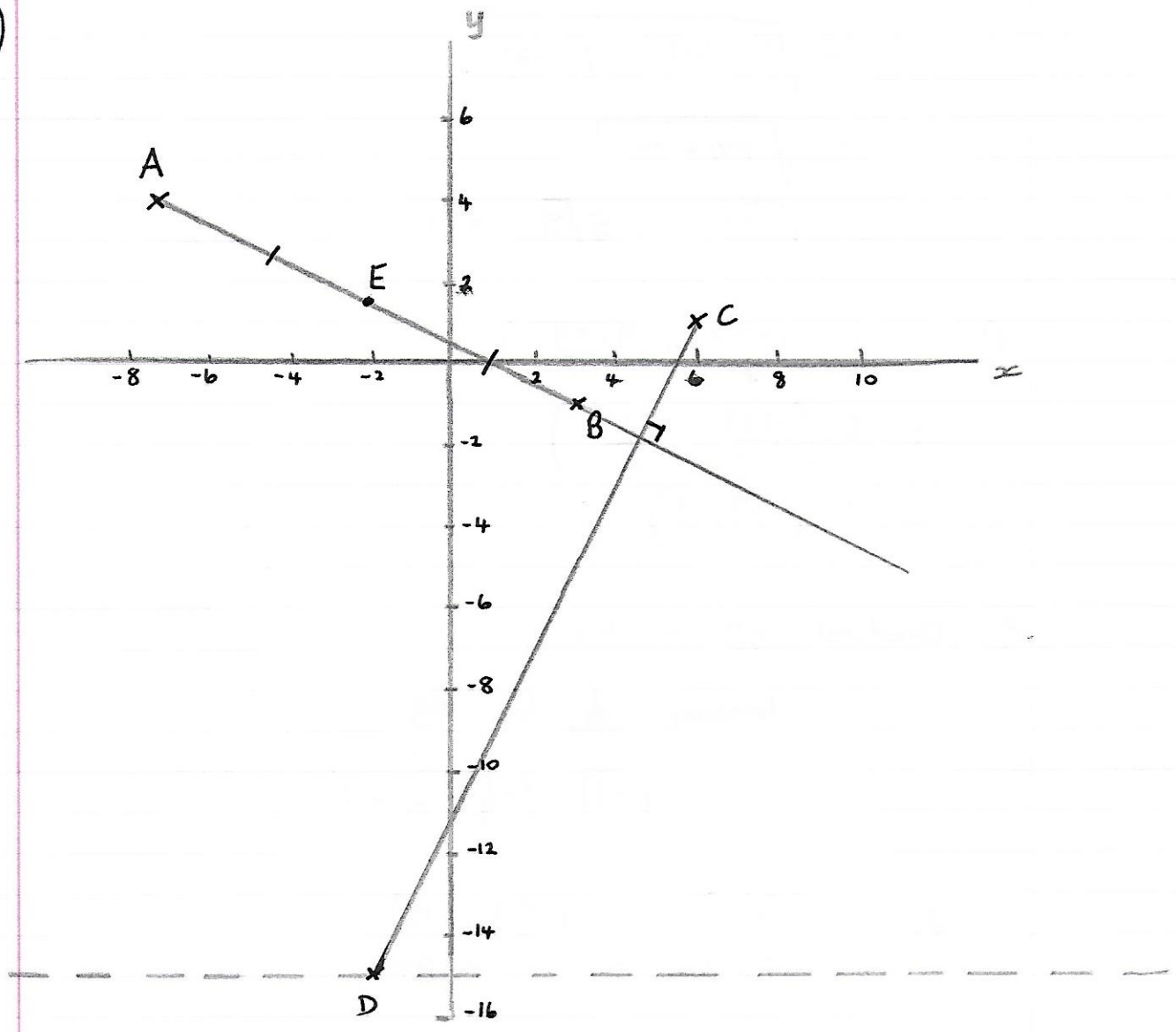
Coordinate Geometry 2 : Answers

5)



a) A $(-7, 4)$ B $(3, -1)$

$x_1\ y_1$ $x_2\ y_2$

AB $\quad m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{-1-4}{3-(-7)}$$

$$m = \frac{-5}{10} = -\frac{1}{2}$$

b) AB

Eqn $\quad y - y_1 = m(x - x_1)$

$$y - 4 = -\frac{1}{2}(x - (-7))$$

$$2y - 8 = -1(x + 7)$$

$$2y - 8 = -x - 7$$

$$2y + x = 1$$

c) $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{[3-(-7)]^2+(-1-4)^2}$

$= \sqrt{100+25}$

$= \sqrt{125} = 5\sqrt{5}$ units

d) $E\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= E\left(\frac{-7+3}{2}, \frac{4+(-1)}{2}\right)$

