YEAR 12 MOCK : ANSWERS

1) a) $\dfrac{(3\sqrt{3}+1)}{(5\sqrt{3}-7)} \times \dfrac{(5\sqrt{3}+7)}{(5\sqrt{3}+7)}$

$$= \frac{45 + 21\sqrt{3} + 5\sqrt{3} + 7}{75 + \cancel{35\sqrt{3}} - \cancel{35\sqrt{3}} - 49}$$

$$= \frac{52 + 26\sqrt{3}}{26}$$

$$= 2 + \sqrt{3}$$

b) $(\sqrt{12} \times \sqrt{24}) + \dfrac{\sqrt{150}}{\sqrt{3}} - \dfrac{36}{\sqrt{2}}$

$\sqrt{12} \times \sqrt{24}$

$= 2\sqrt{3} \times \sqrt{8}\sqrt{3}$

$= 2\sqrt{8} \times 3$

$= 6\sqrt{8}$

$= 6 \times 2\sqrt{2}$

$= 12\sqrt{2}$

$\dfrac{\sqrt{150}}{\sqrt{3}}$

$= \sqrt{50}$

$= 5\sqrt{2}$

$\dfrac{36}{\sqrt{2}}$

$= \dfrac{36}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{36\sqrt{2}}{2}$

$= 18\sqrt{2}$

$\therefore$ Answer

$= 12\sqrt{2} + 5\sqrt{2} - 18\sqrt{2}$

$= -\sqrt{2}$

2) a) x^2+4x-8

$= (x+2)^2 - 4 - 8$

$= (x+2)^2 - 12$

$a = 2 \quad b = -12$

b) $y = x^2 + 4x - 8 \quad — (1)$

$y = 2x+7 \quad — (2)$

sub (2) into (1)

$2x+7 = x^2+4x-8$

$0 = x^2+2x-15$

$0 = (x+5)(x-3)$

either

$x+5=0$ or $x-3=0$

$x=-5 \qquad x=3$

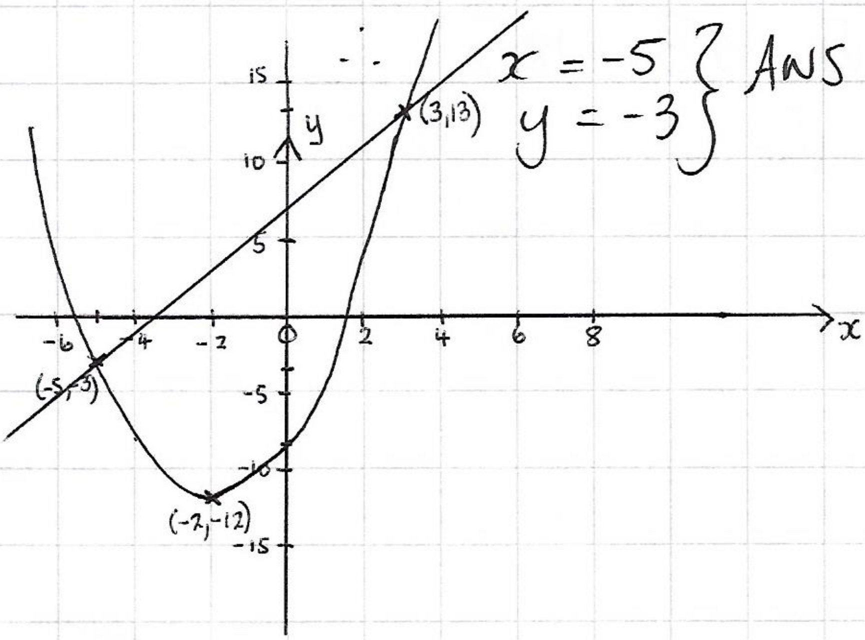
$y = 2(-5)+7 \qquad y = 2(3)+7$

$y = -10+7 \qquad y = 6+7$

$y = -3 \qquad y = 13$

$\left.\begin{array}{l} x=-5 \\ y=-3 \end{array}\right\}$ ANS $\qquad \left.\begin{array}{l} x=3 \\ y=13 \end{array}\right\}$ ANS.

c)



