2005 8. The size P of a population of bacteria at time t days is to be modelled as a continuous variable such that the rate of increase of P is directly proportional to P.

(a) Write down a differential equation that is satisfied by P. [1]

(b) Given that the initial size of the population is P_0, show that $P = P_0\,e^{kt}$, where k is a positive constant. [5]

(c) Two days after the start, the population is $1{\cdot}2P_0$. Find when the population will be $2P_0$. [4]

2006 8. Water leaks from a hole at the bottom of a large water tank. The depth of the water at time t minutes is x metres. The rate of decrease of x is directly proportional to $\sqrt{x}$.

(a) Write down a differential equation that is satisfied by x. [1]

(b) Given that the depth of water in the tank when $t = 0$ is 9 metres, show that

$$kt = 6 - 2\sqrt{x},$$

where k is a positive constant. [4]

(c) Given that the depth of water in the tank is 4 metres when $t = 20$, find the time taken for the tank to empty. [3]

2007 8. The price £P of an item at time t years is to be modelled as a continuous variable such that the rate of increase of P is directly proportional to P.

(a) Write down a differential equation that is satisfied by P. [1]

(b) Given that the price of the item at $t = 0$ is £50, show that $P = 50e^{kt}$, where k is a positive constant. [5]

(c) After seven years the price of the item is £65. Find the price of the item after sixteen years. [4]

2008 7. A neglected large lawn contains a certain type of weed. The area of the lawn covered by the weed at time t years is $W\,\text{m}^2$. The rate of increase of W is directly proportional to W.

(a) Write down a differential equation that is satisfied by W. [1]

(b) The area of the lawn covered by the weed initially is $0{\cdot}10\,\text{m}^2$ and one year later the area covered is $2{\cdot}01\,\text{m}^2$. Find an expression for W in terms of t. [6]

2009 7. The value of an electronic component may be modelled as a continuous variable. The value of the component at time t years is £P. The rate of decrease of P is directly proportional to P^3.

(a) Write down a differential equation that is satisfied by P. [1]

(b) The value of the component when $t = 0$ is £20. Show that

$$\frac{1}{P^2} = \frac{1}{400} + At,$$

where A is a positive constant. [5]

(c) Given that the value of the component when $t = 1$ is £10, find the time when the value is £5. [4]