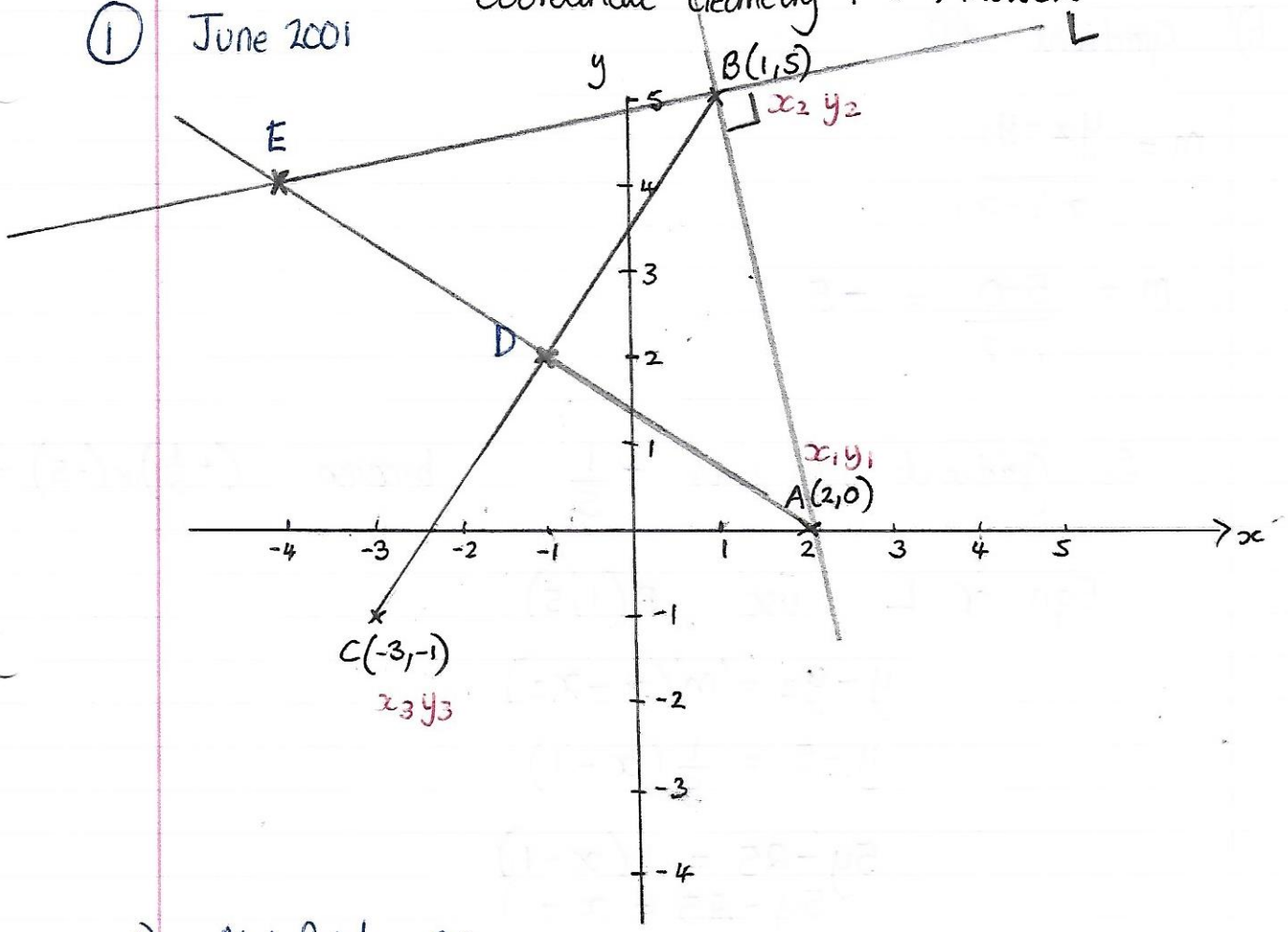
Coordinate Geometry 1 : Answers

① June 2001



a) Mid Point BC

$$D\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$$

$$= D\left(\frac{1+(-3)}{2}, \frac{5+(-1)}{2}\right)$$

$$= D(-1, 2)$$

x_4 y_4

Gradient AD

$$m = \frac{y_4-y_1}{x_4-x_1}$$

$$m = \frac{2-0}{-1-2}$$

$$m = -\frac{2}{3}$$

Equation AD Use A(2,0)

$$y-y_1 = m(x-x_1)$$

$$y-0 = -\frac{2}{3}(x-2)$$

$$3y = -2(x-2)$$

$$3y = -2x+4$$

$$3y+2x-4 = 0$$

b) Gradient AB

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5-0}{1-2} = -5$$

$\therefore$ Gradient L is $+\frac{1}{5}$ because $(+\frac{1}{5}) \times (-5) = -1$

