

By ALGEBRA

Solving Quadratic Inequalities : Answers

1) $x^2 + 3x - 4 > 0$
 $(x+4)(x-1) > 0$

either $(+) \times (+) = (+)$
or $(-) \times (-) = (+)$

either $x+4 > 0$ and $x-1 > 0$
 $x > -4$ $x > 1$

$x > 1$

or $x+4 < 0$ and $x-1 < 0$
 $x < -4$ $x < 1$

$x < -4$

2) $x^2 - 2x - 15 \geq 0$
 $(x-5)(x+3) \geq 0$

either $(+) \times (+) = (+)$
or $(-) \times (-) = (+)$

either $x-5 \geq 0$ and $x+3 \geq 0$
 $x \geq 5$ $x \geq -3$

$x \geq 5$

or $x-5 \leq 0$ and $x+3 \leq 0$
 $x \leq 5$ $x \leq -3$

$x \leq -3$

3) $x^2 + x - 6 < 0$
 $(x+3)(x-2) < 0$
 either $(+) \times (-) = (-)$
 or $(-) \times (+) = (-)$

either $x+3 > 0$ and $x-2 < 0$
 $x > -3$ $x < 2$

$-3 < x < 2$

or $x+3 < 0$ and $x-2 > 0$
 $x < -3$ $x > 2$

IMPOSSIBLE

4) $2x^2 + x - 6 \leq 0$
 $(2x-3)(x+2) \leq 0$
 either $(+) \times (-) = (-)$
 or $(-) \times (+) = (-)$

either $2x-3 \geq 0$ and $x+2 \leq 0$
 $2x \geq 3$ $x \leq -2$
 $x \geq \frac{3}{2}$

IMPOSSIBLE

or $2x-3 \leq 0$ and $x+2 \geq 0$
 $x \leq \frac{3}{2}$ $x \geq -2$

$-2 \leq x \leq \frac{3}{2}$

5) $6x^2 - 7x - 3 > 0$
 $(2x-3)(3x+1) > 0$
 either $(+) \times (+) = (+)$
 or $(-) \times (-) = (+)$

either $2x-3 > 0$ and $3x+1 > 0$
 $x > \frac{3}{2}$ $x > -\frac{1}{3}$

$x > \frac{3}{2}$

or $2x-3 < 0$ and $3x+1 < 0$
 $x < \frac{3}{2}$ $x < -\frac{1}{3}$

$x < -\frac{1}{3}$

6) $5x^2 - 9x - 2 < 0$
 $(5x+1)(x-2) < 0$
 either $(+) \times (-) = (-)$
 or $(-) \times (+) = (-)$

either $5x+1 > 0$ and $x-2 < 0$
 $x > -\frac{1}{5}$ $x < 2$

$-\frac{1}{5} < x < 2$

or $5x+1 < 0$ and $x-2 > 0$
 $x < -\frac{1}{5}$ $x > 2$
 IMPOSSIBLE

GRAPHICAL METHODS

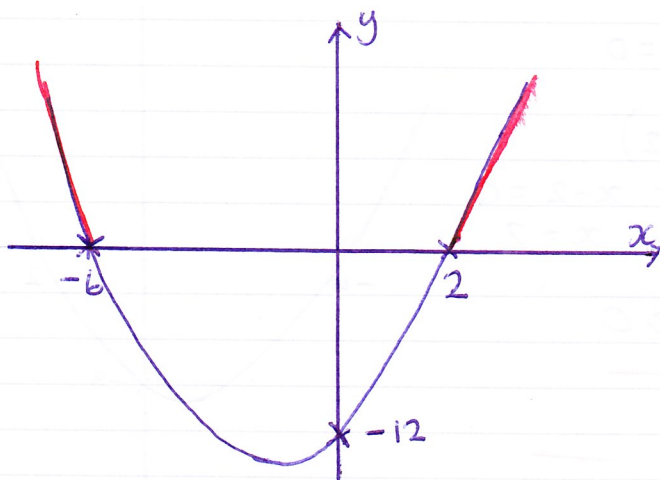
1) $x^2 + 4x - 12 > 0$

Sketch $y = x^2 + 4x - 12$ and $y = 0$ (x axis)

Crosses y axis, $x = 0$
 $y = -12$

Crosses x axis, $y = 0$
 $0 = x^2 + 4x - 12$
 $0 = (x+6)(x-2)$

either $x = -6$ or $x = 2$



NOW $x^2 + 4x - 12 > 0$
 ie $y > 0$ —
 x values for —
 $x > 2$ or $x < -6$

2) $x^2 - x - 12 < 0$

plot a sketch of $y = x^2 - x - 12$

$y = 0$ (x axis)

