42}{15}$$

$$\left. \begin{array}{l} x = 42/15 \\ y = 11/5 \end{array} \right\} \text{ANS}$$

$$\downarrow \quad x = \frac{4+2(1)}{3}$$

$$x = 2$$

$$\left. \begin{array}{l} x = 2 \\ y = 1 \end{array} \right\} \text{ANS}$$