$y = x^2+4x-8$

Crosses x axis $y=0$

$0 = x^2+4x-8$

$0 = \cancel{(x-2)}$ No factorising

3) $(k-1)x^2 + 2kx + (7k-4) = 0$

$b^2 - 4ac < 0$ for no real roots

$$(2k)^2 - 4(k-1)(7k-4) < 0$$

$$4k^2 - 4\left[7k^2 - 4k - 7k + 4\right] < 0$$

$$4k^2 - 28k^2 + 16k + 28k - 16 < 0$$

$$-24k^2 + 44k - 16 < 0$$

$$0 < 24k^2 - 44k + 16$$

$$0 < 6k^2 - 11k + 4$$

$$6k^2 - 11k + 4 > 0$$

$$(3k-4)(2k-1) > 0$$

either $(+) \times (+) \Rightarrow (+)$

or $(-) \times (-) \Rightarrow (+)$

either

$3k - 4 > 0$ and $2k - 1 > 0$

$k > 4/3$ $\quad$ $k > 1/2$

$$k > 4/3$$

or $3k - 4 < 0$ and $2k - 1 < 0$

$k < 4/3$ $\quad$ $k < 1/2$

$$k < 1/2$$

4) $2x^2 + 11x + 12 \geq 0$

$(2x + 3)(x + 4) \geq 0$

either $(+) \times (+) \Rightarrow (+)$

or $(-) \times (-) \Rightarrow (+)$

either $2x+3 \geq 0$ and $x + 4 \geq 0$

$x \geq -3/2$ $x \geq -4$

$x \geq -3/2$

OR

$2x + 3 \leq 0$ and $x + 4 \leq 0$

$x \leq -\frac{3}{2}$ $x \leq -4$

$x \leq -4$

5) Let $y = 9x^{5/4} - 8x^{-1/3}$

$$\frac{dy}{dx} = 9\left(\frac{5}{4}\right)x^{1/4} - 8\left(-\frac{1}{3}\right)x^{-4/3}$$

$$= \frac{45}{4}x^{1/4} + \frac{8}{3\sqrt[3]{x^4}}$$

6). $y = x^2 - 8x + 14$

a) $\frac{dy}{dx} = 2x - 8$

When $x = 6$ at $(6, 2)$

$$\frac{dy}{dx} = 2(6) - 8 = 4$$

$\therefore$ $m = 4$ for tangent at $(6,2)$

$\therefore$ gradient of normal at $(6,2)$ $= \frac{-1}{4}$

because

$$\left(-\frac{1}{4}\right) \times 4 = -1$$

$\therefore$ Equation of normal at $(6, 2)$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{1}{4}(x - 6)$$