Crosses y axis, $x=0$

$$y = -12$$

Crosses x axis, $y=0$

$$0 = x^2 - x - 12$$

$$0 = (x-4)(x+3)$$

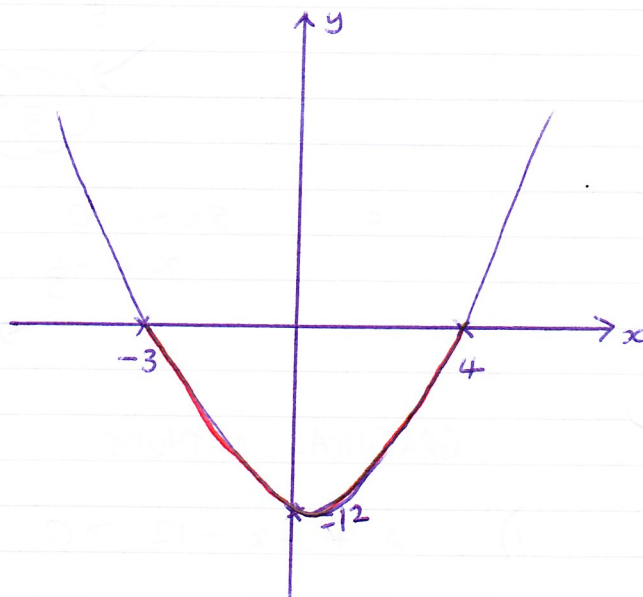
either $x = 4$ or $x = -3$

Now $x^2 - x - 12 < 0$

ie $y < 0$ —

x values for —

$-3 < x < 4$



3) $x^2 + 2x - 8 \geq 0$

sketch $y = x^2 + 2x - 8$

$y = 0$ (x axis)

Crosses y axis, $x=0$
 $y = -8$

Crosses x axis, $y=0$

$$0 = x^2 + 2x - 8$$

$$0 = (x+4)(x-2)$$

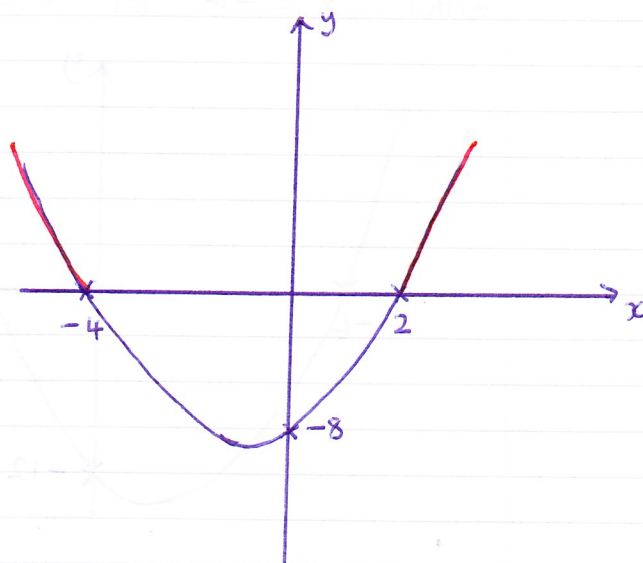
either $x+4=0$ or $x-2=0$
 $x = -4$ or $x = 2$

Now $x^2 + 2x - 8 \geq 0$

ie $y \geq 0$ —

x values for —

$x \leq -4$ or $x \geq 2$



4) $3x^2 + x - 2 \leq 0$

sketch $y = 3x^2 + x - 2$

$y = 0$ (x axis)

