Plotting Cubic Graphs as sketches

In each case find where the curve crosses the x and y axis and then sketch the curve.

① $y = x^3 - 4x$

Curve crosses y axis when $x = 0$

$y = 0^3 - 2(0)$

$y = 0$

Curve crosses x axis when $y = 0$

$0 = x^3 - 4x$ common factor of x

$0 = x(x^2 - 4)$

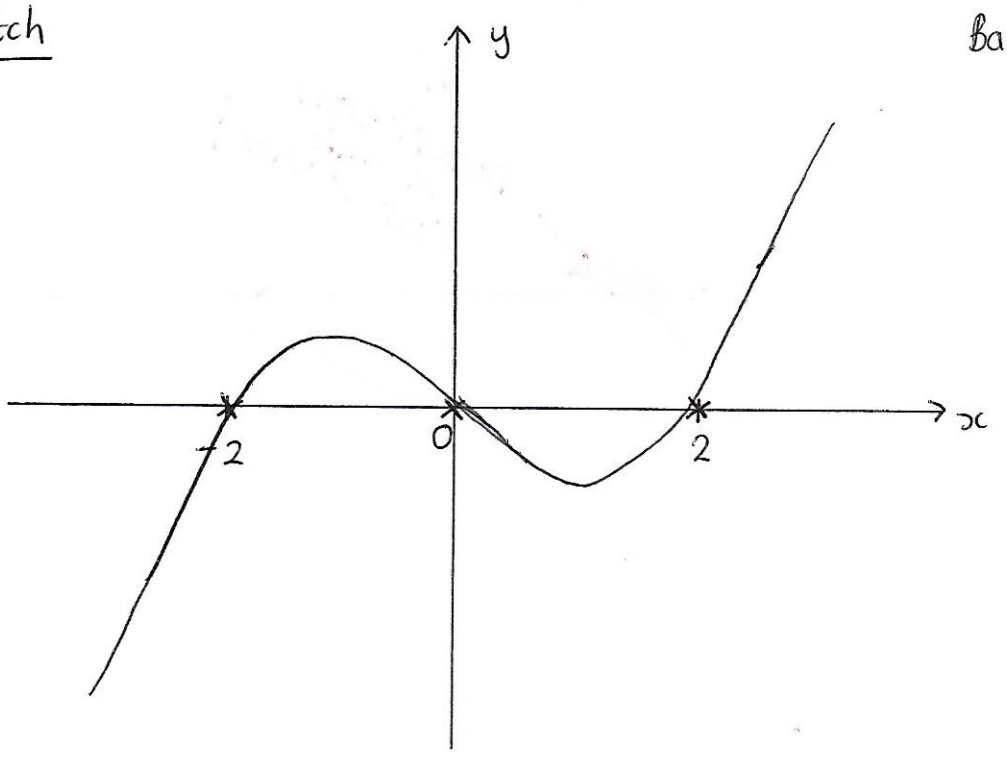
$0 = x(x-2)(x+2)$ difference of 2 squares

