Stationary Points 3 : Answers

136) $y = x^3 + 3x^2 - 1$

a) $\frac{dy}{dx} = 3x^2 + 6x = 0$ at SP's

$x^2 + 2x = 0$

$x(x+2) = 0$

either $x = 0$ or $x + 2 = 0$

$x = -2$

$y = 0^3 + 3(0^2) - 1$

$y = -1$

$(0,-1)$

$y = (-2)^3 + 3(-2)^2 - 1$

$y = -8 + 12 - 1$

$y = 3$

$(-2,3)$

$\frac{d^2y}{dx^2} = 6x + 6$

$x = 0$

$\frac{d^2y}{dx^2} = 6(0) + 6$

$= 6$

$\therefore (0,-1)$ is LOCAL MIN

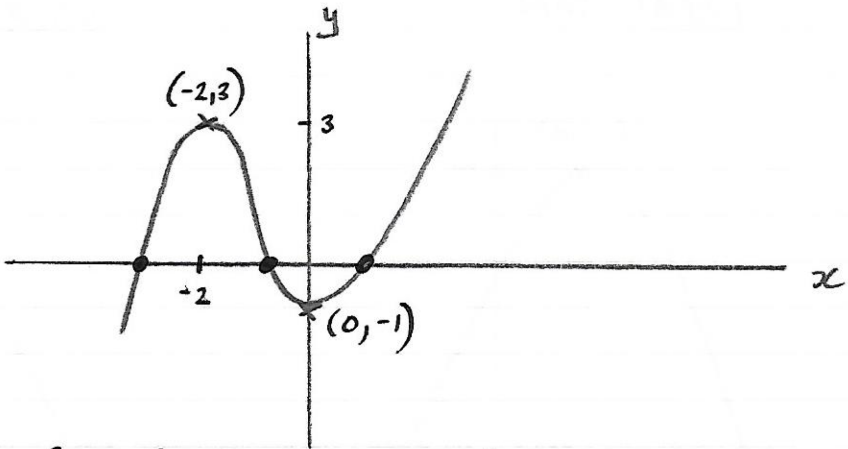
$x = -2$

$\frac{d^2y}{dx^2} = 6(-2) + 6$

$= -6$

$\therefore (-2,3)$ is LOCAL MAX

b)



c) $x^3 + 3x^2 - 1 = 0$ has 3 real roots as there are 3 intersection points of the curve and the x axis $(y=0)$

137) $y = x^3 - 3x^2 - 9x + 14$

a) $\frac{dy}{dx} = 3x^2 - 6x - 9 = 0$ at SP's

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

either $x - 3 = 0$ or $x + 1 = 0$

$x = 3$ $\quad$ $x = -1$

$\downarrow$ $\quad$ $\downarrow$

$y = 3^3 - 3(3)^2 - 9(3) + 14$ $\quad$ $y = (-1)^3 - 3(-1)^2 - 9(-1) + 14$

$y = 27 - 27 - 27 + 14$ $\quad$ $y = -1 - 3 + 9 + 14$

$y = -13$ $\quad$ $y = 19$

$(3, -13)$ $\quad$ $(-1, 19)$

$\frac{d^2y}{dx^2} = 6x - 6$

$x = 3$

$\frac{d^2y}{dx^2} = 18 - 6$

$= 12$

$\therefore (3, -13)$ is

LOCAL MIN

$x = -1$

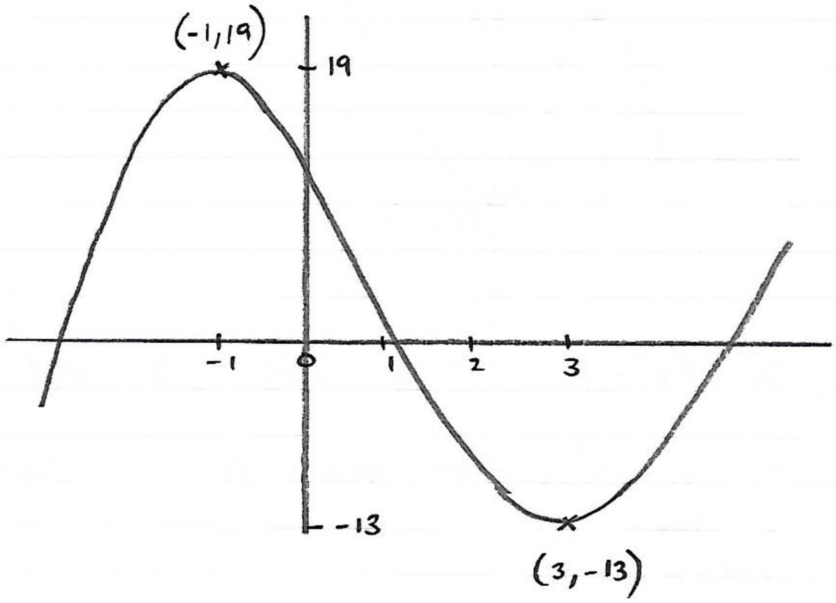
$\frac{d^2y}{dx^2} = 6(-1) - 6$

$= -12$

$\therefore (-1, 19)$ is

LOCAL MAX

b)