Eqn. of L use $B(1,5)$

$$y - y_2 = m(x - x_2)$$
$$y - 5 = \frac{1}{5}(x - 1)$$
$$5y - 25 = 1(x-1)$$
$$5y - 25 = x - 1$$
$$5y = x + 24$$

c) Eqn L $5y = x + 24$ — (1)
Eqn AD $3y + 2x - 4 = 0$ — (2)

Solve (1) and (2) simultaneously to find intersection point

(1) $\Rightarrow$ $5y - 24 = x$ (*)

Sub into (2)

$$3y + 2(5y - 24) - 4 = 0$$
$$3y + 10y - 48 - 4 = 0$$
$$13y = 52$$
$$y = \frac{52}{13} = 4$$

(*) $5(4) - 24 = x$
$20 - 24 = x$
$-4 = x$

$\therefore$ $E(-4, 4)$

d) Area $\triangle ABE$

$$A = \frac{AB \times BE}{2}$$

AB

$A(2,0)$ $B(1,5)$

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(1-2)^2+(5-0)^2}$$
$$= \sqrt{(-1)^2+(5)^2}$$
$$= \sqrt{26}$$

BE

$B(1,5)$ $E(-4,4)$

$$BE = \sqrt{(x_5-x_2)^2+(y_5-y_2)^2}$$
$$= \sqrt{(4-5)^2+(-4-1)^2}$$
$$= \sqrt{(-1)^2+(-5)^2}$$
$$= \sqrt{26}$$

