Differentiation : 2 : Answers.

June 2005

(6) a) (i) $y = e^{2x-5}$

$\frac{dy}{dx} = 2e^{2x-5}$

(ii) $y = x^2 \ln x$

$u = x^2 \qquad v = \ln x$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= x^2 \frac{1}{x} + 2x \ln x$

$= x + 2x \ln x$

$= x(1 + 2\ln x)$

(iii) $y = (3x^2+2)^4$

$\frac{dy}{dx} = 4(3x^2+2)^3 \times 6x$

$= 24x(3x^2+2)^3$

b) $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$

$u = \sin x \qquad v = \cos x$

$\frac{du}{dx} = \cos x \qquad \frac{dv}{dx} = -\sin x$

$\frac{dy}{dx} = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$

$= \frac{1}{\cos^2 x}\left[\cos x \cos x - \sin x(-\sin x)\right]$

$= \frac{1}{\cos^2 x}\left[\cos^2 x + \sin^2 x\right]$

$= \frac{1}{\cos^2 x}[1]$

$= \sec^2 x$

c) $y = \tan^{-1} x$

$x = \tan y$

$\frac{dx}{dy} = \sec^2 y$

$\frac{dy}{dx} = \frac{1}{\sec^2 y}$

$= \frac{1}{1+\tan^2 y}$

$\frac{dy}{dx} = \frac{1}{1+x^2}$

NOW $1 + \tan^2 y = \sec^2 y$

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(6) a) $y = e^{2x}\sin x$

$u = e^{2x}$ $\quad v = \sin x$

$\frac{du}{dx} = 2e^{2x}$ $\quad \frac{dv}{dx} = \cos x$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= e^{2x}\cos x + 2e^{2x}\sin x$$

$$= e^{2x}(\cos x + 2\sin x)$$

b) $y = \frac{2x^2 - 4}{x^2 + 3}$

$u = 2x^2 - 4$ $\quad v = x^2 + 3$

$\frac{du}{dx} = 4x$ $\quad \frac{dv}{dx} = 2x$

$$\frac{dy}{dx} = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$$

$$= \frac{1}{(x^2+3)^2}\left[4x(x^2+3) - 2x(2x^2-4)\right]$$

$$= \frac{1}{(x^2+3)^2}\left[4x^3 + 12x - 4x^3 + 8x\right]$$

$$= \frac{20x}{(x^2+3)^2}$$

c) $y = \tan(4x^2+3)$

$$\frac{dy}{dx} = 8x\sec^2(4x^2+3)$$

(8) a) $y = \tan^{-1} x$

$x = \tan y$

$\frac{dx}{dy} = \sec^2 y$

$\frac{dy}{dx} = \frac{1}{\sec^2 y}$

Now $1 + \tan^2 y = \sec^2 y$

$\therefore \frac{dy}{dx} = \frac{1}{1+\tan^2 y}$

$= \frac{1}{1+x^2}$

b) $y = \ln(x^2+1)$

$\frac{dy}{dx} = \frac{1}{(x^2+1)} \times 2x$

$= \frac{2x}{(x^2+1)}$

c) $\int \frac{3+x}{1+x^2}\, dx$

$= \int \frac{3}{1+x^2}\, dx + \frac{1}{2}\int \frac{2x}{1+x^2}\, dx$

$= 3\int \frac{1}{1+x^2}\, dx + \frac{1}{2}\int \frac{2x}{1+x^2}\, dx$

$= 3\tan^{-1} x + \frac{1}{2}\ln(x^2+1) + C$

↑ from (a) ↑ from (b)