CIRCLES 1 : Answers

85) C $x^2+y^2+2x-8y+8=0$

a) $x^2+2x+y^2-8y=-8$

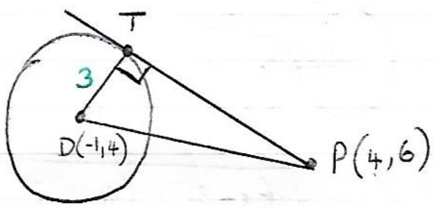
$(x+1)^2-1+(y-4)^2-16=-8$

$(x+1)^2+(y-4)^2=9$

$\therefore$ Centre $D(-1,4)$

radius $=3$

b) (i) A general sketch



$DT = 3 = $ radius

Find $DP = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(4+1)^2+(6-4)^2}$

$= \sqrt{25+4}$

$= \sqrt{29}$ units

$\therefore$ from $\triangle DPT$ $\quad h^2=a^2+b^2$ $\quad$ using Pythag

$29 = 3^2+PT^2$

$20 = PT^2$

$\sqrt{20} = PT$

(ii) Circle needed

centre $(4,6)$ radius $=\sqrt{20}$

$(x-4)^2+(y-6)^2=20$

or $\quad x^2-8x+16+y^2-12y+36=20$

$x^2+y^2-8x-12y+32=0$

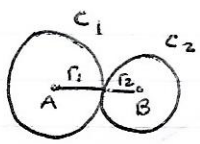
8b) $C_1 \quad x^2+y^2-6x+8y-75=0$

a)
$$x^2-6x+y^2+8y = 75$$
$$(x-3)^2-9+(y+4)^2-16 = 75$$
$$(x-3)^2+(y+4)^2 = 100$$
centre $A\ (3,-4)$

radius $= 10$

b) C_2 centre $B(-6,8)$ radius $= 5$

(i) C_1 and C_2 will touch if $r_1 + r_2 =$ dist apart of centres. AB



$$\therefore AB = \sqrt{(-6-3)^2+(8+4)^2}$$
$$= \sqrt{81+144}$$
$$= \sqrt{225}$$
$$= 15$$

Now $r_1 + r_2 = 10 + 5 = 15$

$\therefore r_1 + r_2 = AB$

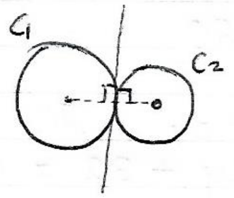
$\therefore C_1$ and C_2 touch

(ii) Gradient of radius line AP

$A\ (3,-4)$ $\quad P\ (-3,4)$

$x_1\ y_1 \qquad x_2\ y_2$

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{4+4}{-3-3} = -\frac{8}{6} = -\frac{4}{3}$$



$\therefore$ Gradient of tangent $= +\frac{3}{4}$

because it is $\perp$ to radius and $\left(-\frac{4}{3}\right) \times \frac{3}{4} = -1$

$\therefore$ Eqn tangent is
$$y - y_2 = m(x-x_2)$$
$$y-4 = \frac{3}{4}(x+3)$$
$$4y-16 = 3x+9$$
$$4y = 3x+25$$

87) a) $x^2 + y^2 + 4x - 16y + 18 = 0$

$(x+2)^2 + (y-8)^2 - 4 - 64 + 18 = 0$

$(x+2)^2 + (y-8)^2 = 50$

centre $A\ (-2, 8)$ radius $= \sqrt{50} = 5\sqrt{2} = r$

b) $y = x + 2$ — ①

$x^2 + y^2 + 4x - 16y + 18 = 0$ — ② solve simultaneously

sub ① into ②

$x^2 + (x+2)^2 + 4x - 16(x+2) + 18 = 0$

$x^2 + x^2 + 4x + 4 + 4x - 16x - 32 + 18 = 0$

$2x^2 - 8x - 10 = 0$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

either

$x = 5$ or $x = -1$

$\downarrow \qquad \downarrow$

$y = 5 + 2 \qquad y = -1 + 2$

$y = 7 \qquad y = 1$

$\therefore (5, 7) \qquad (-1, 1)$

These are coords of B and C.

88) C $x^2+y^2-4x+6y-12=0$

a) $x^2-4x+y^2+6y=12$

$(x-2)^2-4+(y+3)^2-9=12$

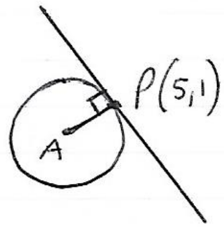
$(x-2)^2+(y+3)^2=25$

centre $A(2,-3)$ radius $=5$

$\quad\quad\quad\; x_1\; y_1$

b) $P(5,1)$

$\quad x_2\; y_2$



Find Gradient of radius AP

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$m=\frac{1+3}{5-2}=\frac{4}{3}$$

$\therefore$ Gradient Tangent $=-\frac{3}{4}$

because $\frac{4}{3}\times\left(-\frac{3}{4}\right)=-1$ as tangt and radius are $\perp$.

$\therefore$ Eqn. of tangent at $P(5,1)$

$$y-y_2=m(x-x_2)$$
$$y-1=-\frac{3}{4}(x-5)$$
$$4y-4=-3x+15$$
$$4y+3x=19$$

c) L $\quad y=x+3$ $\quad$ — ①

C $\quad x^2+y^2-4x+6y-12=0$ $\quad$ — ②

Attempt to solve simultaneously. Sub ① into ②

$$x^2+(x+3)^2-4x+6(x+3)-12=0$$
$$x^2+x^2+6x+9-4x+6x+18-12=0$$
$$2x^2+8x+15=0$$

$$b^2-4ac=8^2-4(2)(15)$$
$$=64-120$$
$$=-VE$$

$\therefore$ No real roots

$\therefore$ L and C do not intersect.