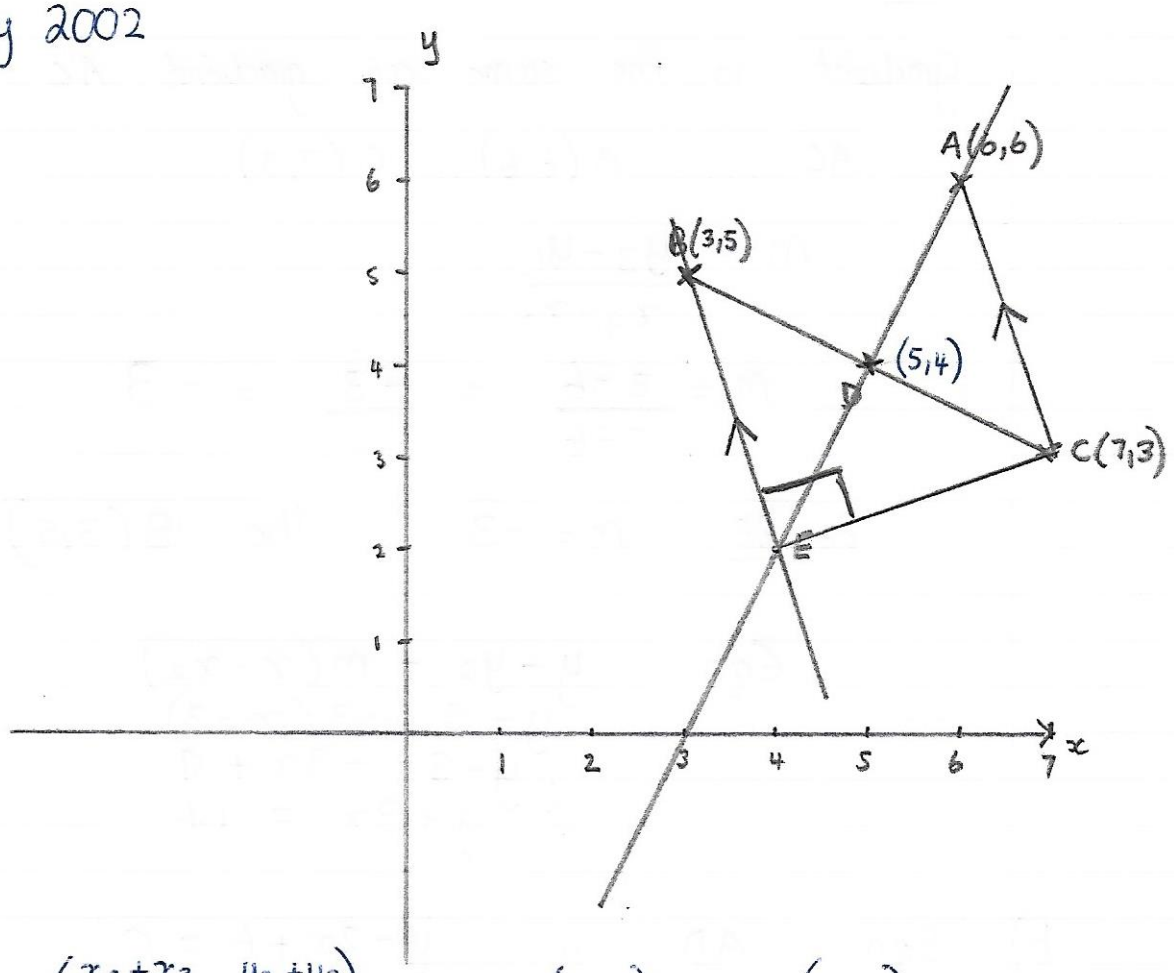
$\therefore$ Area $\triangle ABE$

$$A = \frac{\sqrt{26} \times \sqrt{26}}{2}$$

$$A = \frac{26}{2}$$

$$A = 13 \text{ units}^2$$

2) May 2002



a) $D\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$ B $(3,5)$ $x_2\ y_2$ C $(7,3)$ $x_3\ y_3$

$= D\left(\frac{3+7}{2}, \frac{5+3}{2}\right)$

$= D(5,4)$

AD A $(6,6)$ $x_1\ y_1$ D $(5,4)$ $x_4\ y_4$

$$m = \frac{y_4 - y_1}{x_4 - x_1}$$

$$m = \frac{4-6}{5-6} = \frac{-2}{-1} = 2$$

Equation AD Use A $(6,6)$

$y - y_1 = m(x - x_1)$

$y - 6 = 2(x - 6)$

$y - 6 = 2x - 12$

$y - 2x + 6 = 0$

BE

Gradient is the same as gradient AC (parallel)

AC A(6,6) C(7,3)

$$m = \frac{y_3 - y_1}{x_3 - x_1}$$

$$m = \frac{3-6}{7-6} = \frac{-3}{1} = -3$$

∴ BE $m = -3$ Use $B(3,5)$

Eqn $y - y_2 = m(x - x_2)$

$y - 5 = -3(x-3)$

$y - 5 = -3x + 9$

$y + 3x = 14$

b) Eqn AD is $y - 2x + 6 = 0$ (1)

BE is $y + 3x = 14$ (2)

Solve (1) and (2) simultaneously

(1) $\Rightarrow$ $y = 2x - 6$ (*)

Sub into (2)

$2x - 6 + 3x = 14$

$5x = 20$

$x = 4$

(*) $\Rightarrow$ $y = 2(4) - 6$

$y = 2$

∴ $E(4,2)$

$x_3\ y_3$ $x_5\ y_5$

c) BE EC $C(7,3)$ $E(4,2)$

$m = -3$

$$m = \frac{y_5 - y_3}{x_5 - x_3}$$

$$m = \frac{2-3}{4-7}$$

$$m = \frac{-1}{-3} = +\frac{1}{3}$$

Now $(-3) \times \left(+\frac{1}{3}\right) = -1$

∴ BE and EC are perpendicular

d) Area $\triangle BEC$

$$A = \frac{BE \times EC}{2}$$

BE $B(3,5)$ $E(4,2)$

$$BE = \sqrt{(x_5-x_2)^2 + (y_5-y_2)^2}$$
$$= \sqrt{(4-3)^2 + (2-5)^2}$$
$$= \sqrt{1+9}$$
$$= \sqrt{10}$$