either $x = 0$ or $x - 2 = 0$ or $x + 2 = 0$

$x = 2$ $x = -2$

sketch

Basic x^3 shape



② $y = x^3 - 2x^2$ common factor x^2

$y = x^2(x-2)$

Curve crosses y axis when $x = 0$

$y = 0^3 - 2(0^2)$

$y = 0$

Curve crosses x axis when $y = 0$

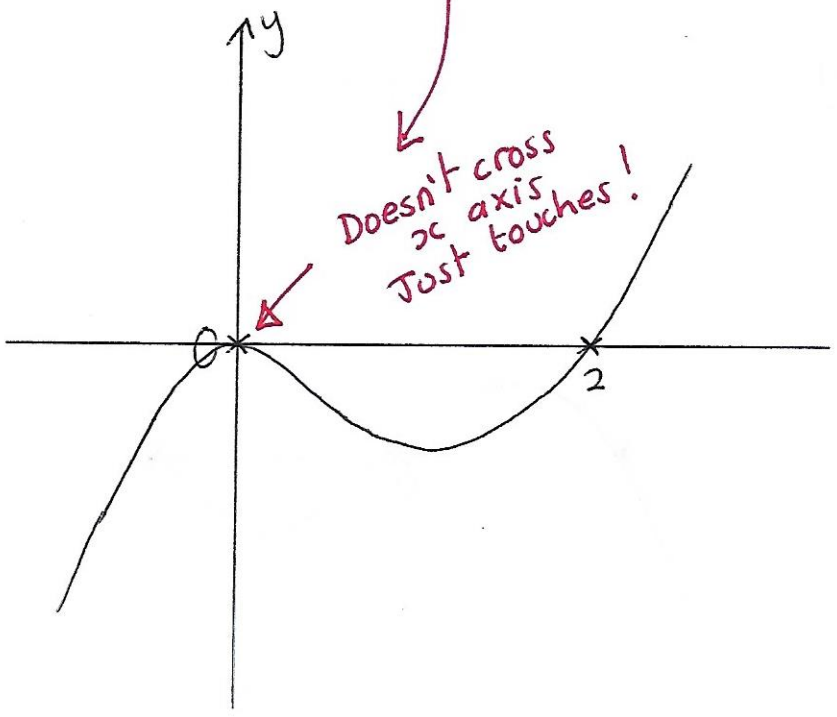
$0 = x^2(x-2)$

either $x^2 = 0$ or $x - 2 = 0$

$x = 0$ $x = 2$

NOTICE only 2 points

Basic x^3 shape is



③ $y = x^3 - 7x + 6$

Curve crosses y axis when $x = 0$

$y = 0^3 - 7(0) + 6$

$y = 6$

Curve crosses x axis when $y = 0$

$0 = x^3 - 7x + 6$

Let $f(x) = x^3 - 7x + 6$

$f(1) = 1 - 7(1) + 6 = 0$

$\therefore$ $(x-1)$ is a factor

Long Division Method

$$
\begin{array}{r}
x^2 + x - 6 \\
x - 1 \overline{)\, x^3 + 0x^2 - 7x + 6} \\
\underline{-x^3 \pm x^2} \\
\quad x^2 - 7x \\
\quad \underline{-x^2 \pm x} \\
\quad\quad -6x + 6 \\
\quad\quad \underline{-6x + 6} \\
\quad\quad\quad 0
\end{array}
$$

