Integration : Areas under Curves 1 : Answers

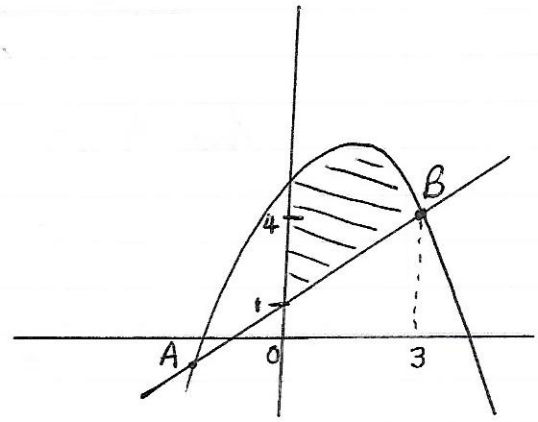
113. a) Solve $y = 7 + 2x - x^2$
$y = x + 1$ simultaneously

$\therefore \quad 7 + 2x - x^2 = x + 1$
$0 = x^2 - x - 6$
$0 = (x-3)(x+2)$

$x = 3$ or $x = -2$

$\therefore \quad y = 3 + 1$
$y = 4$

$B(3,4)$



b) Area shaded $= \displaystyle\int_0^3 7 + 2x - x^2 \, dx$ − area trapezium

$= \left[7x + x^2 - \frac{x^3}{3}\right]_0^3 - \frac{(a+b)h}{2}$

$= (21 + 9 - 9) - (0) - \left[\frac{(1+4)3}{2}\right]$

$= 21 - \frac{15}{2}$

$= \frac{27}{2}$ units^2

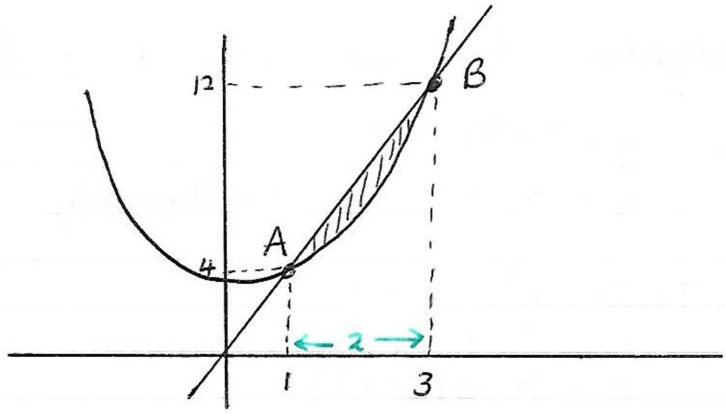
114) b) (i) Solve $y = x^2 + 3$
$y = 4x$ simultaneously

$\therefore \quad x^2 + 3 = 4x$
$x^2 - 4x + 3 = 0$
$(x-3)(x-1) = 0$
either $x = 3$ or $x = 1$

$y = 4(3)$ $y = 4(1)$
$y = 12$ $y = 4$

$A(3,12)$ $B(1,4)$

(ii)