$= E\left(-2, \frac{3}{2}\right)$

e) Gradient $CD = +2$

because $\perp$ to AB

$(+2) \times \left(-\frac{1}{2}\right) = -1$

so $C(6,1)$ $D(k,-15)$

$x_3\ y_3$ $x_4\ y_4$

CD $m = \frac{y_4-y_3}{x_4-x_3}$

$+2 = \frac{-15-1}{k-6}$

$\times(k-6)$

$2(k-6) = -16$

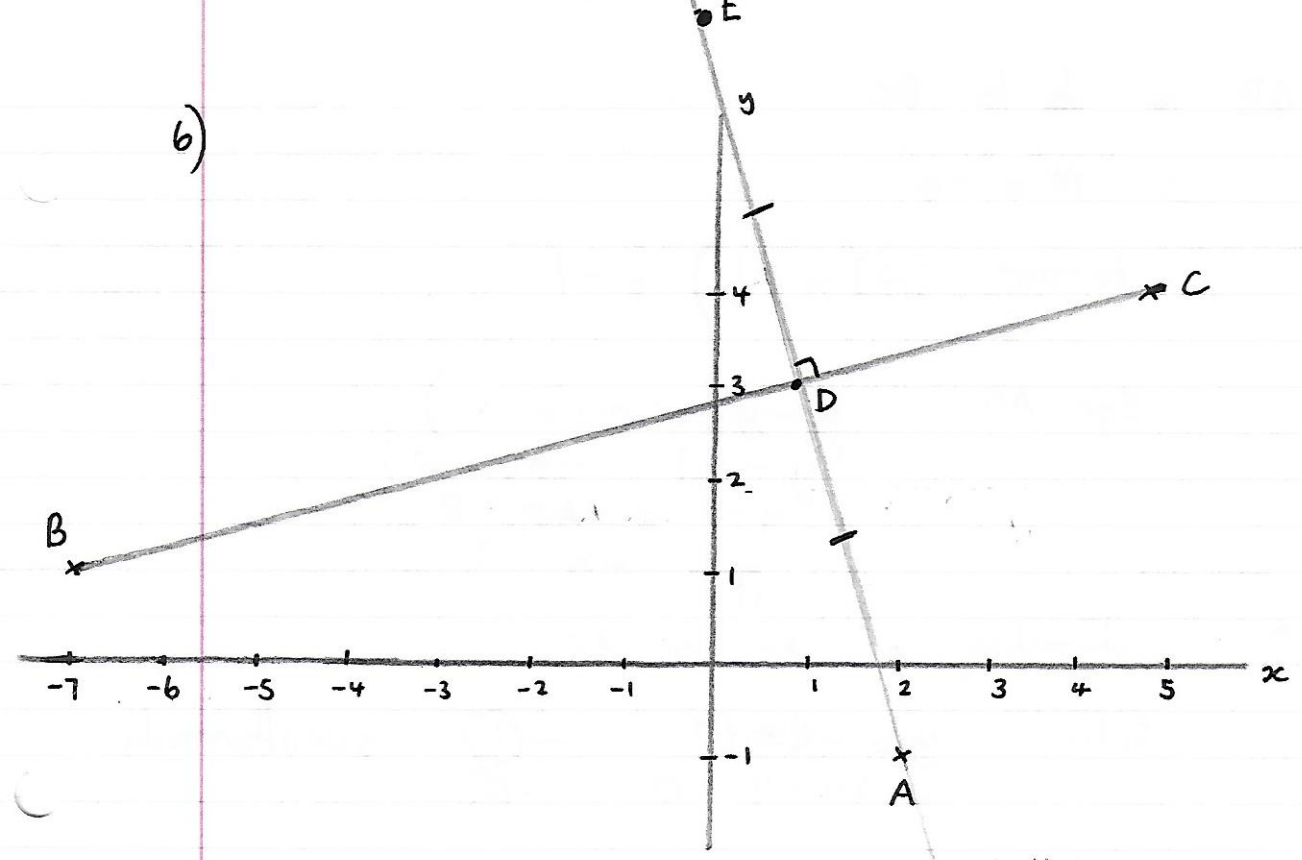
$2k-12 = -16$

$2k = -4$

$k = -2$

$\therefore D(-2,-15)$

6)