EC $C(7,3)$ $E(4,2)$

$$EC = \sqrt{(x_5-x_3)^2 + (y_5-y_3)^2}$$
$$= \sqrt{(4-7)^2 + (2-3)^2}$$
$$= \sqrt{9+1}$$
$$= \sqrt{10}$$

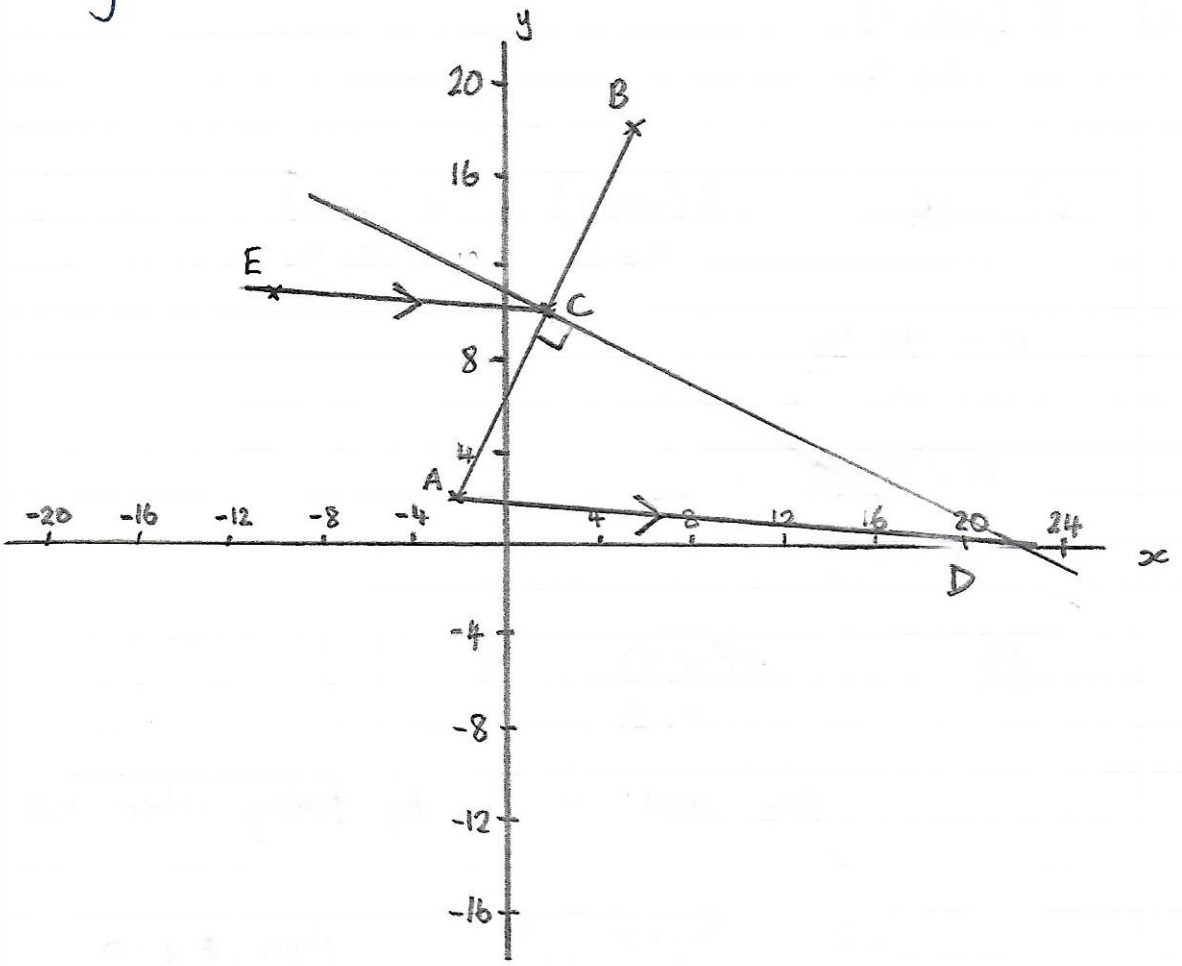
∴ Area $\triangle BEC$

$$A = \frac{\sqrt{10} \times \sqrt{10}}{2}$$

$$A = \frac{10}{2}$$

$$A = 5 \text{ units}^2$$

3) May 2003



a) AB A(-2, 2) B(6, 18)
x_1, y_1 x_2, y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{18 - 2}{6 - (-2)}$$

$$m = \frac{16}{8} = +2$$

b) $C\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= C\left(\frac{-2+6}{2}, \frac{2+18}{2}\right)$

