Integration 4 : Area Under Curves : Answers

1) Find equation of curve first

Crosses x axis $x = -1$ $x = 0$ $x = 2$

$\therefore$ Curve is $y = (x+1)x(x-2)$

$y = x(x^2 - x - 2)$

$y = x^3 - x^2 - 2x$

$\therefore$ Area needed is in 2 parts.

first
$$\int_{-1}^{0} x^3 - x^2 - 2x \, dx$$
$$= \left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_{-1}^{0}$$
$$= (0-0-0) - \left(\frac{1}{4} + \frac{1}{3} - 1\right)$$
$$= 0 - \left(-\frac{5}{12}\right)$$
$$= +\frac{5}{12} \text{ units}^2$$

+ sign shows above x axis as diagram shows

$$\int_{0}^{2} x^3 - x^2 - 2x \, dx$$
$$= \left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_{0}^{2}$$
$$= \left(4 - \frac{8}{3} - 4\right) - (0-0-0)$$
$$= -\frac{8}{3} \text{ units}^2$$

− sign shows below x axis as diagram shows

$\therefore$ Area needed
$$= \frac{5}{12} + \frac{8}{3}$$
$$= \frac{5}{12} + \frac{32}{12}$$
$$= \frac{37}{12} \text{ units}^2$$

2) First find equation of curve

Curve is

$$y = (x+2)(x+1)(x-1)$$
$$y = (x+2)(x^2-1)$$
$$y = x^3 - x + 2x^2 - 2$$
$$y = x^3 + 2x^2 - x - 2$$

BUT  shape $\therefore$ $-x^3$ graph

$\therefore$ $y = -x^3 - 2x^2 + x + 2$ is equation

because $-x^3 - 2x^2 + x + 2$ crosses x axis in same places !!

$\therefore$ Area below $= \displaystyle\int_{-2}^{-1} -x^3 - 2x^2 + x + 2 \, dx$

$$= \left[-\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2} + 2x \right]_{-2}^{-1}$$

$$= \left(-\frac{1}{4} + \frac{2}{3} + \frac{1}{2} - 2 \right) - \left(-4 + \frac{16}{3} + 2 - 4 \right)$$

$$= \left(-\frac{13}{12} \right) - \left(-\frac{2}{3} \right)$$

$$= -\frac{13}{12} + \frac{8}{12}$$

$$= -\frac{5}{12}$$ $-$ shows below x axis

Area above $= \displaystyle\int_{-1}^{1} -x^3 - 2x^2 + x + 2 \, dx$

$$= \left[-\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2} + 2x \right]_{-1}^{1}$$

$$= \left(\cancel{-\frac{1}{4}} - \frac{2}{3} + \cancel{\frac{1}{2}} + 2 \right) - \left(\cancel{-\frac{1}{4}} + \frac{2}{3} + \cancel{\frac{1}{2}} - 2 \right)$$

$$= \left(\frac{4}{3} \right) - \left(-\frac{4}{3} \right)$$

$$= +\frac{8}{3}$$ $+$ shows above x axis

$\therefore$ Total area $= \dfrac{5}{12} + \dfrac{8}{3}$

$$= \frac{5}{12} + \frac{32}{12}$$

$$= \frac{37}{12} \text{ units}^2$$

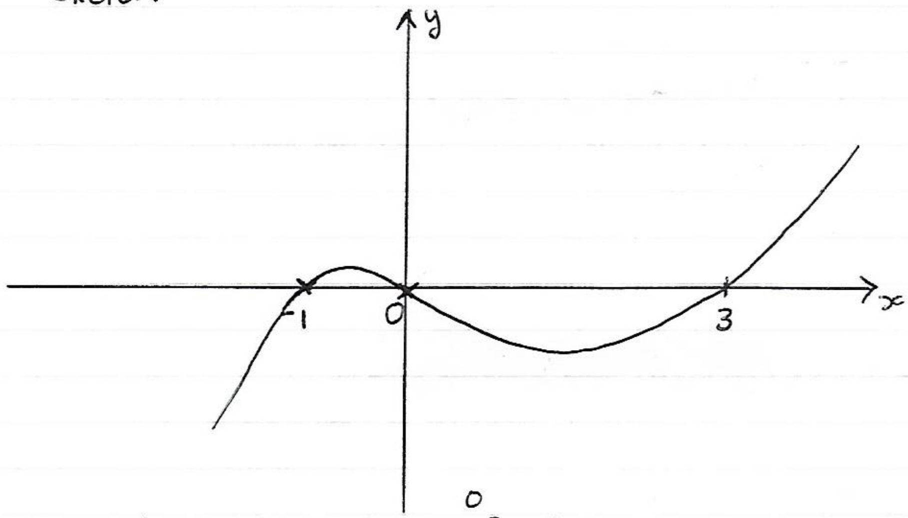
3) $y = x^3 - 2x^2 - 3x$

Curve crosses x axis $y = 0$

$$0 = x^3 - 2x^2 - 3x$$
$$0 = x(x^2 - 2x - 3)$$
$$0 = x(x-3)(x+1)$$

$\therefore x = 0$ or $x - 3 = 0$ or $x + 1 = 0$
$x = 3$ $\quad x = -1$

Sketch



$\therefore$ Area **ABOVE** is $\displaystyle\int_{-1}^{0} x^3 - 2x^2 - 3x \, dx$

$$= \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{3x^2}{2}\right]_{-1}^{0}$$
$$= (0 - 0 - 0) - \left(\frac{1}{4} + \frac{2}{3} - \frac{3}{2}\right)$$
$$= 0 - \left(\frac{3}{12} + \frac{8}{12} - \frac{18}{12}\right)$$
$$= 0 - \left(-\frac{7}{12}\right) = +\frac{7}{12} \text{ units}^2$$

Area BELOW is $\displaystyle\int_{0}^{3} x^3 - 2x^2 - 3x \, dx$

$$= \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{3x^2}{2}\right]_{0}^{3}$$
$$= \left(\frac{81}{4} - 18 - \frac{27}{2}\right) - (0)$$
$$= \frac{81}{4} - \frac{72}{4} - \frac{54}{4}$$
$$= -\frac{45}{4} \text{ units}^2$$

$\therefore$ Total area $= \frac{7}{12} + \frac{135}{12} = \frac{142}{12} = \frac{71}{6}$ units^2.