Trigonometry : 4 : Answers

8i) a) $\sin\theta + 12\cos^2\theta = 6$

$\sin\theta + 12(1-\sin^2\theta) = 6$

$\sin\theta + 12 - 12\sin^2\theta = 6$

$0 = 12\sin^2\theta - \sin\theta - 6$

$0 = (3\sin\theta + 2)(4\sin\theta - 3)$

either

$3\sin\theta + 2 = 0$

$\sin\theta = -2/3$

$\alpha = 41.8^\circ$

Sin -ve 3rd + 4th

$\theta = 221.8^\circ, 318.2^\circ$

or

$4\sin\theta - 3 = 0$

$\sin\theta = \frac{3}{4}$

$\alpha = 48.6^\circ$

Sin +ve 1st + 2nd

$\theta = 48.6^\circ, 131.4^\circ$

$\therefore \theta = 48.6^\circ, 131.4^\circ, 221.8^\circ, 318.2^\circ$

b) $\cos(2x - 35^\circ) = 0.891$ $\quad 0^\circ$ to 180°

$\alpha = 27^\circ$

Cos +ve 1st + 4th

$2x - 35^\circ = 27^\circ, 333^\circ$

$2x = 62^\circ, 368^\circ$

$x = 31^\circ, 184^\circ$

c) $\sin\phi + \cos\phi = 0$

$\sin\phi = -\cos\phi$

$\div \cos\phi \quad \frac{\sin\phi}{\cos\phi} = -1$

$\tan\phi = -1$

$\alpha = 45^\circ$

tan -ve 2nd + 4th

$\phi = 135^\circ, 315^\circ$

82) a) $10\sin^2\theta + 7\cos\theta = 5\cos^2\theta + 8$

$10(1-\cos^2\theta) + 7\cos\theta = 5\cos^2\theta + 8$

$10 - 10\cos^2\theta + 7\cos\theta = 5\cos^2\theta + 8$

$0 = 15\cos^2\theta - 7\cos\theta - 2$

$0 = (5\cos\theta + 1)(3\cos\theta - 2)$

either

$5\cos\theta + 1 = 0$

$\cos\theta = -\frac{1}{5}$

$\alpha = 78.4^\circ$

Cos -ve 2nd + 3rd

$\theta = 101.6^\circ, 258.4^\circ$

or

$3\cos\theta - 2 = 0$

$\cos\theta = \frac{2}{3}$

$\alpha = 48.2^\circ$

Cos +ve 1st + 4th

$\theta = 48.2^\circ, 311.8^\circ$

$\therefore \theta = 48.2^\circ, 101.6^\circ, 258.4^\circ, 311.8^\circ$

b) $\sin(x - 50^\circ) = -0.682$

$\alpha = 43^\circ$

Sin -ve 3rd + 4th

$x - 50^\circ = 223^\circ, 317^\circ$

$x = 273^\circ, 367^\circ$

Too Big

$x = 273^\circ$

c) $\sin\phi + \cos\phi = 3$

Max value $\sin\phi = 1$

Max value $\cos\phi = 1$

and they also can't be 1 at the same time!

$\therefore$ Not possible for $\sin\phi + \cos\phi$ to equal 3.

83) a) $10\cos^2\theta + 3\cos\theta = 4\sin^2\theta - 2$

$10\cos^2\theta + 3\cos\theta = 4(1-\cos^2\theta) - 2$

$10\cos^2\theta + 3\cos\theta = 4 - 4\cos^2\theta - 2$

$14\cos^2\theta + 3\cos\theta - 2 = 0$

$(7\cos\theta - 2)(2\cos\theta + 1) = 0$

either $7\cos\theta - 2 = 0$ or $2\cos\theta + 1 = 0$

$\cos\theta = \frac{2}{7}$ $\cos\theta = -\frac{1}{2}$

$\alpha = 73.4^\circ$ $\alpha = 60^\circ$

cos +ve 1st + 4th cos -ve 2nd + 3rd

$\theta = 120^\circ, 240^\circ$

$\theta = 73.4^\circ, 286.6^\circ$

$\therefore \theta = 73.4^\circ, 120^\circ, 240^\circ, 286.6^\circ$

b) 0° to 180°

$\sin(3x - 21^\circ) = -0.809$

$\alpha = 54.0^\circ$

Sin -ve 3rd + 4th

$3x - 21^\circ = 234^\circ, 306^\circ, 594^\circ, 666^\circ$

$3x = 255^\circ, 327^\circ, 615^\circ, 687^\circ, 975^\circ$

$x = 85^\circ, 109^\circ, 205^\circ, 229^\circ, 325^\circ$

Too Big

$x = 85^\circ, 109^\circ$

c) $\cos\phi - 5\sin\phi = 0$

$\cos\phi = 5\sin\phi$

$\div \cos\phi$

$1 = \frac{5\sin\phi}{\cos\phi}$

$\frac{1}{5} = \tan\phi$

$\alpha = 11.3^\circ$

tan +ve 1st + 3rd

$\phi = 11.3^\circ, 191.3^\circ$

84) a) $7\sin^2\theta - \sin\theta = 3\cos^2\theta$

$7\sin^2\theta - \sin\theta = 3(1 - \sin^2\theta)$

$7\sin^2\theta - \sin\theta = 3 - 3\sin^2\theta$

$10\sin^2\theta - \sin\theta - 3 = 0$

$(5\sin\theta - 3)(2\sin\theta + 1) = 0$

either

$5\sin\theta - 3 = 0$ or $2\sin\theta + 1 = 0$

$\sin\theta = \frac{3}{5}$ $\sin\theta = -\frac{1}{2}$

$\alpha = 36.9°$ $\alpha = 30°$

+ve Sin 1st + 2nd Sin -ve 3rd + 4th

$\theta = 36.7°, 143.1°$ $\theta = 210°, 330°$

$\theta = 36.7°, 143.1°, 210°, 330°$

b) $0°$ to $180°$

$\tan(3x - 20°) = 1.28$

$\alpha = 52°$

tan +ve 1st + 3rd

$3x - 20° = 52°, 232°, 412°$

$3x = 72°, 252°, 432°$

$x = 24°, 84°, 144°$