Trigonometry 6 : Arcs and Sectors Answers

Jan 2012
C2

(9)

a)

Sector AOB

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2} \times 5^2 \times \theta$

$A = \frac{25\theta}{2}$

Sector BOC

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2} \times 5^2 \times \phi$

$A = \frac{25\phi}{2}$

$\therefore \quad \frac{25\theta}{2} + \frac{25\phi}{2} = 22.5$

$\times 2 \quad 25\theta + 25\phi = 45$

$\div 25 \quad \theta + \phi = \frac{45}{25}$

$\theta + \phi = 1.8$ — (1)

b)

AB

$S = r\theta$

$S = 5\theta$

BC

$S = r\theta$

$S = 5\phi$

$\therefore \quad 5\theta = 5\phi + 3.5$

$10\theta = 10\phi + 7$ — (2)

(1) $\Rightarrow \quad \theta = 1.8 - \phi$

sub into (2)

(2) $\Rightarrow \quad 10(1.8 - \phi) = 10\phi + 7$

$18 - 10\phi = 10\phi + 7$

$11 = 20\phi$

$\frac{11}{20} = \phi$

$0.55^c = \phi$

(1) $\Rightarrow \quad \theta = 1.8 - 0.55$

$\theta = 1.25^c$

MAY 2017

C2 (9) a) AB

$s = r\theta$

$s = R\theta$

CD

$s = r\theta$

$s = r\theta$

(i) Now $R\theta + r\theta = L$ (1)

(ii) Area $\hat{AOB}$

$A = \frac{1}{2}R^2\theta$

Area $\hat{COD}$

$A = \frac{1}{2}r^2\theta$

$\therefore$ Area shaded

$k = \frac{1}{2}R^2\theta - \frac{1}{2}r^2\theta$

$k = \frac{\theta}{2}(R^2 - r^2)$ (2)

b) Now $AC = x$

$\therefore R - r = x$

From (2) $k = \frac{\theta}{2}(R-r)(R+r)$

$k = \frac{\theta}{2}(x)(R+r)$

But also from (1)

$\theta(R+r) = L$

$(R+r) = \frac{L}{\theta}$

$\therefore k = \frac{x}{2}L$

May 2011
C2

PQ

$S = r\theta$

$7.6 = r\theta$ *

POQ

$A = \frac{1}{2}r^2\theta$

$36.1 = \frac{r^2\theta}{2}$

$72.2 = r^2\theta$

sub * into above

$72.2 = r(r\theta)$

$72.2 = r(7.6)$

$\frac{72.2}{7.6} = r$

$9.5\text{cm} = r$

$\therefore$ *

$7.6 = 9.5\theta$

$\frac{76}{95} = \theta$

$0.8^c = \theta$

May 2012
C2

(9) Area /// = area $\overset{\text{sector}}{AOQ}$ − area $\triangle POQ$

$10.35 = \frac{1}{2}r^2(1.12) - \frac{1}{2}ab\sin C$

$10.35 = 0.56r^2 - \frac{1}{2}(r)(r)\sin 1.12$

$10.35 = 0.56r^2 - 0.45r^2$

$10.35 = 0.11r^2$

$\frac{1035}{11} = r^2$

$94.09 = r^2$

$9.7\text{cm} = r$

Jan 2014
C2 (9)

Area /// = area sector AOB − area sector COD

$$23.1 = \frac{1}{2} \times 7^2 \times \theta - \frac{1}{2} \times 4^2 \times \theta$$

$$23.1 = 24.5\theta - 8\theta$$

$$23.1 = 16.5\theta$$

$$\frac{23.1}{16.5} = \theta$$

$$1.4^c = \theta$$

Arc AB

$S = r\theta$

$S = 7(1.4)$

$S = 9.8\text{ cm}$

Arc CD

$S = r\theta$

$S = 4(1.4)$

$S = 5.6\text{ cm}$

AC

$AC = 7 - 4$

$AC = 3\text{cm}$

BD

$BD = 3\text{cm}$

∴ Perimeter ACDB = 9.8 + 5.6 + 3 + 3

= 21.4 cm

June 2019
UNIT 3
(12)

Area small segment = Area sector AOB − area △AOB

$$= \frac{1}{2}r^2\theta - \frac{1}{2}ab\sin C$$

$$= \frac{1}{2}r^2\theta - \frac{1}{2}r(r)\sin\theta$$

$$= \frac{1}{2}r^2(\theta - \sin\theta)$$

Area large segment = circle − small segment

$$= \pi r^2 - \frac{1}{2}r^2(\theta - \sin\theta)$$

∴ $2 \times \frac{1}{2}r^2(\theta - \sin\theta) = \pi r^2 - \frac{r^2}{2}(\theta - \sin\theta)$

$\div r^2$ $\quad \theta - \sin\theta = \pi - \frac{1}{2}(\theta - \sin\theta)$

$$\theta - \sin\theta = \pi - \frac{\theta}{2} + \frac{\sin\theta}{2}$$

$$\frac{3\theta}{2} - \pi = \frac{3\sin\theta}{2}$$

$\times 2 \div 3$ $\quad \theta - \frac{2\pi}{3} = \sin\theta$

May 2016
C2

Sector BOC

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2} \times r^2 \times (\pi - 2.15)$

$A = \frac{r^2}{2}(\pi - 2.15)$

Sector AOB

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2} \times r^2 \times 2.15$

$A = 1.075r^2$

NOW

$\frac{r^2}{2}(\pi - 2.15) + 26 = 1.075r^2$

$\frac{0.992r^2}{2} + 26 = 1.075r^2$

$0.496r^2 + 26 = 1.075r^2$

$26 = 1.075r^2 - 0.496r^2$

$26 = 0.579r^2$

$\frac{26}{0.579} = r^2$

$44.91 = r^2$

$6.7\text{cm} = r$

Jan 2013
C2

(a) Sector AOB

$A = \frac{1}{2}r^2\theta$

$43.56 = \frac{1}{2} \times 11 \times 11 \times \theta$

$\frac{43.56 \times 2}{11 \times 11} = \theta$

$0.72^c = \theta$

(b) BC

$S = r\theta$

$S = 11\phi$

CD

$S = r\theta$

$S = 11(\pi - \phi)$

$\therefore \quad 11\phi + 13 = 11(\pi - \phi)$

$11\phi + 13 = 11\pi - 11\phi$

$22\phi = 11\pi - 13$

$22\phi = 21.562$

$\phi = 21.562/22$

$\phi = 0.98^c$