CIRCLES 3 : ANSWERS

93) $x^2 + y^2 - 8x + 2y + 7 = 0$

a)
$$x^2 - 8x + y^2 + 2y = -7$$
$$(x-4)^2 - 16 + (y+1)^2 - 1 = -7$$
$$(x-4)^2 + (y+1)^2 = 10$$

Centre $A(4,-1)$ $\quad r = \sqrt{10}$

b) $P(7,-2)$

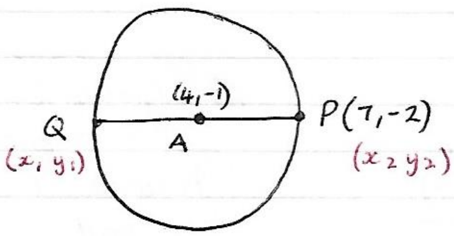
(i) $x = 7 \quad y = -2$

sub. into C eqn.

$$(7-4)^2 + (-2+1)^2 = 10$$
$$3^2 + (-1)^2 = 10$$
$$9 + 1 = 10 \qquad \text{TRUE}$$

$\therefore$ P lies on C

(ii)



For QP, A is the midpoint.

$\therefore \quad 4 = \dfrac{x_1 + x_2}{2} \qquad\qquad -1 = \dfrac{y_1 + y_2}{2}$

$8 = x_1 + 7 \qquad\qquad -2 = y_1 - 2$

$1 = x_1 \qquad\qquad 0 = y_1$

$\therefore \ Q(1,0)$

c) L $\quad y = 2x - 4$

C $\quad x^2 + y^2 - 8x + 2y + 7 = 0 \qquad$ Solve simultaneously

Sub L into C

$$x^2 + (2x-4)^2 - 8x + 2(2x-4) + 7 = 0$$
$$x^2 + 4x^2 - 16x + 16 - 8x + 4x - 8 + 7 = 0$$
$$5x^2 - 20x + 15 = 0$$
$\div 5$
$$x^2 - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 3 \text{ or } x = 1$$

If $x = 3$
$y = 2x - 4$
$y = 6 - 4$
$y = 2$

$(3, 2)$

If $x = 1$
$y = 2 - 4$
$y = -2$

$(1, -2)$

Points of intersection needed

94) $x^2 + y^2 - 2x + 6y - 15 = 0$

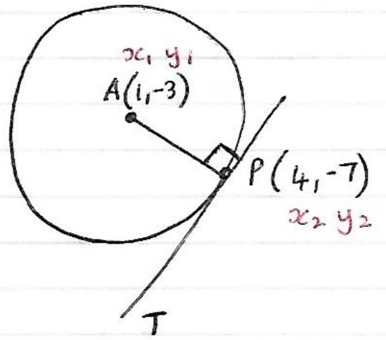
a)(i) $x^2 - 2x + y^2 + 6y = 15$

$(x-1)^2 - 1 + (y+3)^2 - 9 = 15$

$(x-1)^2 + (y+3)^2 = 25$

centre $A\ (1, -3)$ radius $= 5$

(ii)



Gradient AP $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-7-(-3)}{4-1}$

$= -\dfrac{4}{3}$

$\therefore$ Gradient $T = +\dfrac{3}{4}$

because T is $\perp$ to radius and

$\left(-\dfrac{4}{3}\right) \times \dfrac{3}{4} = -1$

Now

Eqn of T $\quad y - y_2 = m(x - x_2)$

$y - (-7) = \dfrac{3}{4}(x - 4)$

$4y + 28 = 3(x-4)$

$4y + 28 = 3x - 12$

$4y = 3x - 40$

b) $y = x + 4 \quad L$

$x^2 + y^2 - 2x + 6y - 15 = 0 \quad C$

Try solving simultaneously

Sub L into C $\quad x^2 + (x+4)^2 - 2x + 6(x+4) - 15 = 0$

$x^2 + x^2 + 8x + 16 - 2x + 6x + 24 - 15 = 0$

$2x^2 + 12x + 25 = 0$

NOW $b^2 - 4ac = 144 - 4(2)(25)$

$= -56 \quad \therefore$ NO REAL ROOTS !!

