Stationary Points 1 : Answers

129) $y = x^3 + 3x^2 - 9x - 13$

a) $\frac{dy}{dx} = 3x^2 + 6x - 9 \quad = 0 \quad$ at SP's

$$x^2 + 2x - 3 = 0$$
$$(x+3)(x-1) = 0$$

either $x + 3 = 0$ or $x - 1 = 0$

$x = -3$ $\qquad$ $x = 1$

$$y = (-3)^3 + 3(-3)^2 - 9(-3) - 13$$
$$y = -27 + 27 + 27 - 13$$
$$y = 14$$
$$(-3, 14)$$

$$y = 1^3 + 3(1^2) - 9(1) - 13$$
$$y = 1 + 3 - 9 - 13$$
$$y = -18$$
$$(1, -18)$$

$(-3, 14)$

$$\frac{d^2y}{dx^2} = 6x + 6$$

$$\frac{d^2y}{dx^2} = 6(-3) + 6$$
$$= -12$$

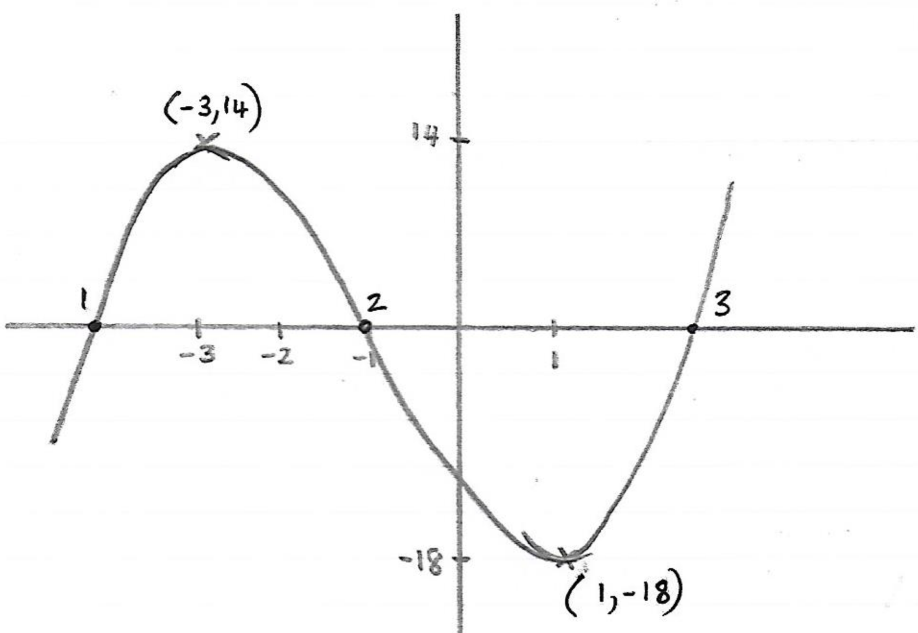
$\therefore$ LOCAL MAX

$(1, -18)$

$$\frac{d^2y}{dx^2} = 6(1) + 6$$
$$= +12$$

$\therefore$ LOCAL MIN

b)



c) Curve crosses x axis ($y = 0$) in 3 places. $\therefore$ 3 real roots

130) $y = x^3 - 3x^2 + 3x + 5$

a) $\frac{dy}{dx} = 3x^2 - 6x + 3 = 0$ at SP's

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$\therefore x - 1 = 0$

$x = 1$ $\therefore$ only 1 S.P.

