Differentiation : 3 : Answers

iii) a) $y = 2x^2 - 5x + 3$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \left[\frac{f(x+\delta x) - f(x)}{\delta x} \right]$$

$$= \lim_{\delta x \to 0} \left[\frac{2(x+\delta x)^2 - 5(x+\delta x) + 3 - (2x^2 - 5x + 3)}{\delta x} \right]$$

$$= \lim_{\delta x \to 0} \left[\frac{2x^2 + 4x\delta x + 2(\delta x)^2 - 5x - 5\delta x + 3 - 2x^2 + 5x - 3}{\delta x} \right]$$

$$= \lim_{\delta x \to 0} \left[\frac{4x\,\delta x^1 + 2(\delta x)^2 - 5\delta x^1}{\delta x_1} \right]$$

$$= \lim_{\delta x \to 0} \left[4x + 2\delta x - 5 \right]$$

$$= 4x - 5$$

b) $y = 2x^2 - 5x + 3$

$\frac{dy}{dx} = 4x - 5$

At $(2,1)$ $\frac{dy}{dx} = 4(2) - 5 = 3$

$\therefore m = 3$ point $(2,1)$

Gradient tangent $= 3$ Gradient normal $= -\frac{1}{3}$ $3 \times (-\frac{1}{3}) = -1$

$\therefore$ Eqn normal

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{1}{3}(x - 2)$$

$$3y - 3 = -1(x - 2)$$

$$3y - 3 = -x + 2$$

$$3y + x = 5$$

112) $y = x^2 - 12x + 10$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \left[\frac{f(x+\delta x) - f(x)}{\delta x} \right]$$

$$= \lim_{\delta x \to 0} \left[\frac{(x+\delta x)^2 - 12(x+\delta x) + 10 - (x^2 - 12x + 10)}{\delta x} \right]$$

$$= \lim_{\delta x \to 0} \left[\frac{x^2 + 2x\delta x + (\delta x)^2 - 12x - 12\delta x + 10 - x^2 + 12x - 10}{\delta x} \right]$$

$$= \lim_{\delta x \to 0} \left[\frac{2x\delta x^1 + (\delta x)^2 - 12\delta x^1}{\delta x_1} \right]$$

$$= \lim_{\delta x \to 0} \left[2x + \delta x - 12 \right]$$

$$= 2x - 12$$

113) (a) $y = 3x^2 - 4x + 7$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \left[\frac{f(x+\delta x) - f(x)}{\delta x} \right]$$

$$= \lim_{\delta x \to 0} \left[\frac{3(x+\delta x)^2 - 4(x+\delta x) + 7 - (3x^2 - 4x + 7)}{\delta x} \right]$$

$$= \lim_{\delta x \to 0} \left[\frac{3x^2 + 6x\delta x + 3(\delta x)^2 - 4x - 4\delta x + 7 - 3x^2 + 4x - 7}{\delta x} \right]$$

$$= \lim_{\delta x \to 0} \left[\frac{6x\delta x^1 + 3(\delta x)^2 - 4\delta x^1}{\delta x_1} \right]$$

$$= \lim_{\delta x \to 0} \left[6x + 3\delta x - 4 \right]$$

$$= 6x - 4$$

b) $y = 5\sqrt{x} - \frac{3}{x^3}$

$$y = 5x^{1/2} - 3x^{-3}$$

$$\frac{dy}{dx} = \frac{5}{2}x^{-1/2} + 9x^{-4}$$

$$= \frac{5}{2\sqrt{x}} + \frac{9}{x^4}$$

114) a) $y = 5x^2 + 3x - 4$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \left[\frac{f(x+\delta x) - f(x)}{\delta x} \right]$$

$$= \lim_{\delta x \to 0} \left[\frac{5(x+\delta x)^2 + 3(x+\delta x) - 4 - (5x^2 + 3x - 4)}{\delta x} \right]$$

$$= \lim_{\delta x \to 0} \left[\frac{5x^2 + 10x\delta x + 5(\delta x)^2 + 3x + 3\delta x - 4 - 5x^2 - 3x + 4}{\delta x} \right]$$

$$= \lim_{\delta x \to 0} \left[\frac{10x\,\delta x + 5(\delta x)^2 + 3\delta x}{\delta x} \right]$$

$$= \lim_{\delta x \to 0} \left[10x + 5\delta x + 3 \right]$$

$$= 10x + 3$$

b) $y = \frac{8}{x} + 3\sqrt{x}$

$y = 8x^{-1} + 3x^{1/2}$

$\frac{dy}{dx} = -8x^{-2} + \frac{3}{2}x^{-1/2}$

$= \frac{-8}{x^2} + \frac{3}{4}x^{-3/2}$

$= \frac{-8}{x^2} + \frac{3}{2\sqrt{x}}$

At $x = 4$

$\frac{dy}{dx} = \frac{-8}{4^2} + \frac{3}{2\sqrt{4}}$

$= \frac{-8}{16} + \frac{3}{4}$

$= \frac{-2}{4} + \frac{3}{4}$

$= \frac{1}{4}$

115) a) $y = 7x^2 + 5x - 2$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \left[\frac{f(x+\delta x) - f(x)}{\delta x} \right]$$

$$= \lim_{\delta x \to 0} \left[\frac{7(x+\delta x)^2 + 5(x+\delta x) - 2 - (7x^2 + 5x - 2)}{\delta x} \right]$$

$$= \lim_{\delta x \to 0} \left[\frac{7x^2 + 14x\delta x + 7(\delta x)^2 + 5x + 5\delta x - 2 - 7x^2 - 5x + 2}{\delta x} \right]$$