Crosses y axis, $x = 0$
 $y = -2$

Crosses x axis, $y = 0$

$0 = 3x^2 + x - 2$

$0 = (3x - 2)(x + 1)$

either

$3x - 2 = 0$

or $x + 1 = 0$

$x = \frac{2}{3}$

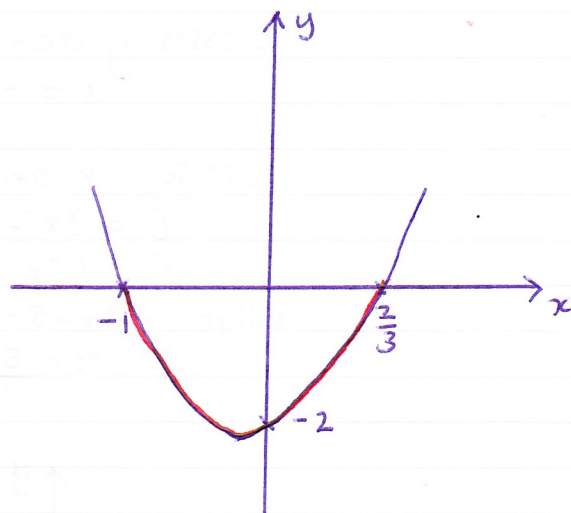
$x = -1$

Now $3x^2 + x - 2 \leq 0$

ie $y \leq 0$ —

x values for —

$-1 \leq x \leq \frac{2}{3}$



5) $4x^2 - 13x + 3 > 0$

Sketch $y = 4x^2 - 13x + 3$

$y = 0$ (x axis)

Crosses y axis, $x = 0$
 $y = 3$

Crosses x axis, $y = 0$

$0 = 4x^2 - 13x + 3$

$0 = (x - 3)(4x - 1)$

either

$x = 3$

or $x = \frac{1}{4}$

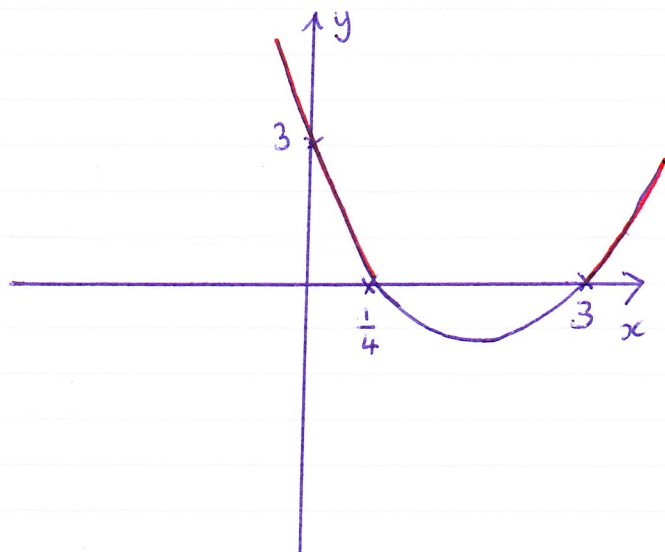
Now $4x^2 - 13x + 3 > 0$

ie $y > 0$ —

x values for —

$x < \frac{1}{4}$

or $x > 3$



$$6) \quad 3x - 2 - \frac{5}{x} < 0$$

$$\underline{x \neq 0} \quad 3x^2 - 2x - 5 < 0$$

$$\text{sketch } y = 3x^2 - 2x - 5$$

$$y = 0 \text{ (x axis)}$$

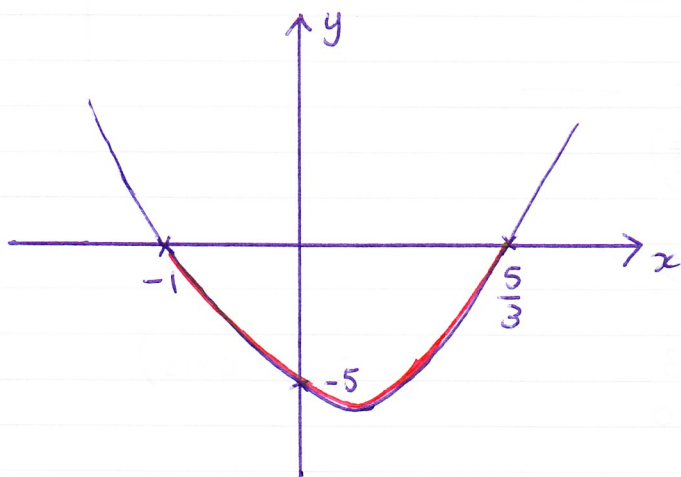
$$\text{Crosses y axis, } x = 0 \\ y = -5$$

$$\text{Crosses x axis, } y = 0$$

$$0 = 3x^2 - 2x - 5$$

$$0 = (3x - 5)(x + 1)$$

$$\text{either } 3x - 5 = 0 \text{ or } x + 1 = 0 \\ x = \frac{5}{3} \quad x = -1$$



$$\text{Now } 3x - 2 - \frac{5}{x} < 0$$

$$3x^2 - 2x - 5 < 0$$

$$\text{ie } y < 0 \text{ ---}$$

x values ---

$$\boxed{-1 < x < \frac{5}{3}}$$