PROOF (10)

05 **10.** Complete the following proof by contradiction to show that $x + \dfrac{25}{x} \geqslant 10$ when x is real and positive.

Assume that $x + \dfrac{25}{x} < 10$ *when x is real and positive.*

Since x is positive, multiplication of both sides of the inequality by x gives $x^2 + 25 < 10x$. [4]

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006 **11.** Complete the following proof by contradiction to show that $\sqrt{2}$ is irrational.

Assume that $\sqrt{2}$ is rational. Then $\sqrt{2}$ may be written in the form $\dfrac{a}{b}$, where a and b are positive integers having no common factor.

$\therefore$ $a^2 = 2b^2$.

$\therefore$ *a^2 has a factor 2.*

$\therefore$ *a has a factor 2 so that $a = 2k$,*

where k is an integer. [4]

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007 **10.** Complete the following proof by contradiction to show that, if n is a positive integer and $3n + 2n^3$ is odd, then n is odd. [2]

It is given that $3n + 2n^3$ is odd.
Assume that n is even so that $n = 2k$.

2008 **10.** Prove by contradiction the following proposition.

When x is real and positive,

$$x + \frac{49}{x} \geqslant 14 .$$

The first line of the proof is given below.

Assume that there is a positive and real value of x such that

$$x + \frac{49}{x} < 14 .$$ [4]

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09 **10.** Complete the following proof by contradiction to show that $\sqrt{3}$ is irrational.

Assume that $\sqrt{3}$ is rational. Then $\sqrt{3}$ may be written in the form $\dfrac{a}{b}$ where a and b are integers having no common factors.

$\therefore$ $a^2 = 3b^2$.
$\therefore$ a^2 has a factor 3.
$\therefore$ a has a factor 3 so that $a = 3k$, where k is an integer. [4]