124) $y = x^3 - 3x^2 - 9x + 2$

$\frac{dy}{dx} = 3x^2 - 6x - 9 \quad = 0$ at SP's

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

either $x - 3 = 0$ or $x + 1 = 0$

$x = 3$ $\qquad$ $x = -1$

$\downarrow$

$y = 3^3 - 3(3^2) - 9(3) + 2$

$y = 27 - 27 - 27 + 2$

$y = -25$

$(3, -25)$

$y = (-1)^3 - 3(-1)^2 - 9(-1) + 2$

$y = -1 - 3 + 9 + 2$

$y = 7$

$(-1, 7)$

$\frac{d^2y}{dx^2} = 6x - 6$

$x = 3$

$\frac{d^2y}{dx^2} = 6(3) - 6$

$= 12$

$\therefore (3, -25)$ is

LOCAL MIN

$x = -1$

$\frac{d^2y}{dx^2} = 6(-1) - 6$

$= -12$

$\therefore (-1, 7)$ is

LOCAL MAX

125) $y = 4x^3 - 12x + 3$

a) $\frac{dy}{dx} = 12x^2 - 12 = 0$ at SP's

$x^2 - 1 = 0$

$(x-1)(x+1) = 0$

either $x = 1$ or $x = -1$

$\downarrow$ $\downarrow$

$y = 4(1^3) - 12(1) + 3$

$y = 4 - 12 + 3$

$y = -5$

$(1, -5)$

$y = 4(-1)^3 - 12(-1) + 3$

$y = -4 + 12 + 3$

$y = 11$

$(-1, 11)$

$\frac{d^2y}{dx^2} = 24x$

$x = 1$

$\frac{d^2y}{dx^2} = 24(1)$

$= +24$

$\therefore (1, -5)$ is LOCAL MIN

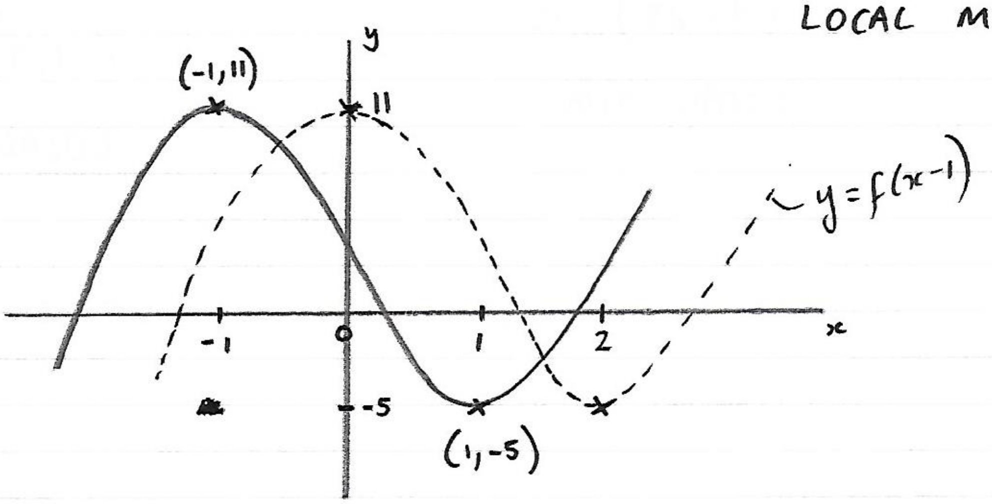
$x = -1$

$\frac{d^2y}{dx^2} = 24(-1)$

$= -24$

$\therefore (-1, 11)$ is LOCAL MAX

b)



c) Drawn above in dotted line

$y = f(x-1)$

$\leftarrow$ move 1 right

12b) $y = x^3 - x^2 - x + 2$

a) $\frac{dy}{dx} = 3x^2 - 2x - 1 = 0$ at SP's

$(3x+1)(x-1) = 0$

either

$3x + 1 = 0$ or $x - 1 = 0$

$x = -\frac{1}{3}$ $\quad$ $x = 1$

$\downarrow$ $\quad$ $\downarrow$

$y = \left(-\frac{1}{3}\right)^3 - \left(-\frac{1}{3}\right)^2 - \left(-\frac{1}{3}\right) + 2$