$$= \lim_{\delta x \to 0} \left[\frac{14x\delta x^1 + 7(\delta x)^2 + 5\delta x^1}{\delta x^1} \right]$$

$$= \lim_{\delta x \to 0} \left[14x + 7\delta x^1 + 5 \right]$$

$$= 14x + 5$$

b) $y = \frac{2}{x^3} + 5x^{2/3}$

$y = 2x^{-3} + 5x^{2/3}$

$$\frac{dy}{dx} = -6x^{-4} + \frac{10}{3}x^{-1/3}$$

$$= \frac{-6}{x^4} + \frac{10}{3\sqrt[3]{x}}$$

116) a) $y = 4x^2 - 5x - 3$

$$\frac{dy}{dx} = \lim_{\delta x \to 0}\left[\frac{f(x+\delta x) - f(x)}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[\frac{4(x+\delta x)^2 - 5(x+\delta x) - 3 - (4x^2 - 5x - 3)}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[\frac{4x^2 + 8x\delta x + 4(\delta x)^2 - 5x - 5\delta x - 3 - 4x^2 + 5x + 3}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[\frac{8x\,\delta x + 4(\delta x)^2 - 5\delta x}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[8x + 4\delta x - 5\right]$$

$$= 8x - 5$$

b) $y = 7x^{3/4} - \frac{2}{x^4}$

$$y = 7x^{3/4} - 2x^{-4}$$

$$\frac{dy}{dx} = \frac{21}{4}x^{-1/4} + 8x^{-5}$$

$$= \frac{21}{4\sqrt[4]{x}} + \frac{8}{x^5}$$

117) a) $y = 3x^2 - 7x - 5$

$$\frac{dy}{dx} = \lim_{\delta x \to 0}\left[\frac{f(x+\delta x) - f(x)}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[\frac{3(x+\delta x)^2 - 7(x+\delta x) - 5 - (3x^2 - 7x - 5)}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[\frac{3x^2 + 6x\delta x + 3(\delta x)^2 - 7x - 7\delta x - 5 - 3x^2 + 7x + 5}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[\frac{6x\,\delta x + 3(\delta x)^2 - 7\delta x}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[6x + 3\delta x - 7\right]$$

$$= 6x - 7$$

b) $y = ax^{5/2}$

$$\frac{dy}{dx} = \frac{5}{2}ax^{3/2} = -2 \quad \text{when } x = 4$$

$$\frac{5a}{2}(4^{3/2}) = -2$$

$$20a = -2$$

$$a = -1/10$$

118) a) $y = -x^2 + 5x - 9$

$$\frac{dy}{dx} = \lim_{\delta x \to 0}\left[\frac{f(x+\delta x) - f(x)}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[\frac{-(x+\delta x)^2 + 5(x+\delta x) - 9 - (-x^2+5x-9)}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[\frac{-x^2 - 2x\delta x - (\delta x)^2 + 5x + 5\delta x - 9 + x^2 - 5x + 9}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[\frac{-2x\delta x - (\delta x)^2 + 5\delta x}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[-2x - \delta x + 5\right]$$

$$= -2x + 5$$

b) $y = \frac{3}{4}x^{1/3} + 12x^{-2}$

$$\frac{dy}{dx} = \frac{1}{4}x^{-2/3} - 24x^{-3}$$

$$= \frac{1}{4\sqrt[3]{x^2}} - \frac{24}{x^3}$$

when $x = 8$

$$\frac{dy}{dx} = \frac{1}{4(4)} - \frac{24}{8^3}$$

$$= \frac{1}{16} - \frac{24}{512}$$

$$= \frac{1}{16} - \frac{3}{64}$$

$$= \frac{1}{64}$$

$$\begin{array}{r} 64 \\ \times_3 8 \\ \hline 512 \end{array}$$

119) (a) $y = 6x^2 + 4x - 9$

$$\frac{dy}{dx} = \lim_{\delta x \to 0}\left[\frac{f(x+\delta x) - f(x)}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[\frac{6(x+\delta x)^2 + 4(x+\delta x) - 9 - (6x^2+4x-9)}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[\frac{6x^2 + 12x\delta x + 6(\delta x)^2 + 4x + 4\delta x - 9 - 6x^2 - 4x + 9}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[\frac{12x\delta x + 6(\delta x)^2 + 4\delta x}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[12x + 6\delta x + 4\right]$$

$$= 12x + 4$$

b) $y = \frac{3}{x^4} - 7x^{1/3}$

$y = 3x^{-4} - 7x^{1/3}$

$\frac{dy}{dx} = -12x^{-5} - \frac{7}{3}x^{-2/3}$

$= -\frac{12}{x^5} - \frac{7}{3\sqrt[3]{x^2}}$

120) a) $y = 7x^2 - 5x + 2$

$$\frac{dy}{dx} = \lim_{\delta x \to 0}\left[\frac{7(x+\delta x)^2 - 5(x+\delta x) + 2 - (7x^2 - 5x + 2)}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[\frac{7x^2 + 14x\delta x + 7(\delta x)^2 - 5x - 5\delta x + 2 - 7x^2 + 5x - 2}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[\frac{14x\delta x + 7(\delta x)^2 - 5\delta x}{\delta x}\right]$$

$$= \lim_{\delta x \to 0}\left[14x + 7\delta x - 5\right]$$

$$= 14x - 5$$

b) $y = 4x^{3/5} - 9x^{-1} - 6$

$\frac{dy}{dx} = \frac{12}{5}x^{-2/5} + 9x^{-2}$

$= \frac{12}{5\sqrt[5]{x^2}} + \frac{9}{x^2}$