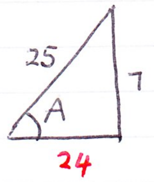
Trigonometry : 2 : Answers

1) $\sin A = \frac{7}{25}$

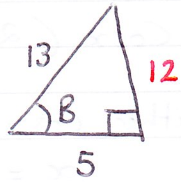
$\cos A = -\frac{24}{25}$

$\tan A = \frac{-7}{24}$



A 2ND QUAD

$\cos B = -\frac{5}{13}$



$\sin B = \frac{-12}{13}$

$\tan B = +\frac{12}{5}$

B is 3rd Quad.

a) $\sin(A+B)$

$= \sin A \cos B + \cos A \sin B$

$= \frac{7}{25}\left(\frac{-5}{13}\right) + \left(\frac{-24}{25}\right)\left(\frac{-12}{13}\right)$

$= \frac{-35}{325} + \frac{288}{325}$

$= \frac{253}{325}$

b) $\cos(A+B)$

$= \cos A \cos B - \sin A \sin B$

$= \left(\frac{-24}{25}\right)\left(\frac{-5}{13}\right) - \left(\frac{7}{25}\right)\left(\frac{-12}{13}\right)$

$= \frac{120}{325} + \frac{84}{325}$

$= \frac{204}{325}$

c) $\tan(A-B)$

$= \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$= \frac{\left(\frac{-7}{24}\right) - \left(\frac{12}{5}\right)}{1 + \left(\frac{-7}{24}\right)\left(\frac{12}{5}\right)}$

$= \left(-\frac{35}{120} - \frac{288}{120}\right) \Big/ \left(1 - \frac{84}{120}\right)$

$= \left(-\frac{323}{120}\right) \Big/ \left(\frac{\cancel{204}\ 36}{120}\right)$

$= \frac{-323}{\cancel{204}\ 36}$

d) $\sin 2A$

$= 2 \sin A \cos A$

$= 2\left(\frac{7}{25}\right)\left(\frac{-24}{25}\right)$

$= -\frac{336}{625}$

e) $\cos 2A$

$= 2\cos^2 A - 1$

$= 2\left(\frac{576}{625}\right) - 1$

$= \frac{1152}{625} - \frac{625}{625}$

$= \frac{527}{625}$

2) $\sin 2x = \cos x$

$2\sin x \cos x = \cos x$

$2\sin x \cos x - \cos x = 0$

$\cos x (2\sin x - 1) = 0$

either $\cos x = 0$ or $2\sin x - 1 = 0$

$x = 90°, 270°$

$\sin x = 1/2$

$\alpha = 30°$

Sin +ve 1st and 2nd

$x = 30°, 150°$

$x = 30°, 90°, 150°, 270°$

3) $3\sin 2\theta = 2\sin\theta$

$3(2\sin\theta\cos\theta) = 2\sin\theta$

$6\sin\theta\cos\theta - 2\sin\theta = 0$

$\div 2$ $3\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(3\cos\theta - 1) = 0$

either or $3\cos\theta - 1 = 0$

$\sin\theta = 0$

$\cos\theta = \frac{1}{3}$

$\theta = 0°, 180°, 360°$

$\alpha = 70.5$

Cos +ve 1st + 4th

$\theta = 70.5°, 289.5°$

$\theta = 0°, 70.5°, 180°, 289.5°, 360°$

4) $2\cos 2\theta = 9\cos\theta + 7$

$2(2\cos^2\theta - 1) = 9\cos\theta + 7$

$4\cos^2\theta - 2 = 9\cos\theta + 7$

$4\cos^2\theta - 9\cos\theta - 9 = 0$

$(4\cos\theta + 3)(\cos\theta - 3) = 0$

either

$4\cos\theta + 3 = 0$ or $\cos\theta - 3 = 0$

$\cos\theta = -\frac{3}{4}$

$\cos\theta = 3$

$\alpha = 41.4°$

IMPOSSIBLE

Cos -ve 2nd + 3rd

$\theta = 138.6°, 221.4°$

5) $\tan 2x = 4\tan x$

$$\frac{2\tan x}{1-\tan^2 x} = 4\tan x$$

$$2\tan x = 4\tan x\,(1-\tan^2 x)$$

$\div 2$ $\quad \tan x = 2\tan x\,(1-\tan^2 x)$

$$0 = 2\tan x(1-\tan^2 x) - \tan x$$

Common factor of $\tan x$

$$0 = \tan x\left[2(1-\tan^2 x) - 1\right]$$

either $\tan x = 0$ or $2(1-\tan^2 x) - 1 = 0$

$x = 0^\circ, 180^\circ$

$$2(1-\tan^2 x) = 1$$
$$1-\tan^2 x = \frac{1}{2}$$
$$1-\frac{1}{2} = \tan^2 x$$
$$\frac{1}{2} = \tan^2 x$$
$$\pm\frac{1}{\sqrt{2}} = \tan x$$
$$\alpha = 35{\cdot}3^\circ$$

ALL Quads

$x = 35{\cdot}3^\circ, 144{\cdot}7^\circ, \cancel{215{\cdot}3^\circ}, \cancel{324{\cdot}7^\circ}$

$\therefore x = 0^\circ, 35{\cdot}3^\circ, 144{\cdot}7^\circ, 180^\circ, \cancel{215{\cdot}3^\circ}, \cancel{324{\cdot}7^\circ}$