$y = -\frac{1}{27} - \frac{1}{9} + \frac{1}{3} + 2$

$y = -\frac{1}{27} - \frac{3}{27} + \frac{9}{27} + \frac{54}{27}$

$y = \frac{59}{27}$

$\left(-\frac{1}{3}, \frac{59}{27}\right)$

$y = 1^3 - 1^2 - 1 + 2$

$y = 1 - 1 - 1 + 2$

$y = 1$

$(1, 1)$

$\frac{d^2y}{dx^2} = 6x - 2$

$x = -\frac{1}{3}$

$\frac{d^2y}{dx^2} = 6\left(-\frac{1}{3}\right) - 2$

$= -2 - 2$

$= -4$

$\left(-\frac{1}{3}, \frac{59}{27}\right)$ is LOCAL MAX

$x = 1$

$\frac{d^2y}{dx^2} = 6(1) - 2$

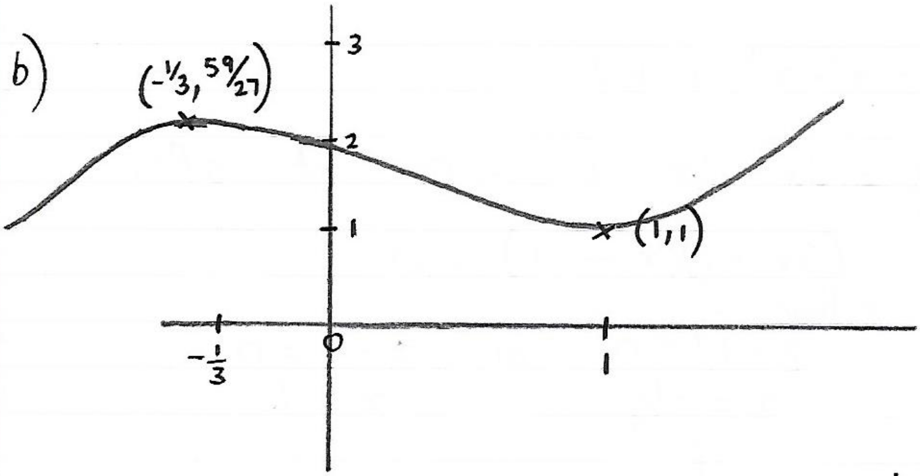
$= +4$

$\therefore (1, 1)$ is LOCAL MIN

b)

PTO

b)



* This curve
was <u>not</u>
asked for !!!

127) $y = x^3 - 12x + 11$

a) $\frac{dy}{dx} = 3x^2 - 12 = 0$ at SP's

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

either $x = 2$ or $x = -2$

$y = 2^3 - 12(2) + 11$

$y = 8 - 24 + 11$

$y = -5$

$(2, -5)$

$y = (-2)^3 - 12(-2) + 11$

$y = -8 + 24 + 11$

$y = 27$

$(-2, 27)$

$\frac{d^2y}{dx^2} = 6x$

$x = 2$

$\frac{d^2y}{dx^2} = 6(2)$

$= +12$

$\therefore (2, -5)$ is LOCAL MIN

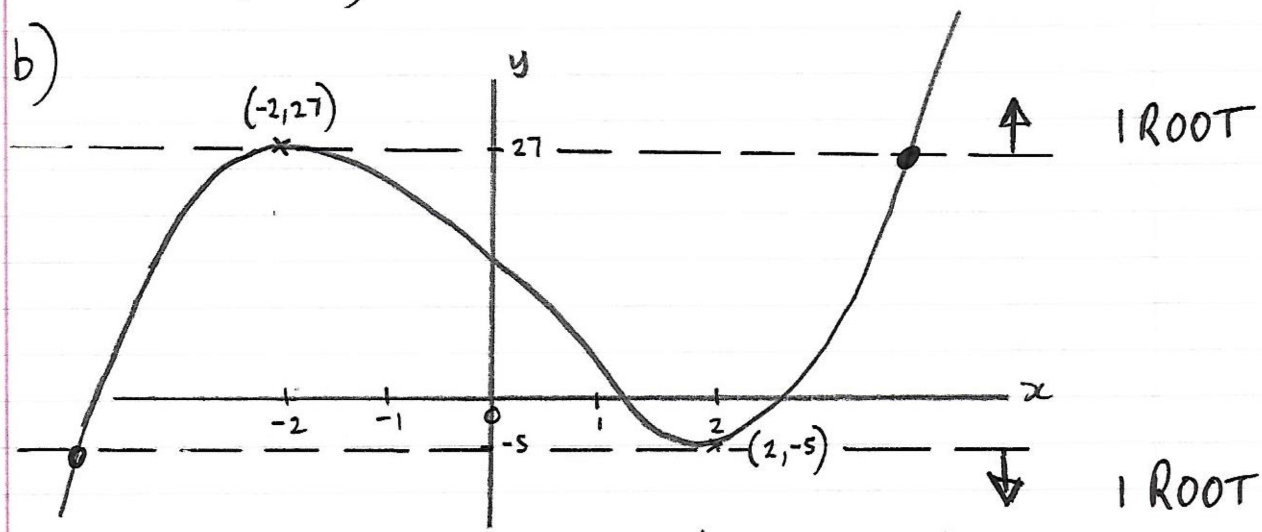
$x = -2$

$\frac{d^2y}{dx^2} = 6(-2)$

$= -12$

$\therefore (-2, 27)$ is LOCAL MAX.

b)



c) $x^3 - 12x + 11 = k$ has only one root

when $k > 27$ or $k < -5$