Cubics : 4 Answers

1) $f(x) = px^3 - x^2 + qx - 6$

$x-3$ is a factor

a) $\therefore \; f(3) = p(3^3) - 3^2 + 3q - 6 = 0$

$$27p - 9 + 3q - 6 = 0$$
$$27p + 3q = 15$$
$$9p + q = 5 \quad — \; ①$$

$\div$ by $(x-2)$ $\quad R = -20$

$$\therefore \; f(2) = p(2^3) - 2^2 + q(2) - 6 = -20$$
$$8p - 4 + 2q - 6 = -20$$
$$8p + 2q = -10$$
$$4p + q = -5$$
$$q = -4p - 5 \quad — \; ②$$

Sub ② into ①

$$① \Rightarrow \; 9p + (-4p-5) = 5$$
$$9p - 4p - 5 = 5$$
$$5p = 10$$
$$p = 2$$

$$② \Rightarrow \; q = -4(2) - 5$$
$$q = -8 - 5$$
$$q = -13$$

b) $\therefore \; 2x^3 - x^2 - 13x - 6 = (x-3)(ax^2 + bx + c)$

Compare x^3: $2 = a$

Compare const: $-6 = -3c$, $\; 2 = c$

Compare x^2: $-1 = -3a + b$, $\; -1 = -6 + b$, $\; 5 = b$

$$\therefore \; 2x^3 - x^2 - 13x - 6 = (x-3)(2x^2 + 5x + 2)$$
$$= (x-3)(2x+1)(x+2)$$

2) $9x^3+6x^2-5x+p$

a) $\div$ by $(x-1)$ $\quad R = 8$

$f(1) = 9+6-5+p = 8$

$p = 8-10$

$p = -2$ $\qquad \therefore f(x) = 9x^3+6x^2-5x-2$

b) Try $f(-1) = -9+6+5-2$

$= 0$

$\therefore$ $(x+1)$ is a factor

$\therefore 9x^3+6x^2-5x-2 = (x+1)(ax^2+bx+c)$

Compare x^3

$9 = a$

Compare consts

$-2 = c$

Compare x^2

$6 = a+b$

$6 = 9+b$

$-3 = b$

$\therefore 9x^3+6x^2-5x-2 = (x+1)(9x^2-3x-2)$

$= (x+1)(3x+1)(3x-2)$

3) a) x^3-5x^2-2x+p

$x-3$ factor

$f(3) = 3^3-5(3^2)-6+p = 0$

$27-45-6+p = 0$

$p = 24$

b) $x^3-5x^2-2x+24 = (x-3)(ax^2+bx+c)$

Compare x^3

$1 = a$

Compare consts

$24 = -3c$

$-8 = c$

Compare x^2

$-5 = -3a+b$

$-5 = -3+b$

$-2 = b$

$\therefore$ Eqn is $(x-3)(x^2-2x-8) = 0$

$(x-3)(x-4)(x+2) = 0$

$\therefore$ either $x-3=0$ or $x-4=0$ or $x+2=0$

$x=3$ $\qquad x=4$ $\qquad x=-2$

c) $f(2) = 2^3-5(2^2)-2(2)+24$

$= 8-20-4+24$

$= 8$

$\therefore$ Remainder $= 8$

4) a) $6x^3 + ax^2 - 3x - 2$

$\div$ by $(x+2)$ $R = -24$

$$f(-2) = 6(-2)^3 + a(-2)^2 - 3(-2) - 2 = -24$$
$$-48 + 4a + 6 - 2 = -24$$
$$4a - 44 = -24$$
$$4a = 20$$
$$a = 5$$

b)