$= C(2, 10)$
$x_3\ y_3$

Gradient of CD is $-\frac{1}{2}$

$\left(-\frac{1}{2}\right) \times (+2) = -1$

∴ Eqn CD $y - y_3 = m(x - x_3)$

$y - 10 = -\frac{1}{2}(x - 2)$

$2y - 20 = -x + 2$

$2y + x = 22$

c) $E(-10, 11)$
$x_5\ y_5$

(i) EC $\quad E(-10, 11)$ $\quad C(2, 10)$
$x_5\ y_5$ $\quad x_3\ y_3$

$$m = \frac{y_5 - y_3}{x_5 - x_3}$$

$$m = \frac{11-10}{-10-2} = -\frac{1}{12}$$

AD $\quad A(-2, 2)$ $\quad D(\ \ )$?
$x_1\ y_1$

Find coords of D by finding where CD crosses x axis.

CD $\quad 2y + x = 22$ $\quad$ when $y = 0$

$$0 + x = 22$$
$$x = 22$$

$\therefore$ D $(22, 0)$
$x_4\ y_4$

$\therefore$ Gradient AD

$$m = \frac{y_4 - y_1}{x_4 - x_1}$$

$$m = \frac{0-2}{22-(-2)}$$

$$m = \frac{-2}{24}$$

$$m = -\frac{1}{12}$$

Gradients of AD and EC are both $-\frac{1}{12}$

$\therefore$ AD and EC are parallel

(ii) EC

$$EC = \sqrt{(x_5 - x_3)^2 + (y_5 - y_3)^2}$$
$$= \sqrt{(-10-2)^2 + (11-10)^2}$$
$$= \sqrt{144 + 1}$$
$$= \sqrt{145}$$

AD

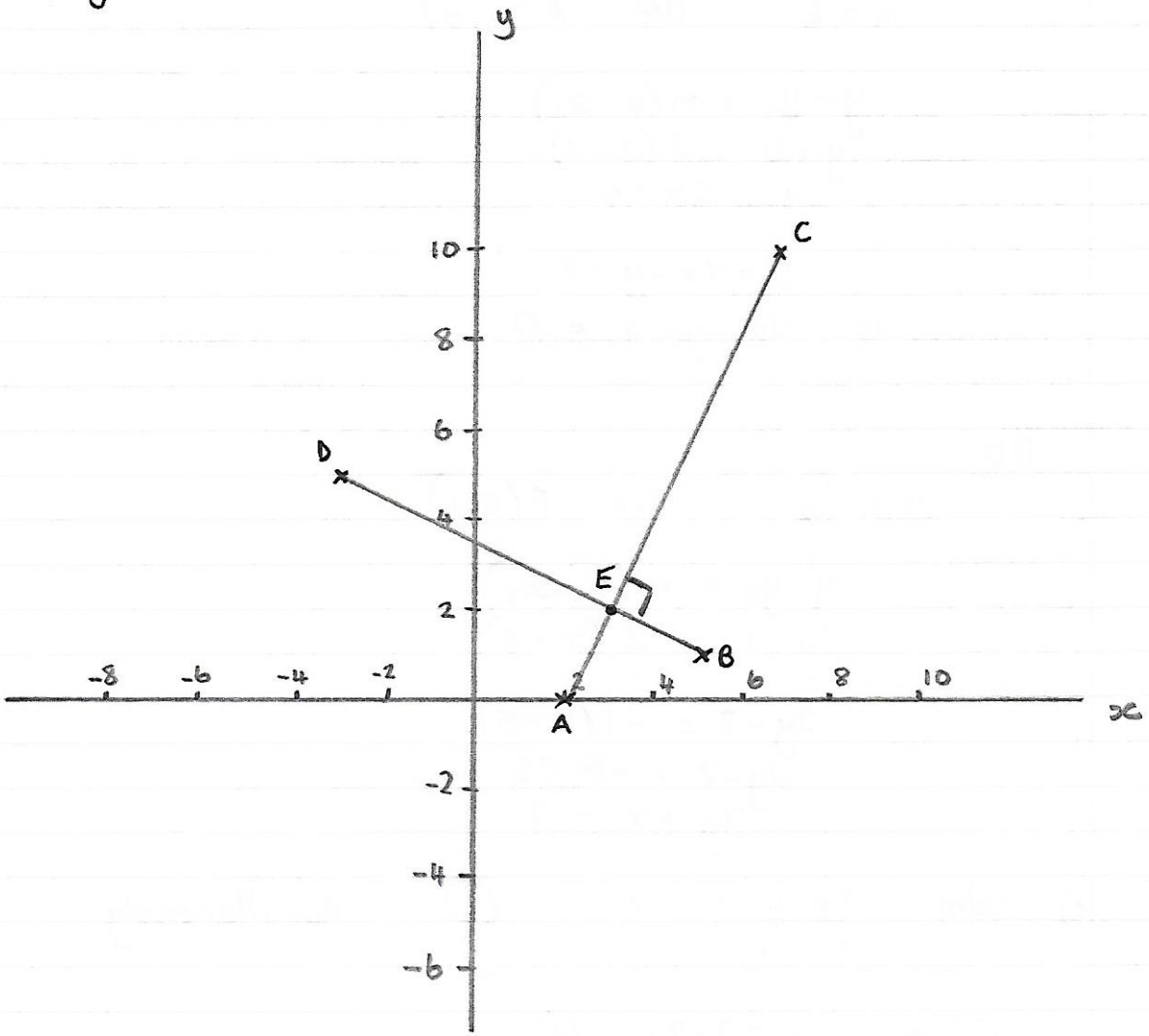
$$AD = \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2}$$
$$= \sqrt{(22-(-2))^2 + (0-2)^2}$$
$$= \sqrt{576 + 4}$$
$$= \sqrt{580}$$