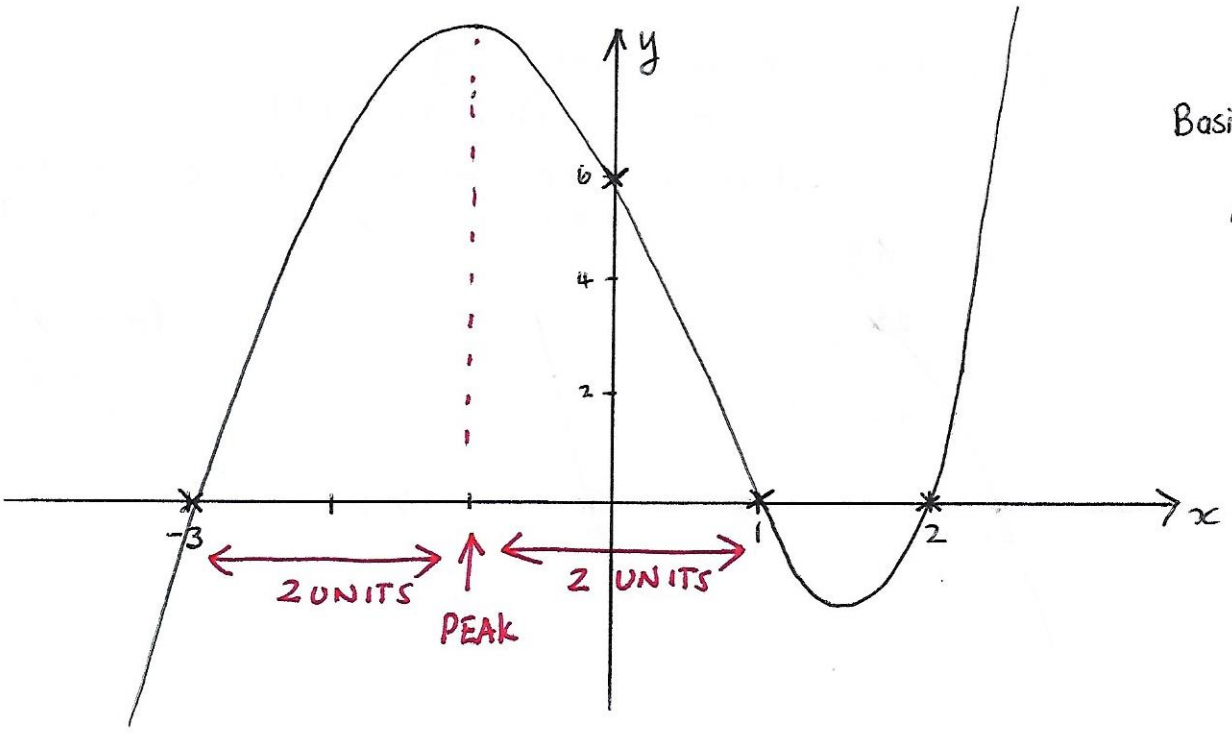
$\therefore$ Crosses x axis

$0 = (x-1)(x^2 + x - 6)$

$0 = (x-1)(x+3)(x-2)$

either $x - 1 = 0$ or $x + 3 = 0$ or $x - 2 = 0$

$x = 1$ $x = -3$ $x = 2$



Basic x^3

(4) $y = x^3 - 2x^2 - x + 2$

Curve crosses y axis when $x = 0$

$y = 0^3 - 2(0^2) - 0 + 2$

$y = 2$

Curve crosses x axis when $y = 0$

$0 = x^3 - 2x^2 - x + 2$

Need to factorise

$f(x) = x^3 - 2x^2 - x + 2$

$f(1) = 1^3 - 2(1^2) - 1 + 2 = 1 - 2 - 1 + 2 = 0$

$\therefore (x-1)$ is a factor

Now $x^3 - 2x^2 - x + 2 = (x-1)(ax^2 + bx + c)$

Coefficients method

Compare x^3: $1 = a$

Compare x: $-1 = c - b$

Compare constants: $2 = -c$

$-2 = c$

$-1 = -2 - b$

$b = -2 + 1$

$b = -1$

$\therefore x^3 - 2x^2 - x + 2 = (x-1)(x^2 - x - 2)$

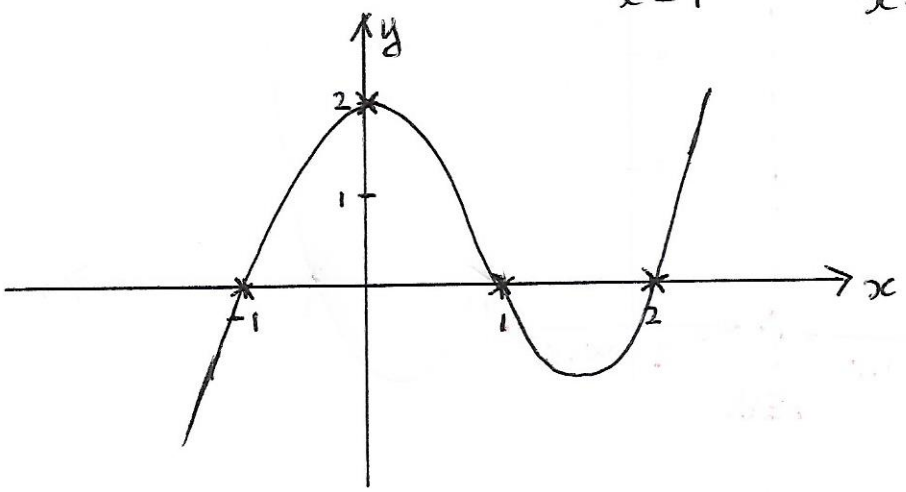
$= (x-1)(x-2)(x+1)$

$\therefore$ Crosses x axis when $y = 0$

$0 = (x-1)(x-2)(x+1)$

