C3 June 2005

6. *(a)* Differentiate each of the following with respect to x and simplify your answers.

(i) e^{2x-5} (ii) $x^2 \ln x$ (iii) $(3x^2 + 2)^4$ [8]

(b) By first writing $\tan x = \dfrac{\sin x}{\cos x}$, show that $\dfrac{d}{dx}(\tan x) = \sec^2 x$. [3]

(c) By first writing $y = \tan^{-1} x$ as $x = \tan y$, show that $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1 + x^2}$. [3]

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6. Differentiate the following with respect to x, simplifying your answers as much as possible.

(a) $e^{2x} \sin x$

(b) $\dfrac{2x^2 - 4}{x^2 + 3}$

(c) $\tan (4x^2 + 3)$

[4], [3], [2]

8. *(a)* Given that $y = \tan^{-1} x$, show that

$$\frac{dy}{dx} = \frac{1}{x^2 + 1}.$$ [3]

(b) Differentiate $\ln(x^2 + 1)$ with respect to x. [2]

(c) Use the results derived in *(a)* and *(b)* to find

$$\int \frac{3 + x}{1 + x^2} dx.$$ [4]