$\therefore$ L and C don't intersect !!

45) $x^2+y^2-4x+2y-20=0$

a) $x^2-4x+y^2+2y=20$

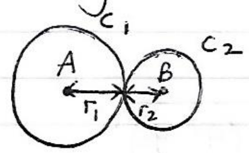
$(x-2)^2-4+(y+1)^2-1=20$

$(x-2)^2+(y+1)^2=25$

C_1 centre $A(2,-1)$ radius $=5$
x_1, y_1

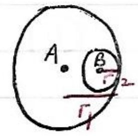
b) C_2 centre $B(8,-9)$ radius $=15$
x_2, y_2

(i) For touching circles externally



Dist $AB = r_1 + r_2$

internally



Dist $AB = r_1 - r_2$

Find AB

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(8-2)^2+(-9+1)^2}$$

$$= \sqrt{6^2+(-8)^2}$$

$$= \sqrt{36+64}$$

$$= \sqrt{100} = 10$$

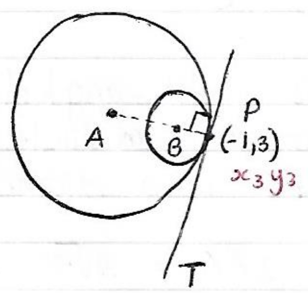
Now $r_1 - r_2$

$= 15 - 5$

$= 10 = AB$

$\therefore$ Circles touch internally.

(ii)



Gradient AP

$$M = \frac{y_3-y_1}{x_3-x_1} = \frac{3+1}{-1-2} = -\frac{4}{3}$$

$\therefore$ Gradient T is $+\frac{3}{4}$ because $(-4/3)\times\frac{3}{4}=-1$

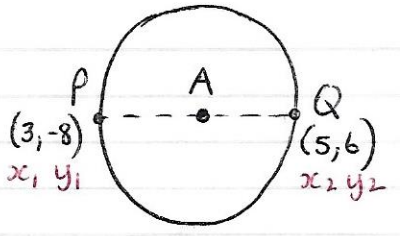
$\therefore$ Eqn T $\quad y-y_3 = m(x-x_3)$

$$y-3=\frac{3}{4}(x+1)$$

$$4y-12=3x+3$$

$$4y=3x+15$$

9b)



a) (i) A is midpoint of PQ

$$A\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= A\left(\frac{3+5}{2}, \frac{-8+6}{2}\right)$$

$$= A(4,-1)$$

$x_3 \quad y_3$

(ii) radius = AQ

$$r = \sqrt{(x_3-x_2)^2+(y_3-y_2)^2}$$

$$r = \sqrt{(4-5)^2+(-1-6)^2}$$

$$r = \sqrt{(-1)^2+(-7)^2}$$

$$r = \sqrt{1+49}$$

$$r = \sqrt{50}$$ QED.

(iii) Eqn of C

$$(x-4)^2+(y+1)^2 = 50$$

b) $R(9,-6)$ sub $x=9$ $y=-6$ into C eqn.

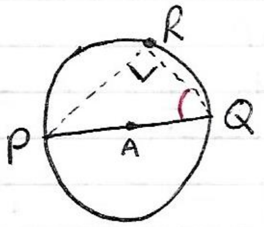
$$(9-4)^2+(-6+1)^2 = 50$$

$$5^2+(-5)^2 = 50$$

$$25+25 = 50$$ TRUE

$\therefore$ R lies on C

c) $P\hat{Q}R$



ΔPQR is right-angled because angles in a semi-circle = 90°
(PQ is a diameter)

Now $PQ = 2AQ = 2\sqrt{50} = 2\times 5\sqrt{2} = 10\sqrt{2}$

ALSO $PR = \sqrt{(9-3)^2+(-6+8)^2}$

$= \sqrt{36+4}$

$= \sqrt{40}$

$$\sin P\hat{Q}R = \frac{PR}{PQ} = \frac{\sqrt{40}}{2\sqrt{50}} = \frac{2\sqrt{10}}{10\sqrt{2}} = \frac{\sqrt{5}}{5}$$

$$P\hat{Q}R = 26.6°$$