NOW $\frac{AD}{EC} = \frac{\sqrt{580}}{\sqrt{145}} = \frac{\sqrt{4 \times 145}}{\sqrt{145}} = \frac{2\sqrt{145}}{\sqrt{145}} = 2$

$\therefore \frac{AD}{2} = EC$

$\therefore EC = \frac{1}{2} AD$

4) May 2004



A(2,0) B(5,1) C(7,10) D(-3,5)

$x_1\ y_1$ $x_2\ y_2$ $x_3\ y_3$ $x_4\ y_4$

a) AC

$$m = \frac{y_3 - y_1}{x_3 - x_1}$$

$$m = \frac{10-0}{7-2} = \frac{10}{5} = +2$$

BD $\quad m = \dfrac{y_4 - y_2}{x_4 - x_2}$

$$= \frac{5-1}{-3-5} = \frac{4}{-8} = -\frac{1}{2}$$

∴ AC and BD are ⊥ because $(+2) \times \left(-\frac{1}{2}\right) = -1$

b) AC

$m = 2$ Use $A(2,0)$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 2(x - 2)$$
$$y = 2x - 4$$
$$0 = 2x - y - 4$$
ie $2x - y - 4 = 0$

BD

$m = -\frac{1}{2}$ Use $B(5,1)$

$$y - y_2 = m(x - x_2)$$
$$y - 1 = -\frac{1}{2}(x - 5)$$
$$2y - 2 = -1(x - 5)$$
$$2y - 2 = -x + 5$$
$$2y + x = 7$$

c) Solve $2x - y - 4 = 0$ — (1)
$2y + x = 7$ — (2) simultaneously

(2) $\Rightarrow$ $x = 7 - 2y$ (*)

(1) $\Rightarrow$ $2(7 - 2y) - y - 4 = 0$
$14 - 4y - y - 4 = 0$
$10 = 5y$
$2 = y$

(*) $x = 7 - 2(2)$
$x = 7 - 4$
$x = 3$ $\therefore$ $E(3,2)$

d) AC

$$AC = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$
$$= \sqrt{(7-2)^2 + (10-0)^2}$$
$$= \sqrt{25 + 100}$$
$$= \sqrt{125}$$
$$= 5\sqrt{5}$$

AE

$$AE = \sqrt{(x_5 - x_1)^2 + (y_5 - y_1)^2}$$
$$= \sqrt{(3-2)^2 + (2-0)^2}$$
$$= \sqrt{1 + 4}$$
$$= \sqrt{5}$$

$\therefore$ $AC = 5AE$