6) $4\cos 2\theta = 1 - 2\sin\theta$

$$4(1-2\sin^2\theta) = 1 - 2\sin\theta$$

$$4 - 8\sin^2\theta + 2\sin\theta - 1 = 0$$

$$0 = 8\sin^2\theta - 2\sin\theta - 3$$
$$0 = (4\sin\theta - 3)(2\sin\theta + 1)$$

either $4\sin\theta - 3 = 0$ or $2\sin\theta + 1 = 0$

$\sin\theta = \frac{3}{4}$ $\qquad$ $\sin\theta = -\frac{1}{2}$

$\alpha = 48{\cdot}6^\circ$ $\qquad$ $\alpha = 30^\circ$

Sin +ve 1st, 2nd $\qquad$ Sin −ve 3rd + 4th

$\theta = 48{\cdot}6^\circ, 131{\cdot}4^\circ$ $\qquad$ $\theta = 210^\circ, 330^\circ$

$\therefore \theta = 48{\cdot}6^\circ, 131{\cdot}4^\circ, 210^\circ, 330^\circ$

7) $8\cos 2x + 6 = \cos^2 x + \cos x$

$8(2\cos^2 x - 1) + 6 = \cos^2 x + \cos x$

$16\cos^2 x - 8 + 6 = \cos^2 x + \cos x$

$15\cos^2 x - \cos x - 2 = 0$

$(5\cos x - 2)(3\cos x + 1) = 0$

either $5\cos x - 2 = 0$ or $3\cos x + 1 = 0$

$\cos x = \frac{2}{5}$ $\quad$ $\cos x = -\frac{1}{3}$

$\alpha = 66.4^\circ$ $\quad$ $\alpha = 70.5^\circ$

Cos +ve 1st, 4th $\quad$ Cos −ve 2nd + 3rd

$x = 66.4^\circ, 293.6^\circ$ $\quad$ $x = 109.5^\circ, 250.5^\circ$

$\therefore x = 66.4^\circ, 109.5^\circ, 250.5^\circ, 293.6^\circ$

8) $2\sin\theta(5\cos 2\theta + 1) = 3\sin 2\theta$

$2\sin\theta\left[5(2\cos^2\theta - 1) + 1\right] = 3(2\sin\theta\cos\theta)$

$2\sin\theta\left[10\cos^2\theta - 5 + 1\right] = 6\sin\theta\cos\theta$

$2\sin\theta\left[10\cos^2\theta - 4\right] = 6\sin\theta\cos\theta$

$2\sin\theta\left[10\cos^2\theta - 4\right] - 6\sin\theta\cos\theta = 0$

$\div 2$ $\quad$ $\sin\theta\left[10\cos^2\theta - 4\right] - 3\sin\theta\cos\theta = 0$

Common factor $\sin\theta$ $\quad$ $\sin\theta\left[10\cos^2\theta - 4 - 3\cos\theta\right] = 0$

either $\sin\theta = 0$ or $10\cos^2\theta - 3\cos\theta - 4 = 0$

$\theta = 0^\circ, 180^\circ, 360^\circ$ $\quad$ $(5\cos\theta - 4)(2\cos\theta + 1) = 0$

either $5\cos\theta - 4 = 0$ or $2\cos\theta + 1 = 0$

$\cos\theta = 4/5$ $\quad$ $\cos\theta = -\frac{1}{2}$

$\alpha = 36.9^\circ$ $\quad$ $\alpha = 60^\circ$

Cos +ve 1st, 4th $\quad$ 2nd, 3rd

$\theta = 36.9^\circ, 323.1^\circ$ $\quad$ $\theta = 120^\circ, 240^\circ$

$\theta = 0^\circ, 36.9^\circ, 120^\circ, 240^\circ, 323.1^\circ, 360^\circ$

180°

9) $4\tan\theta\tan 2\theta = 1$

$4\tan\theta \times \dfrac{2\tan\theta}{(1-\tan^2\theta)} = 1$

$\times(1-\tan^2\theta)$

$8\tan^2\theta = 1(1-\tan^2\theta)$

$8\tan^2\theta = 1-\tan^2\theta$

$9\tan^2\theta - 1 = 0$

$(3\tan\theta - 1)(3\tan\theta + 1) = 0$

either $\tan\theta = \frac{1}{3}$ or $\tan\theta = -\frac{1}{3}$

$\alpha = 18{\cdot}4^\circ$ $\qquad$ $\alpha = 18{\cdot}4^\circ$

tan +ve 1st, 3rd $\qquad$ tan −ve 2nd, 4th

$\theta = 18{\cdot}4^\circ, 198{\cdot}4^\circ$ $\qquad$ $\theta = 161{\cdot}6^\circ, 341{\cdot}6^\circ$

$\theta = 18{\cdot}4^\circ, 161{\cdot}6^\circ, 198{\cdot}4^\circ, 341{\cdot}6^\circ$

10) $3\cot 2\theta + \cot\theta = 1$

$\dfrac{3(1-\tan^2\theta)}{2\tan\theta} + \dfrac{1}{\tan\theta} = 1$

NOTICE

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\therefore \cot 2\theta = \dfrac{1-\tan^2\theta}{2\tan\theta}$

$\times 2\tan\theta$

$3(1-\tan^2\theta) + 2 = 2\tan\theta$

$3 - 3\tan^2\theta + 2 = 2\tan\theta$

$0 = 3\tan^2\theta + 2\tan\theta - 5$

$0 = (3\tan\theta + 5)(\tan\theta - 1)$

either $3\tan\theta + 5 = 0$ or $\tan\theta - 1 = 0$

$\tan\theta = -5/3$ $\qquad$ $\tan\theta = 1$

$\alpha = 45^\circ$

$\alpha = 59{\cdot}0^\circ$ $\qquad$ tan +ve 1st, 3rd

tan −ve 2nd, 4th $\qquad$ $\theta = 45^\circ, 225^\circ$

$\theta = 121^\circ, 301^\circ$

$\theta = 45^\circ, 121^\circ, 225^\circ, 301^\circ$