$\times 4$ $\quad 4y - 8 = -1(x - 6)$

$$4y - 8 = -x + 6$$

$$4y + x = 14$$

b) $\frac{dy}{dx} = 2x - 8$

gradient value

at Q $\frac{dy}{dx} = 2$ because we know $y = 2x + c$

$\therefore$ $2x - 8 = 2$

$2x = 10$

$\underline{x = 5}$

$\therefore$ $y = 5^2 - 8(5) + 14$

$y = 25 - 40 + 14$

$\underline{y = -1}$

$Q(5, -1)$

$\therefore$ For $y = 2x + c$

$-1 = 10 + c$ $\qquad -11 = c$

7) a) $y = -5x^2 - 7x + 13$

$\frac{dy}{dx} = -10x - 7$

b) $y = 6x^{3/4} + 5x^{-3} - 9$

$\frac{dy}{dx} = 6\left(\frac{3}{4}\right)x^{-1/4} - 15x^{-4} + 0$

$= \frac{9}{2}x^{-1/4} - \frac{15}{x^4}$

8) (a) $(x-3)$ is a factor of $px^3 - 13x^2 - 19x + 12$

$\therefore f(3) = (3^3)p - 13(3^2) - 19(3) + 12 = 0$

$27p - 117 - 57 + 12 = 0$

$27p = 162$

$p = \frac{162}{27}$

$p = 6$ QED

(b) $6x^3 - 13x^2 - 19x + 12 = 0$ USE COEFFICIENTS METHOD

LHS

$6x^3 - 13x^2 - 19x + 12 = (x-3)(ax^2 + bx + c)$

Compare x^3

$6 = a$

Compare x^2

$-13 = b - 3a$

$-13 = b - 18$

$5 = b$

Compare consts

$12 = -3c$

$-4 = c$

$\therefore$ Eqn becomes $(x-3)(6x^2 + 5x - 4) = 0$

$(x-3)(3x+4)(2x-1) = 0$

either $x - 3 = 0$ or $3x + 4 = 0$ or $2x - 1 = 0$

$x = 3$ $x = -\frac{4}{3}$ $x = \frac{1}{2}$

9) $6x^3 - 13x^2 + 4 = 0$

Try $f(1) = 6 - 13 + 4 \neq 0$

$f(-1) = -6 - 13 + 4 \neq 0$

$f(2) = 48 - 52 + 4 = 0$

$\therefore (x-2)$ is a factor

USE LONG DIVISION METHOD

$$
\begin{array}{r|l}
 & 6x^2 - x - 2 \\
\hline
x-2 & 6x^3 - 13x^2 + 0x + 4 \\
 & \underline{- \; 6x^3 \mp 12x^2} \\
 & \qquad -x^2 + 0x \\
 & \qquad \underline{\pm x^2 \mp 2x} \\
 & \qquad\qquad -2x + 4 \\
 & \qquad\qquad \underline{\pm 2x \mp 4} \\
 & \qquad\qquad\quad 0 + 0
\end{array}
$$

$\therefore$ Eqn becomes

$$(x-2)(6x^2 - x - 2) = 0$$

$$(x-2)(3x-2)(2x+1) = 0$$

either

$x - 2 = 0$ or $3x - 2 = 0$ or $2x + 1 = 0$

$x = 2$ $\quad x = \frac{2}{3}$ $\quad x = -\frac{1}{2}$

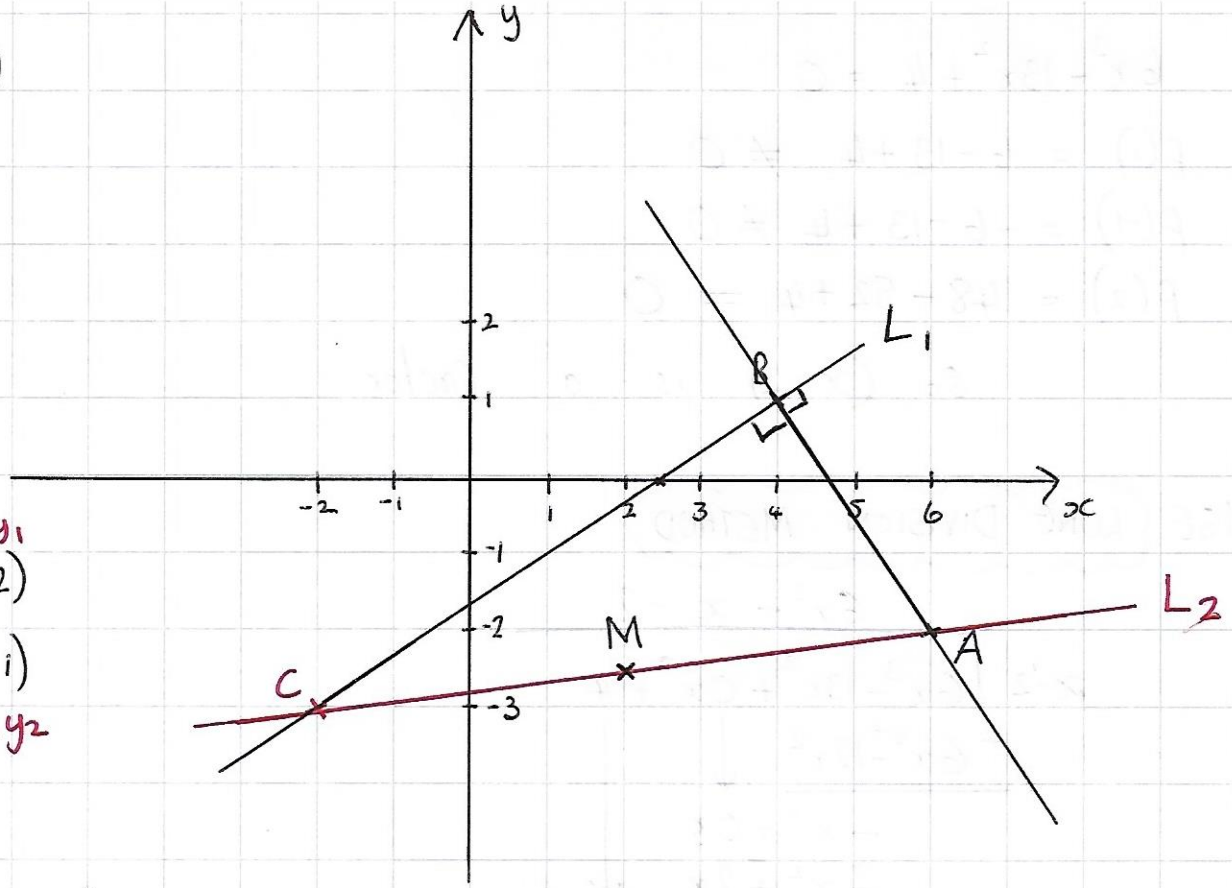
10)

