Stationary Points 2 : Answers

133) $y = x^3 + kx^2 - 9x - 10$

a) $\frac{dy}{dx} = 3x^2 + 2kx - 9 = 0$ at SP's.

If Q has x value -1

$$3(-1)^2 + 2k(-1) - 9 = 0$$
$$3 - 2k - 9 = 0$$
$$-6 = 2k$$
$$-3 = k$$

b) $k = -3$ $\therefore$ $\frac{dy}{dx} = 3x^2 - 6x - 9 = 0$

$$x^2 - 2x - 3 = 0$$
$$(x+1)(x-3) = 0$$

either $x + 1 = 0$ or $x - 3 = 0$

$x = -1$ $x = 3$

$k = -3$ Curve $y = x^3 - 3x^2 - 9x - 10$

$y = (-1)^3 - 3(-1)^2 - 9(-1) - 10$

$y = -1 - 3 + 9 - 10$

$y = -5$

$Q(-1, -5)$

$y = (+3)^3 - 3(3^2) - 9(3) - 10$

$y = 27 - 27 - 27 - 10$

$y = -37$

$R(3, -37)$

c) $\frac{d^2y}{dx^2} = 6x - 6$

$x = -1$

$\frac{d^2y}{dx^2} = -6 - 6$

$= -12$

$Q(-1, -5)$ LOCAL MAX

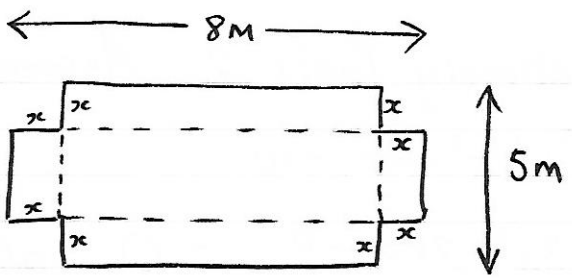
$x = 3$

$\frac{d^2y}{dx^2} = 18 - 6$

$= +12$

$R(3, -37)$ LOCAL MIN

134)



a) $V = (8-2x)(5-2x)x$

$V = (40-16x-10x+4x^2)x$

$V = (40-26x+4x^2)x$

$V = 4x^3-26x^2+40x$

b) $\frac{dV}{dx} = 12x^2-52x+40 = 0$ at SP's

$3x^2-13x+10 = 0$

$(3x-2)(x-5) = 0$

either

$3x-2=0$ $\quad$ $x-5=0$

$x = +2/3$ m. $\quad$ $x = 5$ m

~~IMPOSSIBLE~~ $\quad$ IMPOSSIBLE

$\frac{d^2V}{dx^2} = 24x-52$

when $x = \frac{2}{3}$

$\frac{d^2V}{dx^2} = 24\left(\frac{2}{3}\right)-52$

$= 16-52$

$= -36$

∴ Max Volume when $x = 2/3$

$V_{MAX} = 4\left(\frac{2}{3}\right)^3-26\left(\frac{2}{3}\right)^2+40\left(\frac{2}{3}\right)$

$= 4\times\frac{8}{27} - 26\times\frac{4}{9} + \frac{80}{3}$

$= \frac{32}{27} - \frac{312}{27} + \frac{720}{27}$

$= \frac{440}{27}$ m^3

135) $y = x^3 - 6x^2 + 12x - 9$

a) $\frac{dy}{dx} = 3x^2 - 12x + 12 = 0$ at SP's

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

either $x-2=0$ or $x-2=0$

$x = 2$

only 1 stationary point

$x = 2$

$y = 2^3 - 6(2^2) + 12(2) - 9$

$y = 8 - 24 + 24 - 9$

$y = -1$ $\therefore$ SP is $(2, -1)$

b) $\frac{d^2y}{dx^2} = 6x - 12$

$x = 2$

$\frac{d^2y}{dx^2} = 6(2) - 12$

$= 0$

$\therefore$ $(2, -1)$ is a point of inflection.