a) BC

B(-7, 1) $x_2\ y_2$ c(5,4) $x_3\ y_3$

$$m = \frac{y_3 - y_2}{x_3 - x_2}$$

$$m = \frac{4-1}{5-(-7)}$$

$$m = \frac{3}{12} = \frac{1}{4}$$

$\therefore$ Eqn $y - y_2 = m(x - x_2)$

$$y - 1 = \frac{1}{4}(x - (-7))$$

$$4y - 4 = 1(x + 7)$$

$$4y - 4 = x + 7$$

$$0 = x - 4y + 11$$

or

$$x - 4y + 11 = 0$$

AD is $\perp$ to BC

$\therefore \quad m = -4$

because $(-4) \times \left(\frac{1}{4}\right) = -1$

$\therefore$ Eqn AD $\quad y - y_1 = m(x - x_1)$

$y - (-1) = -4(x - 2)$

$y + 1 = -4x + 8$

$y = -4x + 7$

b) D is intersection of BC and AD

Solve $\quad y = -4x + 7 \quad -(1)$ simultaneously

$x - 4y + 11 = 0 \quad -(2)$

sub (1) into (2)

(2) $\Rightarrow \quad x - 4(-4x + 7) + 11 = 0$

$x + 16x - 28 + 11 = 0$

$17x = 17$

$x = 1$

$\therefore$ (1) $\Rightarrow \quad y = -4(1) + 7$

$y = -4 + 7$

$y = 3$

$\therefore$ D $(1, 3)$

$x_4 \; y_4$

c) CD

$CD = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$

$= \sqrt{(1 - 5)^2 + (3 - 4)^2}$

$= \sqrt{16 + 1}$

$= \sqrt{17}$ units

d) AE

D $\left(\frac{x_1 + x_S}{2}, \frac{y_1 + y_S}{2}\right)$

D $(1, 3)$

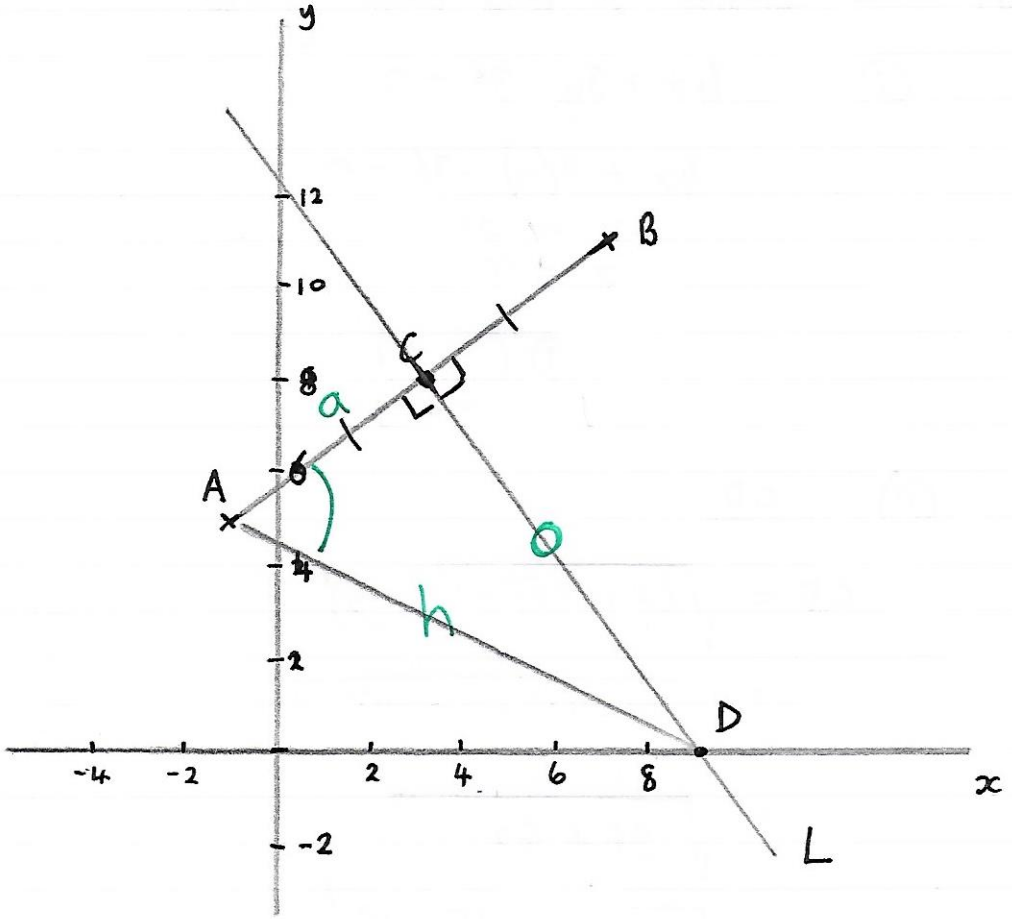
$\therefore \quad 1 = \frac{2 + x_S}{2}$ and $3 = \frac{-1 + y_S}{2}$