$$6x^3 + ax^2 - 3x - 2$$
$$\Rightarrow \quad 6x^3 + 5x^2 - 3x - 2$$

LONG DIVISION METHOD

$$f(1) = 6 + 5 - 3 - 2 \neq 0$$
$$f(-1) = -6 + 5 + 3 - 2 = 0$$

$\therefore$ $(x+1)$ is a factor

$$\begin{array}{r} 6x^2 - x - 2 \\ x+1 \;\overline{)\; 6x^3 + 5x^2 - 3x - 2} \\ \underline{\mp 6x^3 \mp 6x^2} \\ -x^2 - 3x \\ \underline{\pm x^2 \pm x} \\ -2x - 2 \\ \underline{\pm 2x \pm 2} \\ 0 \end{array}$$

$$\therefore \quad 6x^3 + 5x^2 - 3x - 2 = (x+1)(6x^2 - x - 2)$$
$$= (x+1)(3x-2)(2x+1)$$

5) $4x^3 + px^2 - 11x + q$

a) $(x-2)$ factor

$$f(2) = 4(2^3) + p(2^2) - 11(2) + q = 0$$
$$32 + 4p - 22 + q = 0$$
$$4p + q = -10$$
$$q = -10 - 4p \quad -①$$

$\div (x+1) \quad R = 9$

$$f(-1) = 4(-1)^3 + p(-1)^2 - 11(-1) + q = 9$$
$$-4 + p + 11 + q = 9$$
$$q = 9 - 7 - p$$
$$q = 2 - p \quad -②$$

sub ① into ②

$$② \Rightarrow \quad -10 - 4p = 2 - p$$
$$-12 = 3p$$
$$-4 = p$$

$$① \Rightarrow \quad q = -10 - 4(-4)$$
$$q = -10 + 16$$
$$q = 6$$

b)

$$4x^3 - 4x^2 - 11x + 6 = (x-2)(ax^2 + bx + c)$$

Compare x^3
$4 = a$

Compare consts
$6 = -2c$
$-3 = c$

Compare x^2
$-4 = b - 2a$
$-4 = b - 8$
$4 = b$

$$\therefore \quad 4x^3 - 4x^2 - 11x + 6 = (x-2)(4x^2 + 4x - 3)$$
$$= (x-2)(2x-1)(2x+3)$$

6) a) $ax^3 - 12x^2 - 6x + 5$

$\div (x+1) \quad R = -3$

$$f(-1) = a(-1)^3 - 12(-1)^2 - 6(-1) + 5 = -3$$

$$\Rightarrow -a - 12 + 6 + 5 = -3$$

$$-1 + 3 = a$$

$$2 = a$$

b) $8x^3 - 14x^2 - 7x + 6$

$f(1) = 8 - 14 - 7 + 6 \neq 0$

$f(-1) \neq 0$ ~~from before~~ $-8 - 14 + 7 + 6$

$f(2) = 8(8) - 14(4) - 7(2) + 6 = 64 - 56 - 14 + 6 = 0$

$\therefore (x-2)$ is a factor

Use LONG DIVISION METHOD

$$\begin{array}{r|l} & \;\; 8x^2 + 2x - 3 \\ \hline x - 2 & \;\; 8x^3 - 14x^2 - 7x + 6 \\ & -8x^3 \pm 16x^2 \\ \hline & \qquad\quad 2x^2 - 7x \\ & \qquad -2x^2 \pm 4x \\ \hline & \qquad\qquad -3x + 6 \\ & \qquad\qquad \pm 3x \mp 6 \\ \hline & \qquad\qquad\quad 0 \end{array}$$

