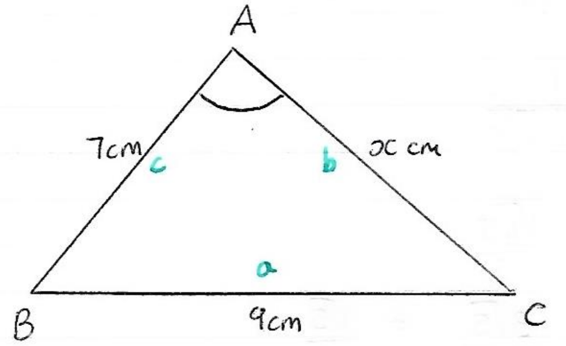
Triangle Trig 2 : Answers

62)



$\cos B\hat{A}C = \frac{2}{7}$

* Must be acute angle. because Cos +ve

a)

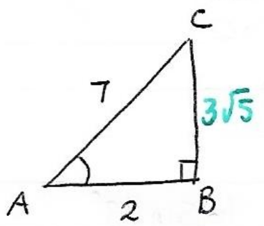
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$9^2 = x^2 + 7^2 - 2(x)(7)\cos B\hat{A}C$$
$$81 = x^2 + 49 - 14x\left(\tfrac{2}{7}\right)$$

$$81 = x^2 + 49 - 4x$$
$$0 = x^2 - 4x - 32$$
$$0 = (x - 8)(x + 4)$$

either $x - 8 = 0$ or ~~$x + 4 = 0$~~
$x = 8\text{cm}$ ~~$x = -4$~~
IMPOSSIBLE

b) (i) $\sin B\hat{A}C$

$\cos B\hat{A}C = \frac{2}{7}$



$$7^2 = 2^2 + BC^2$$
$$49 = 4 + BC^2$$
$$45 = BC^2$$
$$3\sqrt{5} = BC$$

$\therefore \sin B\hat{A}C = \frac{o}{h}$

$= \frac{3\sqrt{5}}{7}$ or $\frac{\sqrt{45}}{7}$ *

ie $m = 45$
$n = 7$

(ii) Now $B\hat{A}C$ is Acute from (*) and since it is opposite the longest side all angles are acute.

$\therefore A\hat{C}B$ is acute.

$\sin A\hat{C}B$

From starting △

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{9}{\sqrt{45}/7} = \frac{8}{\sin B} = \frac{7}{\sin A\hat{C}B}$$

$$\frac{9 \times 7}{\sqrt{45}} = \frac{7}{\sin A\hat{C}B}$$

$\div 7$

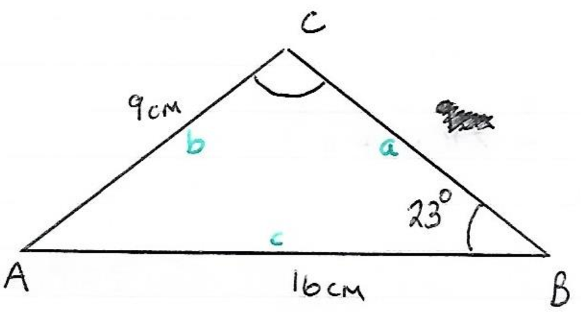
$$\frac{9}{\sqrt{45}} = \frac{1}{\sin A\hat{C}B}$$

$$\sin A\hat{C}B = \frac{\sqrt{45}}{9}$$

$$\sin A\hat{C}B = \frac{3\sqrt{5}}{9} = \frac{\sqrt{5}}{3}$$

where $p = 5$

63)



a) $$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{9}{\sin 23^\circ} = \frac{16}{\sin A\hat{C}B}$$

$$\sin A\hat{C}B = \frac{16 \sin 23^\circ}{9}$$

$$\sin A\hat{C}B = 0.695$$

$$\alpha = 44.0^\circ$$

$\therefore$ Sin +ve 1st or 2nd Quad

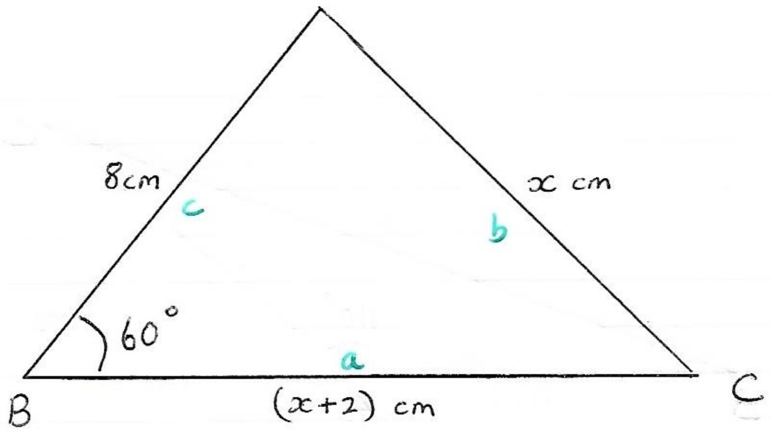
$$A\hat{C}B = 44.0^\circ \quad \text{OR} \quad 136.0^\circ$$
$$\approx 44^\circ \qquad \approx 136^\circ$$

b) $B\hat{A}C$ is acute $\therefore$ $A\hat{C}B = 136^\circ$

(i) $$B\hat{A}C = 180 - 136 - 23$$
$$= 21^\circ$$

(ii) $$\text{Area} = \tfrac{1}{2} bc \sin A$$
$$= \tfrac{1}{2}(9)(16) \sin 21^\circ$$
$$= 25.8 \text{ cm}^2$$

