

# BINOMIAL THEOREM 2 : ANSWERS

$$78) a) (1+x)^6 = 1^6 + {}^6C_1 1^5 x + {}^6C_2 1^4 x^2 + {}^6C_3 1^3 x^3 + {}^6C_4 1^2 x^4 + {}^6C_5 1 x^5 + x^6$$

$$= 1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$$

$$b) 0.99^6 = (1 - 0.01)^6 \quad \text{Let } x = -0.01 \text{ in expansion above}$$

$$0.99^6 = 1 + 6(-0.01) + 15(-0.01)^2 + 20(-0.01)^3 + \dots$$

$$= 1 - 0.06 + 0.0015 - 0.00002 + \dots$$

$$= 0.9415 \text{ to 4 d.p.}$$

$$79) (1 + \sqrt{3})^5 = 1^5 + {}^5C_1 1^4 \sqrt{3} + \frac{{}^5C_2 1^3 (\sqrt{3})^2}{\frac{5 \times 4}{2 \times 1} \times 3} + \frac{{}^5C_3 1^2 (\sqrt{3})^3}{\frac{5 \times 4}{2 \times 1} \times 3\sqrt{3}} + \frac{{}^5C_4 1 (\sqrt{3})^4}{5 \times 3 \times 3} + (\sqrt{3})^5$$

$$= 1 + 5\sqrt{3} + 30 + 30\sqrt{3} + 45 + 9\sqrt{3}$$

$$= 76 + 44\sqrt{3}$$

$$a = 76 \quad b = 44$$

$$80) a) (3 + 2x)^4 = 3^4 + {}^4C_1 3^3 (2x) + {}^4C_2 3^2 (2x)^2 + {}^4C_3 3 (2x)^3 + (2x)^4$$

$$= 81 + 216x + 216x^2 + 96x^3 + 16x^4$$

$$b) \left(1 + \frac{x}{4}\right)^n$$

$$\frac{x^2 \text{ term}}{{}^nC_2 1^{n-2} \left(\frac{x}{4}\right)^2}$$

$$= \frac{n(n-1)}{2} \frac{x^2}{16}$$

$$\frac{x \text{ term}}{{}^nC_1 1^{n-1} \left(\frac{x}{4}\right)^1}$$

$$= \frac{nx}{4}$$

$$\therefore \frac{n(n-1)}{32} = 5 \frac{n}{4}$$

$$n(n-1) = 5n \times 8$$

$$n^2 - n = 40n$$

$$n^2 - 41n = 0$$

$$n(n-41) = 0$$

$$n = 0 \text{ or } n = 41$$

$$\therefore \underline{\underline{n = +41}}$$

$$81) \quad a) \quad \left(x + \frac{3}{x}\right)^4 = x^4 + {}^4C_1 x \left(\frac{3}{x}\right) + {}^4C_2 x^2 \left(\frac{3}{x}\right)^2 + {}^4C_3 x \left(\frac{3}{x}\right)^3 + \left(\frac{3}{x}\right)^4$$

$$= x^4 + 12x^2 + 54 + \frac{108}{x^2} + \frac{81}{x^4}$$

$$b) \quad (1+2x)^n$$

$$\frac{x^2 \text{ term}}{{}^nC_2 1^{n-2} (2x)^2}$$

$$= \frac{n(n-1) \times 1 \times 4x^2}{2 \times 1}$$

$$= 2n(n-1)x^2$$

$$\therefore 2n(n-1) = 760$$

$$n(n-1) = 380$$

$$n^2 - n - 380 = 0$$

$$(n-20)(n+19) = 0$$

$$n = 20 \text{ or } n = -19$$

$$\therefore \underline{n = 20}$$

$$82) \quad (1-2x)^6$$

$$= 1^6 + {}^6C_1 1^5 (-2x) + {}^6C_2 1^4 (-2x)^2 + {}^6C_3 1^3 (-2x)^3 + {}^6C_4 1^2 (-2x)^4 + {}^6C_5 1^1 (-2x)^5 + (-2x)^6$$

$$= 1 - 12x + 60x^2 - 160x^3 + 240x^4 - 192x^5 + 64x^6$$

$$83) \quad (a+4x)^5$$

$$\frac{x^2 \text{ term}}{{}^5C_2 a^3 (4x)^2}$$

$$= 10a^3 16x^2$$

$$= 160a^3 x^2$$

$$\frac{x \text{ term}}{{}^5C_1 a^4 (4x)^1}$$

$$= 5a^4 \times 4x$$

$$= 20a^4 x$$

$$\therefore 160a^3 = 2 \times 20a^4$$

$$\div 4 \quad 40a^3 = 10a^4$$

$$\div 10 \quad 4a^3 = a^4$$

$$0 = a^4 - 4a^3$$

$$0 = a^3(a-4)$$

$$a = 0 \text{ or } a = 4$$

$$a \neq 0 \quad \therefore \boxed{a = 4}$$