Trigonometry : 3 : Proofs : Solutions

1) $$\frac{\cos 2\theta}{\cos\theta + \sin\theta} \equiv \cos\theta - \sin\theta$$

~~LHS =~~ Easiest to say we need to prove

$$\cos 2\theta \equiv (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$$

$\text{LHS} \equiv \cos^2\theta - \sin^2\theta$ $\qquad$ $\text{RHS} \equiv \cos^2\theta - \sin^2\theta$

$\therefore$ LHS = RHS $\therefore$ Proven $\qquad$ QED.

2) $$\tan A + \cot A \equiv 2\operatorname{cosec} 2A$$

$$\text{LHS} \equiv \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \qquad \text{RHS} \equiv \frac{2}{\sin 2A}$$

$$\equiv \frac{\sin^2 A + \cos^2 A}{\cos A \sin A} \qquad \equiv \frac{2}{2\sin A\cos A}$$

$$\equiv \frac{1}{\cos A\sin A} \qquad = \frac{1}{\sin A\cos A}$$

$\therefore$ LHS = RHS

QED.

3)
$$\begin{aligned}
\sin 3A &= \sin(A + 2A) \\
&= \sin A\cos 2A + \cos A\sin 2A \\
&= \sin A(1 - 2\sin^2 A) + \cos A(2\sin A\cos A) \\
&= \sin A - 2\sin^3 A + 2\sin A\cos^2 A \\
&= \sin A - 2\sin^3 A + 2\sin A(1 - \sin^2 A) \\
&= \sin A - 2\sin^3 A + 2\sin A - 2\sin^3 A \\
&= 3\sin A - 4\sin^3 A
\end{aligned}$$

QED.

4) $\cos 3A = \cos(2A + A)$

$= \cos 2A \cos A - \sin 2A \sin A$

$= (2\cos^2 A - 1)\cos A - (2\sin A \cos A)\sin A$

$= 2\cos^3 A - \cos A - 2\sin^2 A \cos A$

$= 2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A$

$= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$

$= 4\cos^3 A - 3\cos A$

QED

5) $\tan\theta + \cot\theta \equiv \dfrac{1}{\sin\theta\cos\theta}$

$$\text{LHS} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$\equiv \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$\equiv \frac{1}{\sin\theta\cos\theta}$$

$= \text{RHS}$ QED

6) $(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1) \equiv 1$

$$\text{LHS} \equiv \sec^2\theta\operatorname{cosec}^2\theta - \sec^2\theta - \operatorname{cosec}^2\theta + 1$$

$$\equiv \frac{1}{\cos^2\theta} \times \frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta} - \frac{1}{\sin^2\theta} + 1$$

$$\equiv \frac{1}{\cos^2\theta\sin^2\theta} - \frac{\sin^2\theta}{\cos^2\theta\sin^2\theta} - \frac{\cos^2\theta}{\cos^2\theta\sin^2\theta} + 1$$

$$\equiv \frac{1 - \sin^2\theta - \cos^2\theta}{\cos^2\theta\sin^2\theta} + 1$$

$$\equiv \frac{1 - (\sin^2\theta + \cos^2\theta)}{\cos^2\theta\sin^2\theta} + 1$$

$$\equiv \frac{1 - 1}{\cos^2\theta\sin^2\theta} + 1 \equiv 0 + 1 = \text{RHS}$$

QED

7) $(\sec x + \tan x)(\sec x - \tan x) \equiv 1$

$$\begin{aligned} LHS &\equiv \sec^2 x - \sec x \tan x + \sec x \tan x - \tan^2 x \\ &\equiv \sec^2 x - \tan^2 x \\ &\equiv 1 + \tan^2 x - \tan^2 x \\ &\equiv 1 \\ &\equiv RHS \qquad QED. \end{aligned}$$

8) $\sec\theta + \operatorname{cosec}\theta \cot\theta \equiv \sec\theta \operatorname{cosec}^2\theta$

$$LHS \equiv \frac{1}{\cos\theta} + \frac{1}{\sin\theta} \times \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin^2\theta}$$

$$= \frac{1}{\cos\theta \sin^2\theta}$$

$$RHS \equiv \frac{1}{\cos\theta} \times \frac{1}{\sin^2\theta}$$

$$\equiv \frac{1}{\cos\theta \sin^2\theta}$$

$\therefore$ LHS $\equiv$ RHS QED.