Differentiation Higher : Year 13 : 1 : Answers

1) Spec 2001/2002

a) $y = \dfrac{2x}{x^2+1}$

$u = 2x \quad v = x^2+1$

$\dfrac{du}{dx} = 2 \quad \dfrac{dv}{dx} = 2x$

$$\frac{dy}{dx} = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$$

$$\frac{dy}{dx} = \frac{1}{(x^2+1)^2}\left[(x^2+1)2 - 2x(2x)\right]$$

$$= \frac{1}{(x^2+1)^2}\left[2x^2+2-4x^2\right]$$

$$= \frac{(2-2x^2)}{(x^2+1)^2}$$

$$= \frac{2(1-x^2)}{(x^2+1)^2}$$

b) $y = x^2 \tan 2x$

$u = x^2 \quad v = \tan 2x$

$\dfrac{du}{dx} = 2x \quad \dfrac{dv}{dx} = 2\operatorname{Sec}^2 2x$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = x^2 2\operatorname{Sec}^2 2x + 2x\tan 2x$$

$$= 2x^2\operatorname{Sec}^2 2x + 2x\tan 2x$$

$$= 2x\left(x\operatorname{Sec}^2 2x + \tan 2x\right)$$

c) $y = \dfrac{1}{\sqrt{3x^2+2}}$

$$y = (3x^2+2)^{-1/2}$$

$$\frac{dy}{dx} = -\frac{1}{2}(3x^2+2)^{-3/2} \times 6x$$

$$= -\frac{3x}{\sqrt{(3x^2+2)^3}}$$

2) June 2001

a) $$y = \frac{e^{2x}}{x+1}$$

$u = e^{2x}$ $\quad v = x+1$

$\frac{du}{dx} = 2e^{2x}$ $\quad \frac{dv}{dx} = 1$

$$\frac{dy}{dx} = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$$

$$\frac{dy}{dx} = \frac{1}{(x+1)^2}\left[(x+1)2e^{2x} - e^{2x}(1)\right]$$

$$= \frac{e^{2x}\left(2(x+1) - 1\right)}{(x+1)^2}$$

$$= \frac{e^{2x}(2x+2-1)}{(x+1)^2}$$

$$= \frac{e^{2x}(2x+1)}{(x+1)^2}$$

b) $$y = x^2 \sin 3x$$

$u = x^2$ $\quad v = \sin 3x$

$\frac{du}{dx} = 2x$ $\quad \frac{dv}{dx} = 3\cos 3x$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = x^2 3\cos 3x + 2x\sin 3x$$

$$= x\left(3x\cos 3x + 2\sin 3x\right)$$

c) $$y = \sqrt{1+\tan x}$$

$$y = (1+\tan x)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}(1+\tan x)^{-1/2} \times \sec^2 x$$

$$= \frac{\sec^2 x}{2\sqrt{1+\tan x}}$$

(3) May 2002

a) (i) $y = (1+e^{2x})^5$

$$\frac{dy}{dx} = 5(1+e^{2x})^4 \times 2e^{2x}$$
$$= 10e^{2x}(1+e^{2x})^4$$

(ii) $y = \dfrac{x^2}{\tan x}$

$u = x^2 \quad v = \tan x$

$\dfrac{du}{dx} = 2x \quad \dfrac{dv}{dx} = \sec^2 x$

$$\frac{dy}{dx} = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$$
$$= \frac{1}{\tan^2 x}\left[2x\tan x - x^2\sec^2 x\right]$$
$$= \frac{x(2\tan x - x\sec^2 x)}{\tan^2 x}$$

b) $y = \cos 2x + x + 1$

$\dfrac{dy}{dx} = -2\sin 2x + 1 \quad = 0$ at SP's

$$1 = 2\sin 2x$$
$$\frac{1}{2} = \sin 2x$$

$\alpha = \pi/6 \quad$ Sin +ve 1st + 2nd.

$\therefore \quad 2x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

$x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$

$$\frac{d^2y}{dx^2} = -4\cos 4x$$

$x = \pi/12 \quad \dfrac{d^2y}{dx^2} = -4\cos \pi/3$

$= -VE$

$\therefore$ LOCAL MAX

$x = 5\pi/12 \quad \dfrac{d^2y}{dx^2} = -4\cos \dfrac{20\pi}{12}$

$= +VE$

$\therefore$ LOCAL MIN

④ May 2003

a) (i) $y = (1+4\tan x)^6$

$$\frac{dy}{dx} = 6(1+4\tan x)^5 \times 4\sec^2 x$$

$$= 24\sec^2 x\,(1+4\tan x)^5$$

(ii) $y = x^2 \ln(3x+1)$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$u = x^2$ $\quad v = \ln(3x^2+1)$

$\frac{du}{dx} = 2x$ $\quad \frac{dv}{dx} = \frac{1}{(3x^2+1)} \times (6x)$

$= \frac{6x}{(3x^2+1)}$

$$= \frac{x^2\, 6x}{(3x^2+1)} + 2x\ln(3x^2+1)$$

$$= 2x\left(\frac{3x^2}{3x^2+1} + \ln(3x^2+1)\right)$$

b) $y = \frac{e^x + 2x}{x+2}$

$u = e^x + 2x$ $\quad v = x+2$

$\frac{du}{dx} = e^x + 2$ $\quad \frac{dv}{dx} = 1$

$$\frac{dy}{dx} = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$$

$$= \frac{1}{(x+2)^2}\left[(x+2)(e^x+2) - 1(e^x+2x)\right]$$

$$= \frac{xe^x + \cancel{2x} + 2e^x + 4 - e^x - \cancel{2x}}{(x+2)^2}$$

$$= \frac{e^x + 4 + xe^x}{(x+2)^2}$$

$$= \frac{e^x(1+x) + 4}{(x+2)^2}$$

QED.

(5) May 2004

a) $y = \dfrac{4x^2}{x^3+1}$

$u = 4x^2 \qquad v = x^3+1$

$\dfrac{du}{dx} = 8x \qquad \dfrac{dv}{dx} = 3x^2$

$$\frac{dy}{dx} = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$$

$$= \frac{1}{(x^3+1)^2}\left[8x(x^3+1) - 4x^2(3x^2)\right]$$

$$= \frac{8x^4+8x-12x^4}{(x^3+1)^2}$$

$$= \frac{8x-4x^4}{(x^3+1)^2}$$

$$= \frac{4x(2-x^3)}{(x^3+1)^2}$$

b) $y = x^2\tan x$

$u = x^2 \qquad v = \tan x$

$\dfrac{du}{dx} = 2x \qquad \dfrac{dv}{dx} = \sec^2 x$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= x^2\sec^2 x + 2x\tan x$$

$$= x(x\sec^2 x + 2\tan x)$$

c) $y = \dfrac{1}{\sqrt{2x^4+3}}$

$$y = (2x^4+3)^{-1/2}$$

$$\frac{dy}{dx} = -\frac{1}{2}(2x^4+3)^{-3/2} \times 8x^3$$

$$= \frac{-4x^3}{\sqrt{(2x^4+3)^3}}$$