$x_1 \; y_1$

$A(6,-2)$

$B(4,1)$

$x_2 \; y_2$



a) (i) <u>AB</u>

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1-(-2)}{4-6} = \frac{3}{-2} = -\frac{3}{2}$$

(ii) <u>L_1</u> is perpendicular to AB pass through $B(4,1)$

$m = +\frac{2}{3}$ because $\left(-\frac{3}{2}\right) \times \frac{2}{3} = -1$

$B\ (4,1)$

Eqn L_1

$$y - y_2 = m(x - x_2)$$

$$y - 1 = \frac{2}{3}(x-4)$$

$$\times 3 \quad 3y - 3 = 2(x-4)$$

$$3y - 3 = 2x - 8$$

$$3y = 2x - 5$$

b) (i) $\underline{L_2}$ $\qquad\qquad$ $\underline{L_1}$

$$x - 8y - 22 = 0 \quad -(1) \qquad 3y = 2x - 5 \quad -(2)$$

L_1 and L_2 intersection.

Solve (1) and (2) simultaneously

$$(1) \Rightarrow x = 8y + 22 \qquad (*)$$

Sub into (2)

$$3y = 2(8y + 22) - 5$$
$$3y = 16y + 44 - 5$$
$$-39 = 13y$$
$$-3 = y$$

from $(*)$ $\quad x = 8(-3) + 22$

$$x = -24 + 22$$
$$x = -2$$

$$\therefore \quad C(\overset{x_3}{-2}, \overset{y_3}{-3})$$

(ii) Mid Point AC

$$M\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$$
$$= M\left(\frac{6 + (-2)}{2}, \frac{-2 + (-3)}{2}\right)$$
$$= M\left(2, -\tfrac{5}{2}\right)$$

(iii) $\underline{\text{Triangle ABC}}$

$$\text{Area} = \frac{bh}{2}$$
$$A = \frac{AB \times BC}{2}$$
$$A = \frac{\sqrt{13} \times 2\sqrt{13}}{2}$$
$$\text{Area} = 13 \text{ units}^2$$

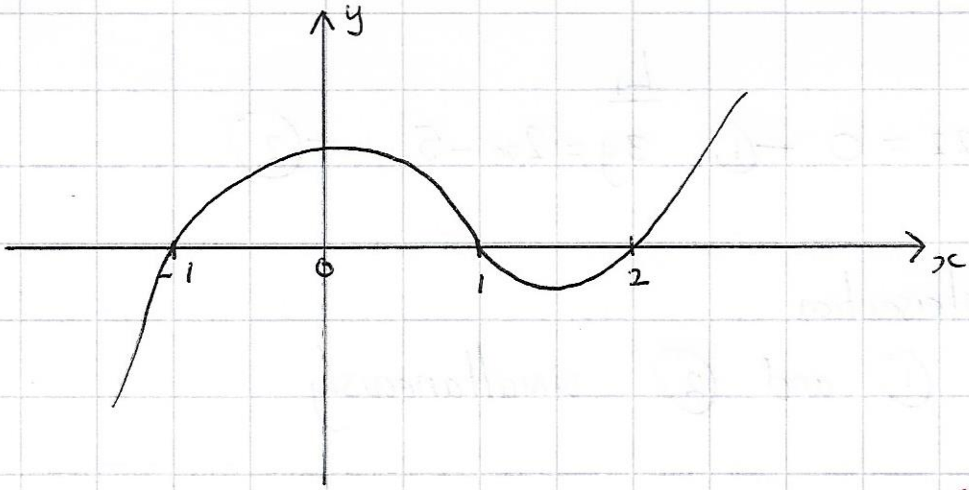
$\underline{AB}$

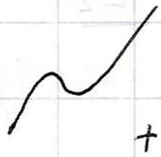
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4 - 6)^2 + (1 + 2)^2}$$
$$= \sqrt{4 + 9}$$
$$= \sqrt{13}$$

$\underline{BC}$

$$BC = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$$
$$= \sqrt{(-2 - 4)^2 + (-3 - 1)^2}$$
$$= \sqrt{36 + 16}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13}$$

11)





+ve x^3

curve crosses
x axis
at
$x=-1 \quad x=1 \quad x=2$

Equation of curve must be

$$y=(x-2)(x-1)(x+1)$$
$$y=(x-2)(x^2-1)$$
$$y=x^3-x-2x^2+2$$
$$y=x^3-2x^2-x+2$$