Implicit Differentiation 3 : Answers

1) $x^4 + 3x^2y - 2y^2 = 15$

$4x^3 + 3x^2\frac{dy}{dx} + y6x - 4y\frac{dy}{dx} = 0$

$3x^2\frac{dy}{dx} - 4y\frac{dy}{dx} = -4x^3 - 6xy$

$\frac{dy}{dx}(3x^2 - 4y) = -4x^3 - 6xy$

$\frac{dy}{dx} = \frac{-4x^3 - 6xy}{(3x^2 - 4y)}$ $\begin{matrix}\times(-1)\\ \times(-1)\end{matrix}$

$\frac{dy}{dx} = \frac{4x^3 + 6xy}{4y - 3x^2}$

$\underline{3x^2y}$

$u = 3x^2$ $\quad v = y$

$\frac{du}{dx} = 6x$ $\quad \frac{dv}{dx} = \frac{dy}{dx}$

$u\frac{dv}{dx} + v\frac{du}{dx} = 3x^2\frac{dy}{dx} + y6x$

2) $x^3 + 2x\cos y + y^2 = 1 + \frac{\pi^2}{4}$

$3x^2 + 2x\left(-\sin y\frac{dy}{dx}\right) + 2\cos y + 2y\frac{dy}{dx} = 0 + 0$

$-2x\sin y\frac{dy}{dx} + 2y\frac{dy}{dx} = -3x^2 - 2\cos y$

$2\frac{dy}{dx}(y - x\sin y) = -3x^2 - 2\cos y$

$\frac{dy}{dx} = -\frac{3x^2 - 2\cos y}{2(y - x\sin y)}$ $\begin{matrix}\times(-1)\\ \times(-1)\end{matrix}$

$\frac{dy}{dx} = \frac{3x^2 + 2\cos y}{2(x\sin y - y)}$

$\underline{2x\cos y}$

$u = 2x$ $\quad v = \cos y$

$\frac{du}{dx} = 2$ $\quad \frac{dv}{dx} = -\sin y\frac{dy}{dx}$

At $(1, \pi/2)$

$\frac{dy}{dx} = \frac{3(1^2) + 2\cos\pi/2}{2(1\sin\pi/2 - \pi/2)}$

$= \frac{3 + 0}{2(1 - \pi/2)}$

$= \frac{3}{2\left(\frac{2-\pi}{2}\right)}$

$= \frac{3}{(2 - \pi)}$

3) $\frac{dy}{dx} = x^2 y$

$\frac{d^2y}{dx^2} = x^2 \frac{dy}{dx} + 2xy$

Now $\frac{dy}{dx} = x^2 y$

$\therefore \frac{d^2y}{dx^2} = x^2(x^2y) + 2xy$

$= x^4y + 2xy$

$= xy(x^3+2)$

x^2y

$u = x^2 \quad v = y$

$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = \frac{dy}{dx}$

$u\frac{dv}{dx} + v\frac{du}{dx}$

4) $x^2y^2 + x^4 + 6 = 2y^3 + 2x$

$x^2 2y\frac{dy}{dx} + 4x^3 + 0 = 6y^2\frac{dy}{dx} + 2$

$2x^2y\frac{dy}{dx} - 6y^2\frac{dy}{dx} = 2 - 4x^3$

$2y\frac{dy}{dx}(x^2 - 3y) = 2(1-2x^3)$

$\div 2 \quad y\frac{dy}{dx}(x^2-3y) = 1-2x^3$

$\frac{dy}{dx} = \frac{(1-2x^3)}{y(x^2-3y)}$

At $(2,3)$

$\frac{dy}{dx} = \frac{1-2(2^3)}{3[2^2-9]}$

$= \frac{1-16}{3(-5)}$

$= \frac{-15}{-15}$

$= +1$

x^2y^2

$u = x^2 \quad v = y^2$

$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = 2y\frac{dy}{dx}$

$u\frac{dv}{dx} + v\frac{du}{dx}$

5) $x^3 + 5x^4y - 2y^3 + 7 = 0$

$$3x^2 + 5x^4\frac{dy}{dx} + 20x^3y - 6y^2\frac{dy}{dx} + 0 = 0$$

$$5x^4\frac{dy}{dx} - 6y^2\frac{dy}{dx} = -3x^2 - 20x^3y$$

$$\frac{dy}{dx}(5x^4 - 6y^2) = -3x^2 - 20x^3y$$

$$\frac{dy}{dx} = \frac{-3x^2 - 20x^3y}{(5x^4 - 6y^2)} \quad \begin{matrix} \times(-1) \\ \times(-1) \end{matrix}$$