either $x - 1 = 0$ or $x - 2 = 0$ or $x + 1 = 0$

$x = 1$ $x = 2$ $x = -1$



Basic x^3

(5) $y = -x^3 - x^2 + 4x + 4$

Curve crosses y axis when $x = 0$

$y = -0^3 - 0^2 + 4(0) + 4$

$y = 4$

Curve crosses x axis when $y = 0$

$0 = -x^3 - x^2 + 4x + 4$

$x^3 + x^2 - 4x - 4 = 0$

Let $f(x) = x^3 + x^2 - 4x - 4$

$f(1) = 1^3 + 1^2 - 4(1) - 4 \neq 0$

$f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4$

$= -1 + 1 + 4 - 4 = 0$

$\therefore (x+1)$ is a factor

Now

$x^3 + x^2 - 4x - 4 = (x+1)(ax^2 + bx + c)$

Compare x^3: $1 = a$

Compare x^2: $1 = b + a$, $1 = b + 1$, $0 = b$

Compare consts: $-4 = c$.

$\therefore x^3 + x^2 - 4x - 4 = (x+1)(x^2 - 4)$

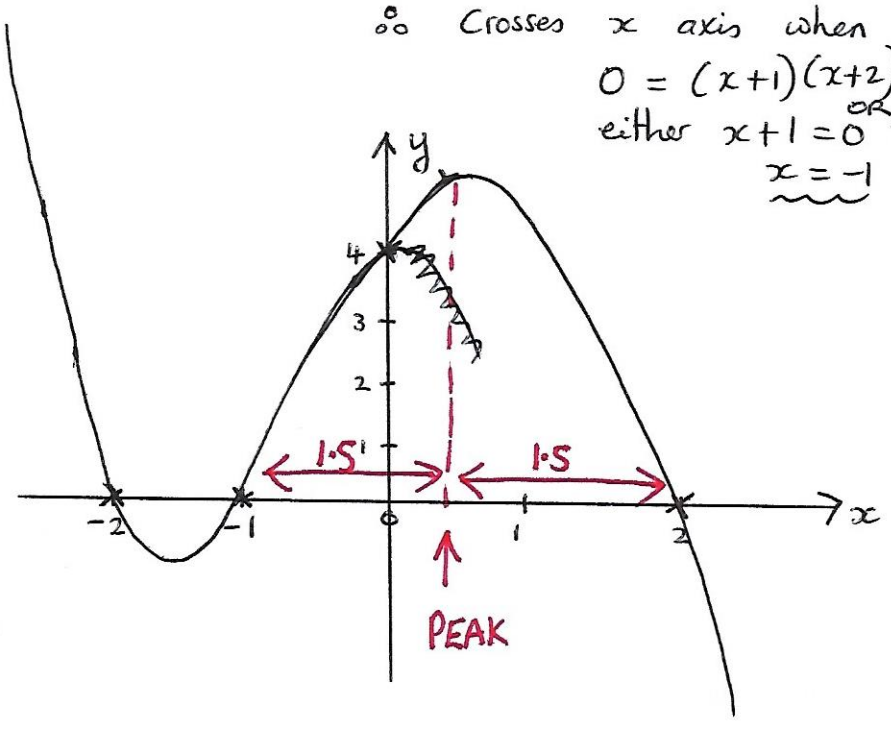
$= (x+1)(x+2)(x-2)$

$\therefore$ Crosses x axis when

$0 = (x+1)(x+2)(x-2)$

either $x+1 = 0$ OR $x+2 = 0$ OR $x-2 = 0$

$x = -1$ $x = -2$ $x = 2$



Now $-x^3$ graph