64)



a)

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$x^2 = (x+2)^2 + 8^2 - 2(x+2)8\cos 60^\circ$$
$$x^2 = x^2 + 4x + 4 + 64 - 8(x+2)$$
$$x^2 = x^2 + 4x + 68 - 8x - 16$$
$$4x = 52$$
$$x = 13\text{cm}$$

b)

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

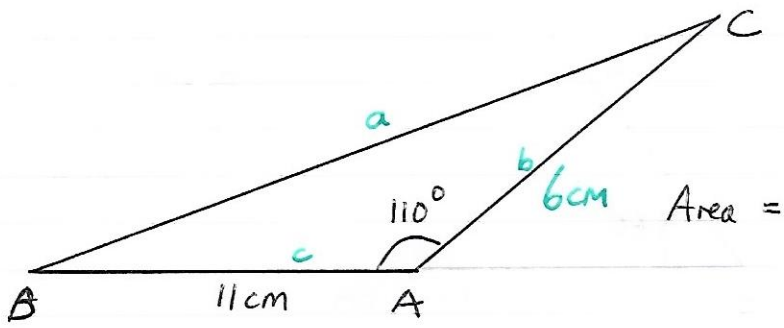
$$\frac{8}{\sin A\hat{C}B} = \frac{13}{\sin 60^\circ}$$

$$\frac{8\sin 60^\circ}{13} = \sin A\hat{C}B$$

$$\frac{4\sqrt{13}}{13} = \sin A\hat{C}B$$

$$32{\cdot}2^\circ = A\hat{C}B$$ (Must be acute as 8cm is smallest side opposite $A\hat{C}B$)

65) (a)



Area $= 31\ cm^2$

$$Area = \frac{1}{2} bc \sin A$$

$$31 = \frac{1}{2}(b)(11)\sin 110°$$

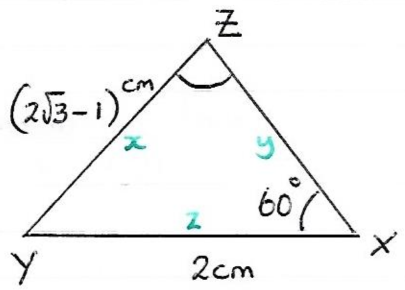
$$\frac{62}{11\sin 110°} = b$$

$$6cm = b = AC$$

Now
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$BC^2 = 6^2 + 11^2 - 2(6)(11)\cos 110°$$

$$BC^2 = 36 + 121 - 132\cos 110°$$
$$BC^2 = 157 + 45.15$$
$$BC^2 = 202.15$$
$$BC = 14.2\ cm.$$

b)



$$\frac{x}{\sin Z\hat{X}Y} = \frac{z}{\sin X\hat{Z}Y}$$

$$\frac{(2\sqrt{3}-1)}{\sin 60°} = \frac{2}{\sin X\hat{Z}Y}$$

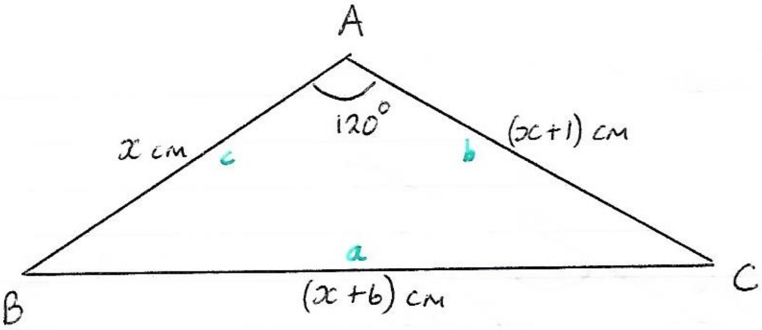
$$\sin X\hat{Z}Y = \frac{2\sin 60°}{(2\sqrt{3}-1)}$$

$$\sin X\hat{Z}Y = \frac{2\frac{\sqrt{3}}{2}}{(2\sqrt{3}-1)} = \frac{\sqrt{3}}{(2\sqrt{3}-1)} \times \frac{(2\sqrt{3}+1)}{(2\sqrt{3}+1)}$$

$$= \frac{6+\sqrt{3}}{12-1} = \frac{6+\sqrt{3}}{11}$$

$m = 6$
$n = 11$

6b)



a) $$a^2 = b^2 + c^2 - 2bc\cos A$$

$$(x+6)^2 = (x+1)^2 + x^2 - 2(x+1)x\cos 120^\circ$$

$$x^2 + 12x + 36 = x^2 + 2x + 1 + x^2 + x(x+1)$$

$$x^2 + 12x + 36 = 2x^2 + 2x + 1 + x^2 + x$$

$$0 = 2x^2 - 9x - 35$$

$$\therefore \quad 0 = (2x+5)(x-7)$$

either $2x + 5 = 0$ or $x = 7$ cm

$x = \cancel{-\frac{5}{2}}$

IMPOSSIBLE

b) $$\text{Area} = \frac{1}{2}bc\sin A$$

$$= \frac{1}{2}(x+1)x\sin 120^\circ$$

$$= \frac{1}{2}(8)(7)\frac{\sqrt{3}}{2}$$

$$= 14\sqrt{3} \text{ units}^2$$

$$= 24\cdot 25 \text{ units}^2$$