Trigonometry : 3 : Answers

76) a) $6\cos^2\theta + \sin\theta = 4$

$6(1-\sin^2\theta) + \sin\theta = 4$

$6 - 6\sin^2\theta + \sin\theta = 4$

$0 = 6\sin^2\theta - \sin\theta - 2$

$0 = (3\sin\theta - 2)(2\sin\theta + 1)$

either $3\sin\theta - 2 = 0$ or $2\sin\theta + 1 = 0$

$\sin\theta = \frac{2}{3}$ $\sin\theta = \frac{-1}{2}$

$\alpha = 41.8^\circ$ $\alpha = 30^\circ$

Sin +ve 1st + 2nd Sin −ve 3rd + 4th

$\theta = 41.8^\circ, 138.2^\circ$ $\theta = 210^\circ, 330^\circ$

$\therefore \theta = 41.8^\circ, 138.2^\circ, 210^\circ, 330^\circ$

b) 0° to 180°

$\tan 3x = -1.54$

$\alpha = 57^\circ$

tan −ve 2nd + 4th

$3x = 123^\circ, 303^\circ, 483^\circ, 663^\circ$

$x = 41^\circ, 101^\circ, 161^\circ, 221^\circ$

Too BIG.

77) a) $5\cos^2\theta + 2 = 3\sin^2\theta - 2\cos\theta$

$5\cos^2\theta + 2 = 3(1-\cos^2\theta) - 2\cos\theta$

$5\cos^2\theta + 2 = 3 - 3\cos^2\theta - 2\cos\theta$

$8\cos^2\theta + 2\cos\theta - 1 = 0$

$(4\cos\theta - 1)(2\cos\theta + 1) = 0$

either $4\cos\theta - 1 = 0$ or $2\cos\theta + 1 = 0$

$\cos\theta = \frac{1}{4}$ $\cos\theta = \frac{-1}{2}$

$\alpha = 75.5^\circ$ $\alpha = 60^\circ$

Cos +ve 1st + 4th Cos −ve 2nd + 3rd

$\theta = 75.5^\circ, 284.5^\circ$ $\theta = 120^\circ, 240^\circ$

$\therefore \theta = 75.5^\circ, 120^\circ, 240^\circ, 284.5^\circ$

b) 0° to 180°

$\sin(2x + 12^\circ) = -0.53$

$\alpha = 32^\circ$

sin −ve 3rd + 4th

$2x + 12^\circ = 212^\circ, 328^\circ$

$2x = 200^\circ, 316^\circ$

$x = 100^\circ, 158^\circ$

78) a) $3 - 7\cos\theta = 6\sin^2\theta$

$3 - 7\cos\theta = 6(1 - \cos^2\theta)$

$3 - 7\cos\theta = 6 - 6\cos^2\theta$

$6\cos^2\theta - 7\cos\theta - 3 = 0$

$(3\cos\theta + 1)(2\cos\theta - 3) = 0$

either $3\cos\theta + 1 = 0$ or $2\cos\theta - 3 = 0$

$\cos\theta = -\frac{1}{3}$ ~~$\cos\theta = \frac{3}{2}$~~

$\alpha = 70.5^\circ$ ~~$\alpha =$~~ ~~$\cos$~~ IMPOSSIBLE

Cos −ve 2nd + 3rd

$\theta = 109.5^\circ, 250.5^\circ$

b) 0° to 180°

$\tan(2x + 45^\circ) = 0.7$

$\alpha = 35.0^\circ$

tan +ve 1st + 3rd

$2x + 45^\circ = 35.0^\circ, 215^\circ, 395.0^\circ$

$2x = -10^\circ, 170^\circ, 350^\circ$

$x = -5^\circ, 85^\circ, 175^\circ$

Too small

c) $4\tan\theta\cos\theta + 1 = 0$

$4\frac{\sin\theta}{\cos\theta}\cos\theta + 1 = 0$

$4\sin\theta + 1 = 0$

$\sin\theta = -\frac{1}{4}$

$\alpha = 14.5^\circ$

Sin −ve 3rd + 4th

$\theta = 194.5^\circ, 345.5^\circ$

79) a) $12\cos^2\theta - 5\sin\theta = 10$

$12(1-\sin^2\theta) - 5\sin\theta = 10$

$12 - 12\sin^2\theta - 5\sin\theta = 10$

$0 = 12\sin^2\theta + 5\sin\theta - 2$

$0 = (4\sin\theta - 1)(3\sin\theta + 2)$

either

$4\sin\theta - 1 = 0$ or $3\sin\theta + 2 = 0$

$\sin\theta = \frac{1}{4}$ $\sin\theta = -\frac{2}{3}$

$\alpha = 14.5^\circ$ $\alpha = 41.8^\circ$

Sin +ve 1st + 2nd Sin -ve 3rd + 4th

$\theta = 14.5^\circ, 165.5^\circ$ $\theta = 221.8^\circ, 318.2^\circ$

$\therefore \theta = 14.5^\circ, 165.5^\circ, 221.8^\circ, 318.2^\circ$

b) 0° to 180°

$\tan 2x = -1.6$

$\alpha = 58.0^\circ$

tan -ve 2nd + 4th

$2x = 122^\circ, 302^\circ, 482^\circ, 662^\circ$

$x = 61^\circ, 151^\circ, 241^\circ, 331^\circ$

c) 0° to 180°

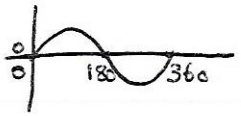
$\tan\phi + 2\sin\phi = 0$

$\frac{\sin\phi}{\cos\phi} + 2\sin\phi = 0$

$\times\cos\phi$ $\sin\phi + 2\sin\phi\cos\phi = 0$

$\sin\phi(1 + 2\cos\phi) = 0$

either $\sin\phi = 0$ or $1 + 2\cos\phi = 0$

From 

$\phi = 0^\circ, 180^\circ, 360^\circ$

$\cos\phi = -\frac{1}{2}$

$\alpha = 60^\circ$

Cos -ve 2nd + 3rd

$\phi = 120^\circ, 240^\circ$

$\phi = 0^\circ, 120^\circ, 180^\circ$

80) a) $7\sin^2\theta + 1 = 3\cos^2\theta - \sin\theta$

$7\sin^2\theta + 1 = 3(1-\sin^2\theta) - \sin\theta$

$7\sin^2\theta + 1 = 3 - 3\sin^2\theta - \sin\theta$

$10\sin^2\theta + \sin\theta - 2 = 0$

$(2\sin\theta + 1)(5\sin\theta - 2) = 0$

either $2\sin\theta + 1 = 0$ or $5\sin\theta - 2 = 0$

$\sin\theta = -\frac{1}{2}$ $\sin\theta = \frac{2}{5}$

$\alpha = 30^\circ$ $\alpha = 23.6^\circ$

Sin -ve 3rd + 4th Sin +ve 1st + 2nd

$\theta = 210^\circ, 330^\circ$ $\theta = 23.6^\circ, 156.4^\circ$

$\therefore \theta = 23.6^\circ, 156.4^\circ, 210^\circ, 330^\circ$

b) 0° to 180°

$\cos(2x + 25^\circ) = -0.454$

$\alpha = 63.0^\circ$

Cos -ve 2nd + 3rd

$2x + 25^\circ = 117^\circ, 243.0^\circ, 477^\circ$

$2x = 92^\circ, 218^\circ, 452^\circ$

$x = 46^\circ, 109^\circ, 226^\circ$

Too BIG