Year 13 A Level Mathematics: Pure

Teacher : Mr Lunt

Answer all questions

Question	Maximum Mark	Actual Mark
1	7	
2	5	
3	6	
4	5	
5	3	
6	6	
7	17	
8	4	
9	10	
10	4	
11	9	
12	5	
13	10	
14	6	
Total	97	

1. Given that

$$f(x) = \frac{2x^2 + 4}{(x-2)^2(x+4)},$$

(a) express $f(x)$ in partial fractions, [4]

(b) hence find the value of $f'(0)$. [3]

2. Find the equation of the normal to the curve

$$2x^3 + 6xy^2 - y^4 = 27$$

at the point $(2, 1)$. [5]

3. Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

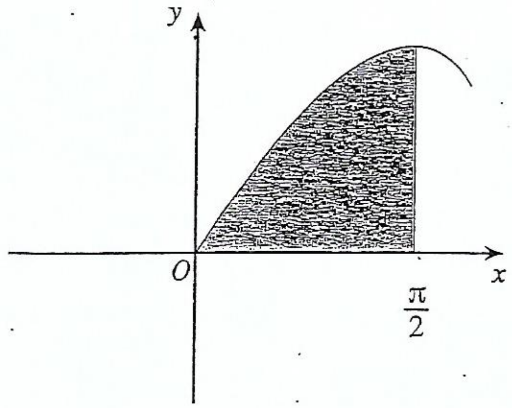
$$2 + 3\cos 2\theta = \cos\theta.$$ [6]

4. (a) Express $4\sin x + 3\cos x$ in the form $R\sin(x + \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$. [3]

(b) Hence find the greatest value of

$$\frac{1}{4\sin x + 3\cos x + 7}.$$ [2]

5.

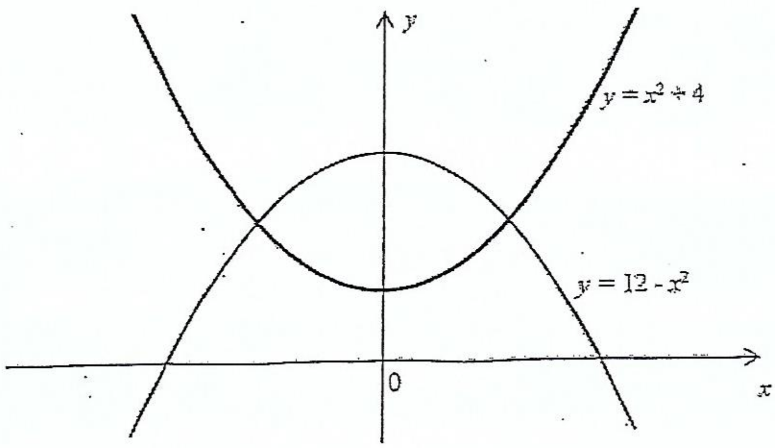


[3]

The diagram shows the shaded region bounded by the curve $y = \sin x$, the x-axis and the line $x = \frac{\pi}{2}$.

Calculate the area shaded.

6.



The diagram above shows a sketch of the curves $y = x^2 + 4$ and $y = 12 - x^2$.

- Find the area of the region bounded by the two curves. [6]

7. The curve $y = ax^4 + bx^3 + 18x^2$ has a point of inflection at (1, 11).

(a) Show that $2a + b + 6 = 0$. [3]

(b) Find the values of the constants a and b and show that the curve has another point of inflection at (3, 27). [8]

(c) Sketch the curve, identifying all the stationary points including their nature. [6]

8. Given that

$$x^2 + x\sin y + y^3 = \pi^3 + 1,$$

find the value of $\frac{dy}{dx}$ at the point $(1, \pi)$. [4]

9. (a) Find $\int xe^{-x}dx$. [4]

(b) Use the substitution $u = 2x - 1$ to evaluate

$$\int_{\frac{1}{2}}^{1} x(2x-1)^9\,dx.$$ [6]

10. (a) Show that $f(x) = \sin^{-1}x - 2x^{\frac{3}{2}} + 1$ has a stationary value when x satisfies

$$9x^3 - 9x + 1 = 0.$$ [4]

11. The parametric equations of the curve C are

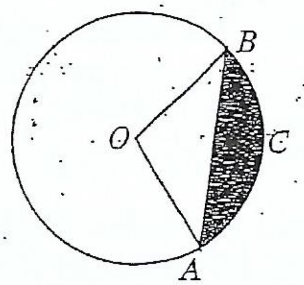
$$x = \frac{1}{t}, \quad y = t^2 .$$

(a) Show that the tangent to C at the point P with parameter p has equation

$$y + 2p^3x - 3p^2 = 0. \qquad [4]$$

(b) The tangent to C at the point P intersects the x-axis at A and the y-axis at B. Show that $PB = 2PA$. [5]

12.



The diagram shows a circle of centre O and radius r cm. The radii OA and OB are such that $A\hat{O}B = 0{\cdot}8$ radians. The length of the minor arc ACB is 4 cm.

(a) Calculate the value of r. [2]

(b) Calculate the area of the shaded segment. [3]

13. (a) Differentiate each of the following with respect to x.

(i) $\sqrt{5x^2 - 3x}$ (ii) $\sin^{-1} 7x$ (iii) $e^{3x}\ln x$ [7]

(b) By first writing $\cot x = \dfrac{\cos x}{\sin x}$, show that $\dfrac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$. [3]

14. (a) Find

(i) $\displaystyle\int \cos\left(\frac{4x+5}{3}\right) dx$, (ii) $\displaystyle\int e^{2x+9}\, dx$, (iii) $\displaystyle\int \frac{3}{(7-2x)^6}\, dx$. [6]