Area //// = trapezium − area under curve

$$= \frac{(a+b)h}{2} - \int_1^3 x^2 + 3 \, dx$$

$$= \frac{(4+12)2}{2} - \left[\frac{x^3}{3} + 3x\right]_1^3$$

$$= 16 - \left[(9+9) - \left(\frac{1}{3} + 3\right)\right]$$

$$= 16 - \left[18 - \frac{10}{3}\right]$$

$$= 16 - \frac{44}{3}$$

$$= \frac{48}{3} - \frac{44}{3}$$

$$= \frac{4}{3} \text{ units}^2$$

115) b)(i) Solve $y = x^2 + 2$
$y = 3x$ simultaneously

$\therefore \quad x^2 + 2 = 3x$
$x^2 - 3x + 2 = 0$
$(x-2)(x-1) = 0$

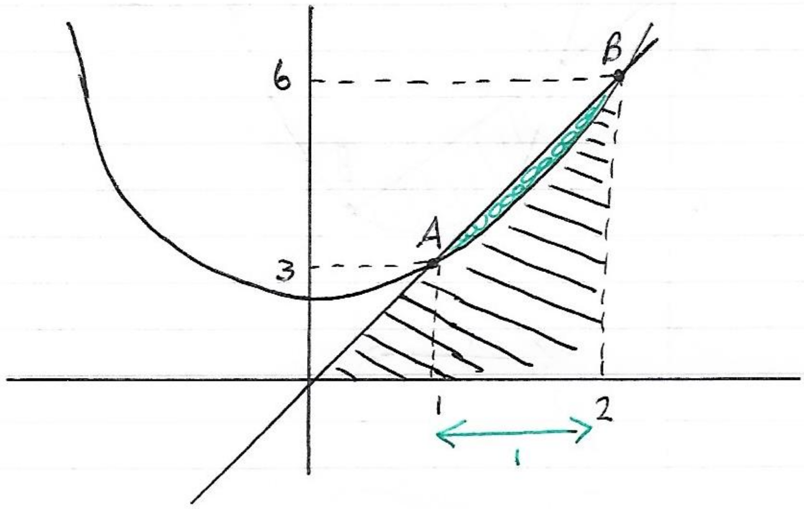
either $x = 2$ or $x = 1$

$\downarrow$ $\quad$ $\downarrow$

$y = 3(2)$ $\quad$ $y = 3(1)$
$y = 6$ $\quad$ $y = 3$

$B(2,6)$ $\quad$ $A(1,3)$

(ii)



Area //// = area large triangle − green area

First: green area = area trapezium − area under curve ($x=1$ to $x=2$)

$$= \frac{(a+b)h}{2} - \int_1^2 x^2+2\,dx$$

$$= \frac{(3+6)1}{2} - \left[\frac{x^3}{3} + 2x\right]_1^2$$

$$= \frac{9}{2} - \left[\left(\frac{8}{3}+4\right) - \left(\frac{1}{3}+2\right)\right]$$

$$= \frac{9}{2} - \left[\frac{7}{3}+2\right]$$

$$= \frac{9}{2} - \frac{13}{3} = \frac{27}{6} - \frac{26}{6} = \frac{1}{6} \text{ units}^2$$

$\therefore$ Area needed //// $= \frac{bh}{2} - \frac{1}{6}$

$$= \frac{2(6)}{2} - \frac{1}{6} = 6 - \frac{1}{6} = \frac{35}{6} \text{ units}^2$$

11b) b)(i) $y = x^2 - 6x + 11$
$y = -x + 7$ solve simultaneously

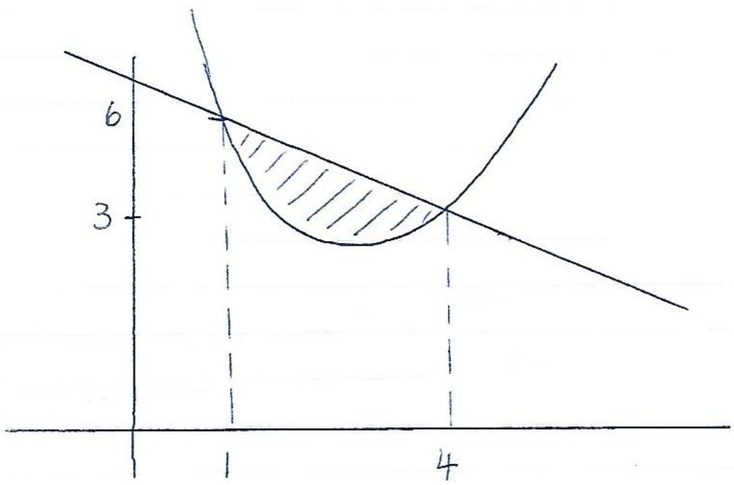
$x^2 - 6x + 11 = -x + 7$
$x^2 - 5x + 4 = 0$
$(x-4)(x-1) = 0$
either $x = 4$ or $x = 1$

$\downarrow$ $\downarrow$

$y = -4 + 7$
$y = 3$ $y = 6$

$B(4,3)$ $A(1,6)$

(ii)



Area //// = trapezium − area under curve $x = 1$ to 4

$$= \frac{(a+b)h}{2} - \int_1^4 x^2 - 6x + 11 \, dx$$

$$= \frac{(6+3)3}{2} - \left[\frac{x^3}{3} - 3x^2 + 11x\right]_1^4$$

$$= \frac{27}{2} - \left[\left(\frac{64}{3} - 48 + 44\right) - \left(\frac{1}{3} - 3 + 11\right)\right]$$

$$= \frac{27}{2} - \left[\frac{63}{3} - 4 - 8\right]$$

$$= \frac{27}{2} - \left[\frac{63}{3} - \frac{36}{3}\right]$$

$$= \frac{27}{2} - 9$$

$$= \frac{27}{2} - \frac{18}{2}$$

$$= \frac{9}{2} \text{ units}^2.$$