$$= \frac{3x^2 + 20x^3y}{(6y^2 - 5x^4)}$$

$$= \frac{x^2(3 + 20xy)}{(6y^2 - 5x^4)}$$

$5x^4y$

$u = 5x^4 \qquad v = y$

$\frac{du}{dx} = 20x^3 \qquad \frac{dv}{dx} = \frac{dy}{dx}$

6)

$$x^4 - 3x^2y + 2y^3 - 4x = 7$$

$$4x^3 - \left(3x^2\frac{dy}{dx} + 6xy\right) + 6y^2\frac{dy}{dx} - 4 = 0$$

$$4x^3 - 3x^2\frac{dy}{dx} - 6xy + 6y^2\frac{dy}{dx} - 4 = 0$$

$$6y^2\frac{dy}{dx} - 3x^2\frac{dy}{dx} = 4 + 6xy - 4x^3$$

$$3\frac{dy}{dx}\left(2y^2 - x^2\right) = 4 + 6xy - 4x^3$$

$$\frac{dy}{dx} = \frac{4 + 6xy - 4x^3}{3(2y^2 - x^2)}$$

$$= \frac{2(2 + 3xy - 2x^3)}{3(2y^2 - x^2)}$$

$3x^2y$

$u = 3x^2 \qquad v = y$

$\frac{du}{dx} = 6x \qquad \frac{dv}{dx} = \frac{dy}{dx}$

$u\frac{dv}{dx} + v\frac{du}{dx}$

7) $x^3y^2 = 128$ curve C

x^3y^2

$u = x^3$ $\quad$ $v = y^2$

$\frac{du}{dx} = 3x^2$ $\quad$ $\frac{dv}{dx} = 2y\frac{dy}{dx}$

a) $x^3 2y\frac{dy}{dx} + 3x^2y^2 = 0$

$$2x^3y\frac{dy}{dx} = -3x^2y^2$$

$$\frac{dy}{dx} = \frac{-3x^2y^2}{2x^3y}$$

$$= \frac{-3y}{2x}$$

b) $P(a,b)$ on C

Now when $x = a$, $y = b$, $\frac{dy}{dx} = 3$

$$\therefore \quad \frac{dy}{dx} = \frac{-3y}{2x}$$

$$3 = \frac{-3b}{2a}$$

$$1 = \frac{-b}{2a}$$

$$2a = -b$$

OR $b = -2a$ QED.

Now from

$$x^3y^2 = 128$$

$$a^3b^2 = 128$$

If $b = -2a$

$$a^3(-2a)^2 = 128$$

$$4a^5 = 128$$

$$a^5 = 32$$

$$a = 2$$

$\therefore$ $b = -2(2)$

$$b = -4$$

8) $x^2 + 3xy + 2y^3 - 2x = 21$

$$2x + 3x\frac{dy}{dx} + 3y + 6y^2\frac{dy}{dx} - 2 = 0$$

$$3x\frac{dy}{dx} + 6y^2\frac{dy}{dx} = 2 - 2x - 3y$$

$$3\frac{dy}{dx}(x + 2y^2) = 2 - 2x - 3y$$

$$\frac{dy}{dx} = \frac{(2 - 2x - 3y)}{3(x + 2y^2)}$$

$3xy$

$u = 3x \quad v = y$

$\frac{du}{dx} = 3 \quad \frac{dv}{dx} = \frac{dy}{dx}$

$u\frac{dv}{dx} + v\frac{du}{dx}$

At $P(-5, 2)$

$$\frac{dy}{dx} = \frac{(2 - (-10) - 6)}{3(-5 + 8)}$$

$$= \frac{6}{3(3)}$$

$$= \frac{6}{9}$$

$$= \frac{2}{3}$$