$y = 1^3 - 3(1^2) + 3(1) + 5$

$y = 1 - 3 + 3 + 5$

$y = 6$

$\therefore (1,6)$

b) $\frac{d^2y}{dx^2} = 6x - 6$

at $x = 1$

$\frac{d^2y}{dx^2} = 6(1) - 6$

$= 0$

$\therefore$ Point of inflection at $(1,6)$

131) $y = x^3 - 6x^2 + 20$

a) $\frac{dy}{dx} = 3x^2 - 12x = 0$ at SP's

$$x^2 - 4x = 0$$
$$x(x-4) = 0$$

either $x = 0$ or $x - 4 = 0$
$x = 4$

$y = 0^3 - 6(0^2) + 20$
$y = 20$
$(0, 20)$

$y = 4^3 - 6(4^2) + 20$
$y = 64 - 96 + 20$
$y = -12$
$(4, -12)$

$$\frac{d^2y}{dx^2} = 6x - 12$$

At $x = 0$

$\frac{d^2y}{dx^2} = 0 - 12$
$= -12$

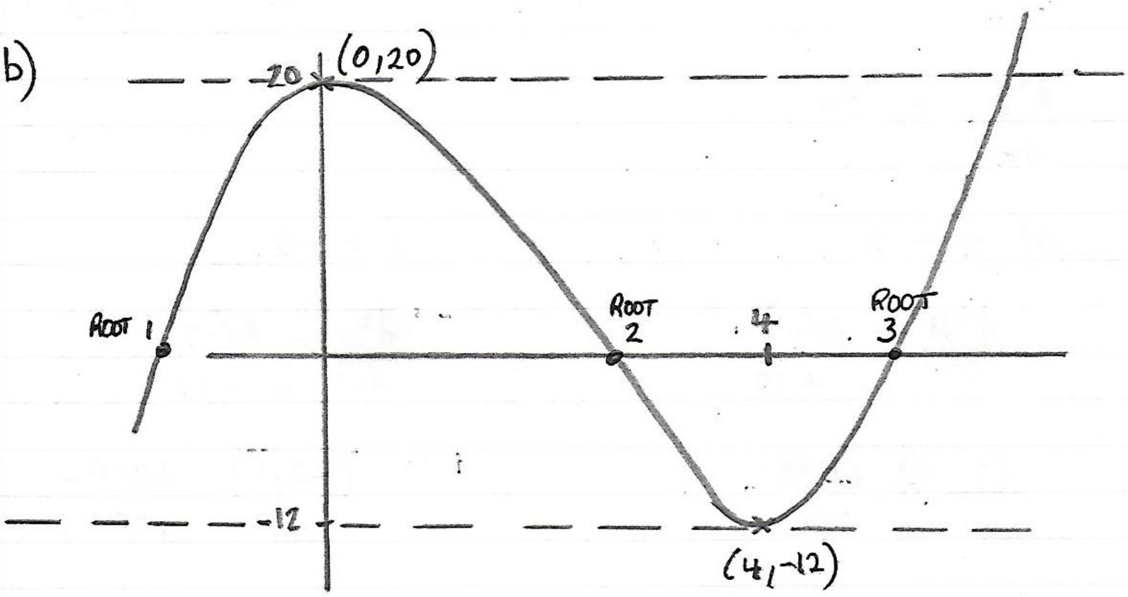
LOCAL MAX
$(0, 20)$

$x = 4$

$\frac{d^2y}{dx^2} = 6(4) - 12$
$= +12$

LOCAL MIN
$(4, -12)$

b)



c) Curve crosses x axis $(y=0)$ in 3 places.

$\therefore$ 3 distinct roots for eqn

$$x^3 - 6x^2 + 20 = 0$$

There will be 3 distinct answers obtained as long as

$-12 < K < 20$ as can be seen from dotted horizontal lines on the graph.

132) $y = \frac{1}{2}x^3 - 6x + 3$

$$\frac{dy}{dx} = \frac{3}{2}x^2 - 6 = 0 \text{ at SP's}$$

$$3x^2 - 12 = 0$$

$$x^2 - 4 = 0$$

$$(x-2)(x+2) = 0$$

either $x - 2 = 0$ or $x + 2 = 0$

$x = 2$ $\qquad$ $x = -2$

$\downarrow$ $\qquad$ $\downarrow$

$y = \frac{1}{2}(2^3) - 6(2) + 3$

$y = 4 - 12 + 3$

$y = -5$

$(2, -5)$

$y = \frac{1}{2}(-2)^3 - 6(-2) + 3$

$y = -4 + 12 + 3$

$y = 11$

$(-2, 11)$

$$\frac{d^2y}{dx^2} = 3x$$

At $x = 2$

$$\frac{d^2y}{dx^2} = 6(2) = +12$$

$x = -2$

$$\frac{d^2y}{dx^2} = 6(-2) = -12$$

$\therefore (2, -5)$ LOCAL MIN

$\therefore (-2, 11)$ LOCAL MAX.