Trigonometry : 2 : Answers

71) a) $\tan x = -0.4$

$\alpha = 21.8^\circ$

tan −ve 2nd + 4th

$x = 158.2^\circ, 338.2^\circ$

b) 0° to 180° $\quad \cos 3x = \frac{1}{2}$

$\alpha = 60^\circ$

Cos +ve 1st + 4th

$3x = 60^\circ, 300^\circ, 420^\circ, 660^\circ, 780^\circ, 1020^\circ$

$x = 20^\circ, 100^\circ, 140^\circ, \cancel{220^\circ}$

TOO BIG

c) $2\cos^2\theta + 3\sin\theta = 0$

$2(1-\sin^2\theta) + 3\sin\theta = 0$

$2 - 2\sin^2\theta + 3\sin\theta = 0$

$0 = 2\sin^2\theta - 3\sin\theta - 2$

$0 = (2\sin\theta + 1)(\sin\theta - 2)$

either

$2\sin\theta + 1 = 0$ or $\sin\theta - 2 = 0$

$\sin\theta = -\frac{1}{2}$ $\quad \cancel{\sin\theta = 2}$

IMPOSSIBLE

$\alpha = 30^\circ$

Sin −ve 3rd + 4th

$\theta = 210^\circ, 330^\circ$

72) a) $10\sin^2 x - 3\sin x = 4\cos^2 x + 1$

$10\sin^2 x - 3\sin x = 4(1-\sin^2 x) + 1$

$10\sin^2 x - 3\sin x = 4 - 4\sin^2 x + 1$

$14\sin^2 x - 3\sin x - 5 = 0$

$(7\sin x - 5)(2\sin x + 1) = 0$

either $7\sin x - 5 = 0$ or $2\sin x + 1 = 0$

$\sin x = \frac{5}{7}$ $\quad \sin x = -\frac{1}{2}$

$\alpha = 45.6^\circ$ $\quad \alpha = 30^\circ$

Sin +ve 1st + 2nd $\quad$ Sin −ve 3rd + 4th

$x = 210^\circ, 330^\circ$

$x = 45.6^\circ, 134.4^\circ$

$\therefore x = 45.6^\circ, 134.4^\circ, 210^\circ, 330^\circ$

b) $\tan(2x + 30^\circ) = \sqrt{3}$ $\quad 0^\circ$ to 180°

$\alpha = 60^\circ$

tan +ve 1st + 3rd

$2x + 30^\circ = 60^\circ, 240^\circ, 420^\circ, 600^\circ$

$2x = 30^\circ, 210^\circ, 390^\circ, 570^\circ$

$x = 15^\circ, 105^\circ, \cancel{195^\circ}, \ldots$ ← TOO BIG

73) a) $\tan 3x = \sqrt{3}$ $\quad 0^\circ$ to 180°

$\alpha = 60^\circ$

tan +ve 1st + 3rd

$3x = 60^\circ, 240^\circ, 420^\circ, 600^\circ, 780^\circ, 960^\circ$

$x = 20^\circ, 80^\circ, 140^\circ, \cancel{200^\circ}, \ldots$ ← TOO BIG

b) $4\cos^2\theta - \cos\theta = 2\sin^2\theta$

$4\cos^2\theta - \cos\theta = 2(1 - \cos^2\theta)$

$4\cos^2\theta - \cos\theta = 2 - 2\cos^2\theta$

$6\cos^2\theta - \cos\theta - 2 = 0$

$(2\cos\theta + 1)(3\cos\theta - 2) = 0$

either $2\cos\theta + 1 = 0$ or $3\cos\theta - 2 = 0$

$\cos\theta = -\frac{1}{2}$ $\quad$ $\cos\theta = \frac{2}{3}$

$\alpha = 60^\circ$ $\quad$ $\alpha = 48.2^\circ$

Cos −ve 2nd + 3rd $\quad$ Cos +ve 1st + 4th

$\theta = 120^\circ, 240^\circ$ $\quad$ $\theta = 48.2^\circ, 311.8^\circ$

$\therefore \quad \theta = 48.2^\circ, 120^\circ, 240^\circ, 311.8^\circ$

74) a) $12\sin^2\theta - 5\cos\theta = 9$

$12(1 - \cos^2\theta) - 5\cos\theta = 9$

$12 - 12\cos^2\theta - 5\cos\theta = 9$

$0 = 12\cos^2\theta + 5\cos\theta - 3$

$0 = (3\cos\theta - 1)(4\cos\theta + 3)$

either $3\cos\theta - 1 = 0$ or $4\cos\theta + 3 = 0$

$\cos\theta = \frac{1}{3}$ $\quad$ $\cos\theta = -\frac{3}{4}$

$\alpha = 70.5^\circ$ $\quad$ $\alpha = 41.4^\circ$

Cos +ve 1st + 4th $\quad$ Cos −ve 2nd + 3rd

$\theta = 70.5^\circ, 289.5^\circ$ $\quad$ $\theta = 138.6^\circ, 221.4^\circ$

$\theta = 70.5^\circ, 138.6^\circ, 221.4^\circ, 289.5^\circ$

b) 0° to 180°

$$\sin(3x + 15^\circ) = 0.5$$
$$\alpha = 30^\circ$$
Sin +ve 1st + 2nd

$$3x + 15^\circ = 30^\circ, 150^\circ, 390^\circ, 510^\circ$$
$$3x = 15^\circ, 135^\circ, 375^\circ, 495^\circ$$
$$x = 5^\circ, 45^\circ, 125^\circ, 165^\circ$$

75) a) $2\sin\theta = 3\cos\theta$

$\div \cos\theta$
$$2\tan\theta = 3$$
$$\tan\theta = \frac{3}{2}$$
$$\alpha = 56.3^\circ$$
tan +ve 1st + 3rd

$$\theta = 56.3^\circ, 236.3^\circ$$

b) 0° to 180°
$$\cos 3x = 0.9$$
$$\alpha = 25.8^\circ$$
Cos +ve 1st + 4th

$$3x = 25.8^\circ, 334.2^\circ, 385.8^\circ, 694.2^\circ$$
$$x = 8.6^\circ, 111.4^\circ, 128.6^\circ, \cancel{231.4}^\circ$$ TOO BIG

c) $\sin^2\theta - 4\cos^2\theta = 8\sin\theta$
$$\sin^2\theta - 4(1 - \sin^2\theta) = 8\sin\theta$$
$$\sin^2\theta - 4 + 4\sin^2\theta = 8\sin\theta$$
$$5\sin^2\theta - 8\sin\theta - 4 = 0$$
$$(5\sin\theta + 2)(\sin\theta - 2) = 0$$

either $5\sin\theta + 2 = 0$ or $\sin\theta - 2 = 0$

$\sin\theta = -\frac{2}{5}$ $\quad$ $\cancel{\sin\theta = 2}$

$\alpha = 23.6^\circ$ $\quad$ IMPOSSIBLE

Sin -ve 3rd + 4th

$$\theta = 203.6^\circ, 336.6^\circ$$