AREAS UNDER CURVES 3 : ANSWERS

121) $y = 3x + \frac{1}{5}x^3$

Curve crosses x axis when $y = 0$

$$0 = 3x + \frac{1}{5}x^3$$

$$0 = x\left(3 + \frac{1}{5}x^2\right)$$

either $x = 0$ or $3 + \frac{1}{5}x^2 = 0$

$$\frac{1}{5}x^2 = -3$$

$$x^2 = -15$$

NO SOLNS.

$\therefore$ Only crosses x axis at $x = 0$

$\therefore$ Area needed $= \int_1^3 3x + \frac{1}{5}x^3 \, dx$

$$= \left[\frac{3x^2}{2} + \frac{x^4}{20}\right]_1^3$$

$$= \left(\frac{27}{2} + \frac{81}{20}\right) - \left(\frac{3}{2} + \frac{1}{20}\right)$$

$$= \frac{24}{2} + \frac{80}{20}$$

$$= 12 + 4$$

$$= 16 \text{ units}^2$$

122) b) (i) $y = 25 - x^2$
$y = -2x + 17$ Solve simultaneously

$$25 - x^2 = -2x + 17$$
$$0 = x^2 - 2x - 8$$
$$0 = (x-4)(x+2)$$

either $x - 4 = 0$ or $x + 2 = 0$

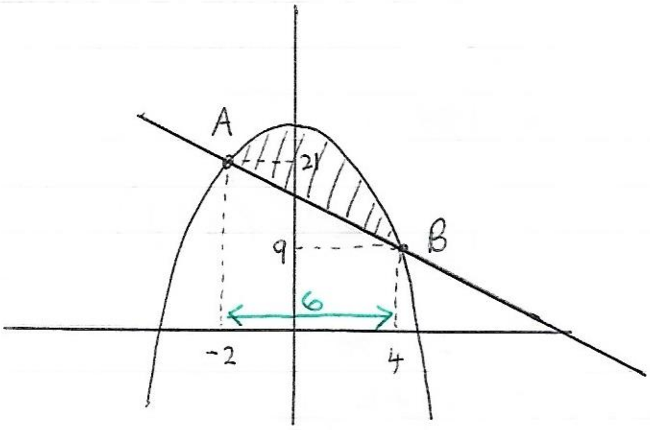
$x = 4$ $x = -2$

$y = -2(4) + 17$ $y = -2(-2) + 17$

$y = 9$ $y = 21$

$B(4, 9)$ $A(-2, 21)$

(ii)



Area //// = area under curve $x = -2$ to $+4$ − area trapezium

$$= \int_{-2}^{4} 25 - x^2 \, dx - \frac{(a+b)h}{2}$$

$$= \left[25x - \frac{x^3}{3}\right]_{-2}^{4} - \frac{(21+9)6}{2}$$

$$= \left(100 - \frac{64}{3}\right) - \left(-50 + \frac{8}{3}\right) - 90$$

$$= 150 - \frac{72}{3} - 90$$

$$= 150 - 24 - 90$$

$$= 36 \text{ units}^2$$

123) (b) (i) $y = x^2 - 4x + 6$

$y = -x + 10$ Solve simultaneously

$x^2 - 4x + 6 = -x + 10$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

either or

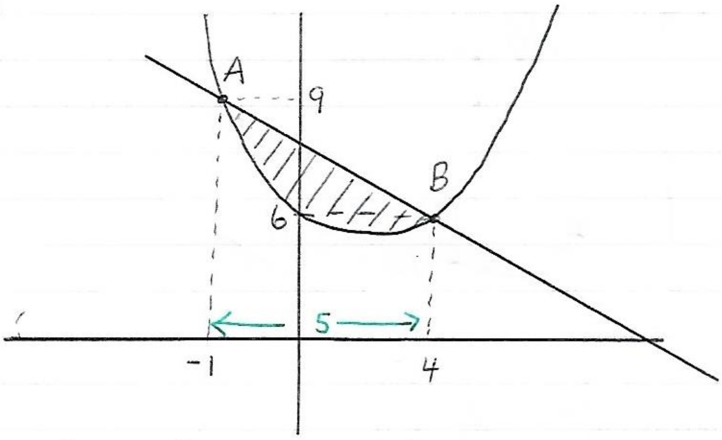
$x = 4$ $x = -1$

$y = -4 + 10$ $y = -1 + 10$

$y = 6$ $y = 9$

$B(4,6)$ $A(-1,9)$

(ii)