$$\therefore 8x^3 - 14x^2 - 7x + 6 = (x-2)(8x^2 + 2x - 3)$$

$$= (x-2)(2x - 1)(4x + 3)$$

7) $f(x) = 2x^3 + 11x^2 + 4x - 5$

a) (i) $f(-2) = 2(-2)^3 + 11(-2)^2 + 4(-2) - 5$
$= -16 + 44 - 8 - 5$
$= 15$

(ii) $(x+2)$ is NOT a factor of $f(x)$

b) $f(1) = 2 + 11 + 4 - 5 \neq 0$

$f(-1) = -2 + 11 - 4 - 5 = 0$

$\therefore$ $(x+1)$ is a factor

$\therefore$ $2x^3 + 11x^2 + 4x - 5 = (x+1)(ax^2 + bx + c)$

Compare x^3 | Compare consts | Compare x^2

$2 = a$ | $-5 = c$ | $11 = a + b$
$11 = 2 + b$
$9 = b$

$\therefore$

$f(x) = (x+1)(2x^2 + 9x - 5)$
$= (x+1)(2x - 1)(x + 5)$

$\therefore$ Solving the eqn $f(x) = 0$

$(x+1)(2x-1)(x+5) = 0$

either $x + 1 = 0$ or $2x - 1 = 0$ or $x + 5 = 0$
$x = -1$ | $x = \frac{1}{2}$ | $x = -5$

8) a) $x^3 - 3$ $\div (x+2)$ find R

Let $f(x) = x^3 - 3$

$f(-2) = (-2)^3 - 3 = -8 - 3 = -11$

$\therefore R = -11$

b) $g(x) = 6x^3 + x^2 - 11x - 6$

$g(1) = 6 + 1 - 11 - 6 \neq 0$

$g(-1) = -6 + 1 + 11 - 6 = 0$

$\therefore (x+1)$ is a factor

LONG DIVISION METHOD

$$
\begin{array}{r|l}
 & 6x^2 - 5x - 6 \\
\hline
x+1 & 6x^3 + x^2 - 11x - 6 \\
 & -6x^3 \mp 6x^2 \\
\hline
 & \quad -5x^2 - 11x \\
 & \quad \pm 5x^2 \pm 5x \\
\hline
 & \qquad -6x - 6 \\
 & \qquad \pm 6x \pm 6 \\
\hline
 & \qquad\quad 0
\end{array}
$$

$\therefore$ Eqn is

$(x+1)(6x^2 - 5x - 6) = 0$

$(x+1)(3x+2)(2x-3) = 0$

either $x + 1 = 0$ or $3x + 2 = 0$ or $2x - 3 = 0$

$x = -1$ $\quad x = -\frac{2}{3}$ $\quad x = \frac{3}{2}$

9) a) $12x^3+kx^2-13x-6$

$(x+2)$ is a factor

$\therefore f(-2) = 12(-2)^3+k(-2)^2-13(-2)-6 = 0$

$-96+4k+26-6=0$

$4k = 76$

$k = 19$

b) $12x^3+19x^2-13x-6 = (x+2)(ax^2+bx+c)$

Compare x^3	Compare consts	Compare x^2
$12 = a$	$-6 = 2c$	$19 = 2a+b$
	$-3 = c$	$19 = 24+b$
		$-5 = b$

