C4 Proof

P3 June 2001

9. Given that $2n^3 + 3n$ is odd, where n is an integer, use **proof by contradiction** to show that n is odd. [4]

P3 June 2002

10. Complete the following proof by contradiction to show that $\sqrt{5}$ is irrational.

Assuming that $\sqrt{5}$ is rational, let

$$\sqrt{5} = \frac{a}{b},$$

where a and b are integers that have no common factors.
Then $5b^2 = a^2 \Rightarrow 5$ is a factor of a^2.

[5]

P3 June 2003

10. Complete the following proof by contradiction to show that $x + \frac{9}{x} \geqslant 6$ when x is real and positive.

Assume that $x + \frac{9}{x} < 6$. Since $x > 0$, $x^2 + 9 < 6x$. [4]

P3 June 2004

10. Complete the following proof by contradiction to show that $\sqrt{2}$ is irrational.

Assuming that $\sqrt{2}$ is rational, let $\sqrt{2} = \frac{a}{b}$, where a and b are integers that have no common factor.

Squaring both sides, we have $2 = \frac{a^2}{b^2}$.

$$\therefore 2b^2 = a^2$$

which implies 2 is a factor of a^2. Then 2 is a factor of a. [4]

C4 June 2005

10. Complete the following proof by contradiction to show that $x + \frac{25}{x} \geqslant 10$ when x is real and positive.

Assume that $x + \frac{25}{x} < 10$, when x is real and positive.

Since x is positive, multiplication of both sides of the inequality by x gives $x^2 + 25 < 10x$. [4]