Area //// = area trapezium − area under curve $x=-1$ to $x=4$

$$= \frac{(a+b)h}{2} - \int_{-1}^{4} x^2 - 4x + 6 \, dx$$

$$= \frac{(9+6)5}{2} - \left[\frac{x^3}{3} - 2x^2 + 6x\right]_{-1}^{4}$$

$$= \frac{75}{2} - \left[\left(\frac{64}{3} - 32 + 24\right) - \left(-\frac{1}{3} - 2 - 6\right)\right]$$

$$= \frac{75}{2} - \left[\frac{64}{3} \cancel{-8} + \frac{1}{3} \cancel{+8}\right]$$

$$= \frac{75}{2} - \left[\frac{65}{3}\right]$$

$$= \frac{225}{6} - \frac{130}{6}$$

$$= \frac{95}{6} \text{ units}^2$$

124) (i) $y = 4 - x^2$ crosses x axis when $y = 0$

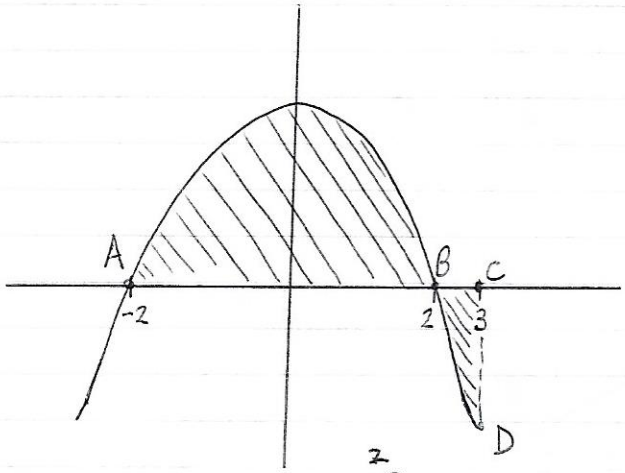
$0 = 4 - x^2$

$0 = (2-x)(2+x)$

either $x = 2$ or $x = -2$

$\therefore A(-2,0) \quad B(2,0)$ ← NOT ASKED FOR

(ii)



Area above x axis $= \displaystyle\int_{-2}^{2} 4 - x^2 \, dx$

$= \left[4x - \dfrac{x^3}{3}\right]_{-2}^{2}$

$= \left(8 - \dfrac{8}{3}\right) - \left(-8 + \dfrac{8}{3}\right)$

$= 8 - \dfrac{8}{3} + 8 - \dfrac{8}{3}$

$= +16$ units^2 ie above x axis

Area below x axis $= \displaystyle\int_{+2}^{+3} 4 - x^2 \, dx$

$= \left[4x - \dfrac{x^3}{3}\right]_{2}^{3}$

$= (12 - 9) - \left(8 - \dfrac{8}{3}\right)$

$= 3 - \left(\dfrac{16}{3}\right)$

$= \dfrac{9}{3} - \dfrac{16}{3} = -\dfrac{7}{3}$ units^2 ie below x axis

$\therefore$ Total area $= 16 + \dfrac{7}{3}$

$= \dfrac{48}{3} + \dfrac{7}{3}$

$= \dfrac{55}{3}$ units^2

125) (b) (i) $y = 36 - x^2$

$y = 5x$ Solve simultaneously

$36 - x^2 = 5x$

$0 = x^2 + 5x - 36$

$0 = (x+9)(x-4)$

either $x = -9$ or $x = 4$

Not needed here

$y = 5(4)$

$y = 20$

$\therefore A(4, 20)$

To find B coords

$y = 36 - x^2$ crosses x axis when $y = 0$

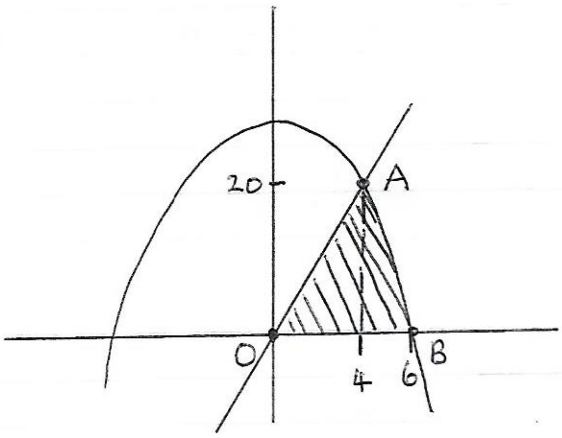
$0 = 36 - x^2$

$x^2 = 36$

$x = \pm 6$

$\therefore$ $B(6, 0)$

(ii)



Area //// = $\triangle$ 0 4 + area under curve $x = 4$ to 6

$= \frac{bh}{2} + \int_4^6 36 - x^2 \, dx$

$= \frac{4 \times 20}{2} + \left[36x - \frac{x^3}{3}\right]_4^6$

$= 40 + \left[\left(216 - \frac{216}{3}\right) - \left(144 - \frac{64}{3}\right)\right]$

$= 40 + \left[72 - \frac{152}{3}\right]$

$= 40 + \frac{216}{3} - \frac{152}{3}$

$= \frac{120}{3} + \frac{64}{3}$

$= \frac{184}{3}$ units^2