$\therefore 12x^3+19x^2-13x-6 = (x+2)(12x^2-5x-3)$

$= (x+2)(4x-3)(3x+1)$

c) Find $f(1/2)$ if $(2x-1)$ is used to $\div$

$= 12(1/2)^3+19(1/2)^2-13(1/2)-6$

$= \frac{3}{2}+\frac{19}{4}-\frac{13}{2}-6$

$= \frac{6}{4}+\frac{19}{4}-\frac{26}{4}-\frac{24}{4}$

$= -\frac{25}{4}$

10) $px^3 - x^2 - 31x + q$

$(x+2)$ is a factor

$f(-2) = p(-2)^3 - (-2)^2 - 31(-2) + q = 0$

$-8p - 4 + 62 + q = 0$

$q = 8p - 58 \quad — ①$

$\div (x-1) \quad R = -36$

$f(1) = p - 1 - 31 + q = -36$

$q = -4 - p \quad — ②$

Sub ① into ②

② $\Rightarrow \quad 8p - 58 = -4 - p$

$9p = 54$

$p = 6$

② $\Rightarrow \quad \cancel{q = -4-6}$

$q = -10$

b) $6x^3 - x^2 - 31x - 10 = (x+2)(ax^2 + bx + c)$

Compare x^3

$6 = a$

Compare consts

$-10 = 2c$

$-5 = c$

Compare x^2

$-1 = 2a + b$

$-1 = 12 + b$

$-13 = b$

$\therefore 6x^3 - x^2 - 31x - 10 = (x+2)(6x^2 - 13x - 5)$

$= (x+2)(3x+1)(2x-5)$

11) a) $ax^3 - 21x - 10$

$\div (x-3) \quad R = 35$

$$f(3) = a(3^3) - 21(3) - 10 = 35$$

$$27a - 63 - 10 = 35$$
$$27a = 108$$
$$a = 4$$

b) $4x^3 - 21x - 10$

Try $f(1) = 4 - 21 - 10 \neq 0$

$f(-1) = -4 + 21 - 10 \neq 0$

$f(2) = 32 - 42 - 10 \neq 0$

$f(-2) = -32 + 42 - 10 = 0$

$\therefore$ $(x+2)$ is a factor

LONG DIVISION METHOD

$$\begin{array}{r|l}
 & 4x^2 - 8x - 5 \\ \hline
x+2 & 4x^3 + 0x^2 - 21x - 10 \\
 & -4x^3 \mp 8x^2 \\ \hline
 & \quad -8x^2 - 21x \\
 & \quad \pm 8x^2 \pm 16x \\ \hline
 & \qquad -5x - 10 \\
 & \qquad \pm 5x \pm 10 \\ \hline
 & \qquad\qquad 0
\end{array}$$

$$\therefore \; 4x^3 - 21x - 10 = (x+2)(4x^2 - 8x - 5)$$
$$= (x+2)(2x+1)(2x-5)$$

12) a) $f(x) = 6x^3 - 19x^2 + 11x + 6$

$f(1) = 6 - 19 + 11 + 6 \neq 0$

$f(-1) = -6 - 19 - 11 + 6 \neq 0$

$f(2) = 48 - 76 + 22 + 6 = 0$

$\therefore$ $(x-2)$ is a factor

$\therefore \quad 6x^3 - 19x^2 + 11x + 6 = (x-2)(ax^2 + bx + c)$

Compare x^3: $6 = a$

Compare consts: $6 = -2c$, $-3 = c$

Compare x^2: $-19 = -2a + b$, $-19 = -12 + b$, $-7 = b$

$\therefore \quad 6x^3 - 19x^2 + 11x + 6 = (x-2)(6x^2 - 7x - 3)$
$= (x-2)(3x+1)(2x-3)$

$\therefore$ eqn is
$(x-2)(3x+1)(2x-3) = 0$

either $x - 2 = 0$ or $3x + 1 = 0$ or $2x - 3 = 0$
$x = 2$. $x = -1/3$ $x = 3/2$

b) $x^3 - 53$
$\div$ by $(x-a)$ $R = 11$

$f(a) = a^3 - 53 = 11$
$a^3 = 64$
$a = \sqrt[3]{64}$

$a = 4$

13) a) $px^3 + 18x^2 - 4x - 8$

$(x+2)$ is a factor

$\therefore \quad f(-2) = p(-2)^3 + 18(-2)^2 - 4(-2) - 8 = 0$

$$-8p + 72 + 8 - 8 = 0$$
$$72 = 8p$$
$$9 = p$$

b) $9x^3 + 18x^2 - 4x - 8 = (x+2)(ax^2 + bx + c)$

Compare x^3

$9 = a$

Compare consts

$-8 = 2c$

$-4 = c$

Compare x^2

$18 = 2a + b$

$18 = 18 + b$

$0 = b$

$$\therefore \quad 9x^3 + 18x^2 - 4x - 8 = (x+2)(9x^2 - 4)$$
$$= (x+2)(3x+2)(3x-2)$$

$\therefore$ Eqn becomes

$$(x+2)(3x+2)(3x-2) = 0$$

either $x+2=0$, $x=-2$ or $3x+2=0$, $x = -2/3$ or $3x-2=0$, $x = 2/3$