$2 = 2 + x_S \qquad 6 = -1 + y_S$

$0 = x_S \qquad 7 = y_S$

$\therefore$ E $(0, 7)$

7)



a) AB A(-1, 5) B(7, 11)
$x_1\ y_1$ $x_2\ y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{11-5}{7-(-1)}$$

$$m = \frac{6}{8} = \frac{3}{4}$$

b) $C\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= C\left(\frac{-1+7}{2}, \frac{5+11}{2}\right)$

$= C(3, 8)$
$x_3\ y_3$

c) L $m = -\frac{4}{3}$ because it is ⊥ to AB.

$$\left(-\frac{4}{3}\right) \times \frac{3}{4} = -1$$

∴ Eqn of L $y - y_3 = m(x - x_3)$

$y - 8 = -\frac{4}{3}(x - 3)$

$3y - 24 = -4x + 12$

$4x + 3y - 36 = 0$

d) L crosses x axis when $y = 0$

(i) $4x + 3y - 36 = 0$

$4x + 3(0) - 36 = 0$

$4x = 36$

$x = 9$

$\therefore D(9, 0)$

$x_4 \; y_4$

(ii) CD

$$CD = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$$

$$= \sqrt{(9-3)^2 + (0-8)^2}$$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100} = 10 \text{ units}$$

(iii) Use $\triangle CAD$

$$\tan \hat{CAD} = \frac{o}{a}$$

$$\tan \hat{CAD} = \frac{CD}{AC}$$

First AC $= \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$$= \sqrt{[3 - (-1)]^2 + (8-5)^2}$$

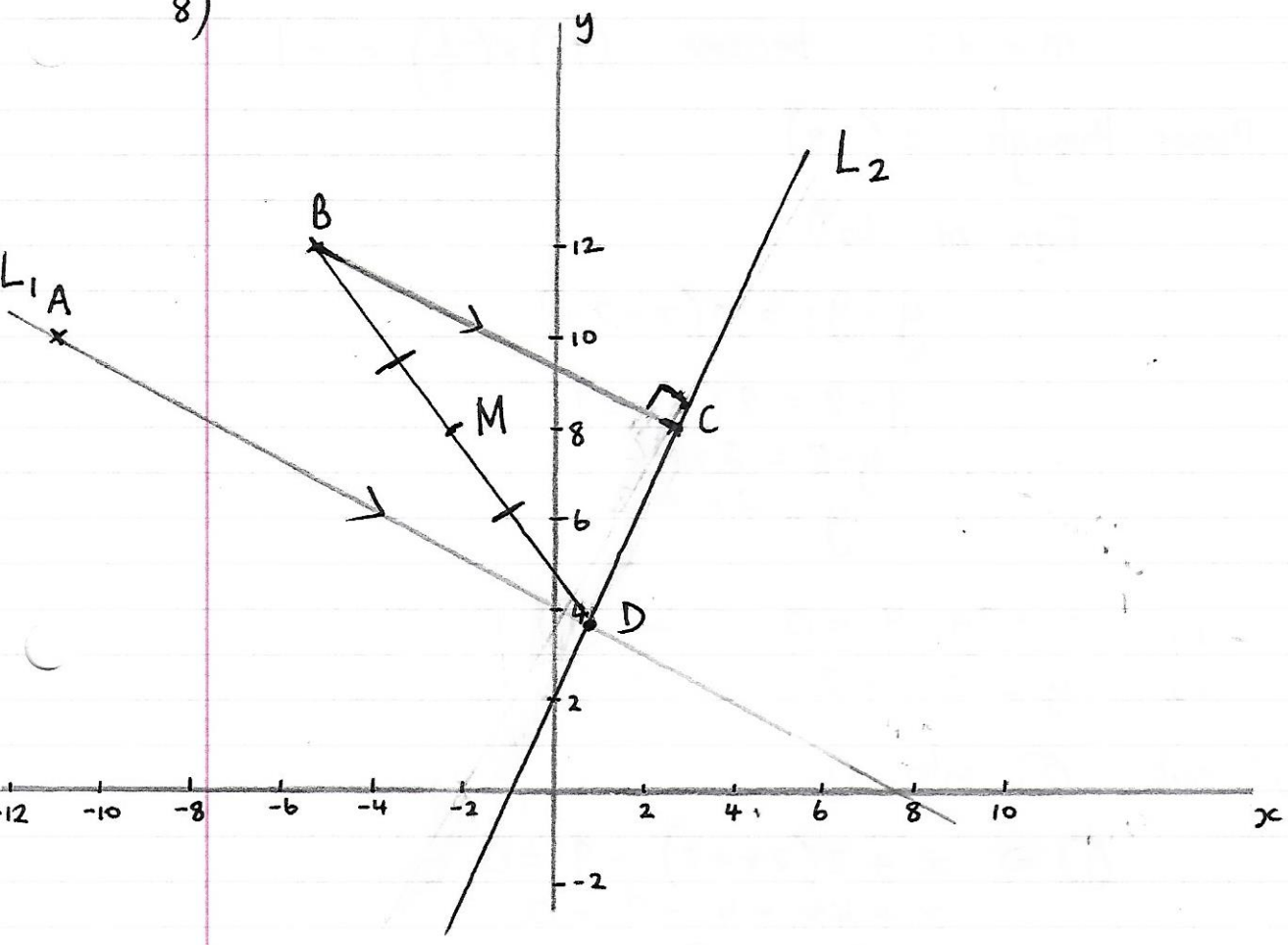
$$= \sqrt{16 + 9}$$

$$= \sqrt{25} = 5 \text{ units}$$

$$\therefore \tan \hat{CAD} = \frac{10}{5}$$

$$= 2$$

8)



a) B $(-5, 12)$ C $(3, 8)$
$x_2\ y_2$ $x_3\ y_3$

BC $\quad m = \dfrac{y_3 - y_2}{x_3 - x_2}$

$$m = \frac{8-12}{3-(-5)}$$

$$m = \frac{-4}{8} = -\frac{1}{2}$$

b) (i) L_1 $\quad m = -\frac{1}{2}$ because parallel to BC

Use A$(-11, 10)$ Eqn $\quad y - y_1 = m(x - x_1)$

$$y - 10 = -\frac{1}{2}(x - (-11))$$

$$2y - 20 = -1(x + 11)$$

$$2y - 20 = -x - 11$$

$$x + 2y - 9 = 0$$

(ii) L_2 is $\perp$ to BC

$m = +2$ because $(+2) \times (-\frac{1}{2}) = -1$

Passes through $c(3,8)$

Eqn of L_2

$$y - y_3 = m(x - x_3)$$
$$y - 8 = 2(x - 3)$$
$$y - 8 = 2x - 6$$
$$y = 2x + 2$$

c) L_1 $x + 2y - 9 = 0$ — (1)

L_2 $y = 2x + 2$ — (2)

(i) sub (2) into (1)

(1) $\Rightarrow$ $x + 2(2x+2) - 9 = 0$
$x + 4x + 4 - 9 = 0$
$5x = 5$
$x = 1$

(2) $\Rightarrow$ $y = 2(1) + 2$
$y = 4$ $\therefore$ $D(1,4)$
$x_4\ y_4$

(ii) BD

$$B = \sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2}$$
$$= \sqrt{[1 - (-5)]^2 + (4 - 12)^2}$$
$$= \sqrt{36 + 64}$$
$$= \sqrt{100} = 10 \text{ units}$$

(iii) $M\left(\frac{x_2 + x_4}{2}, \frac{y_2 + y_4}{2}\right)$ mid-point BD

$= M\left(\frac{-5+1}{2}, \frac{12+4}{2}\right)$

$= M(-2, 8)$