Differentiation Revision 4 Solutions

1) $\frac{dy}{dx} = 3x^2 - 4$

2) $f'(x) = 24x^2$

3) $y = 8x^{1/2}$

$\frac{dy}{dx} = 4x^{-1/2}$

$= \frac{4}{\sqrt{x}}$

4) $f(x) = 2x^{-3} + 7x$

$f'(x) = -6x^{-4} + 7$

$= \frac{-6}{x^4} + 7$

5) Easiest to expand (not product rule)

$y = 3x^2 + 3x$

$\frac{dy}{dx} = 6x + 3$

$= 3(2x+1)$

6) $f'(x) = 2\cos x$

7) $f(x) = 6x^{2/3}$

$f'(x) = 6(\frac{2}{3})x^{-1/3}$

$= 4x^{-1/3}$

$= \frac{4}{\sqrt[3]{x}}$

8) $y = 4x^{-1}$

$\frac{dy}{dx} = -4x^{-2}$

$= \frac{-4}{x^2}$

9) Easiest to expand and not use product rule

$y = x^2 + x - 2x - 2$

$y = x^2 - x - 2$

$\frac{dy}{dx} = 2x - 1$

10) Let $y = (3x^2 - 2)^4$

$\frac{dy}{dx} = 4(3x^2-2)^3 \times 6x$

$= 24x(3x^2-2)^3$

11) Let $y = 2(x^2-2)^{-1/2}$

$$\frac{dy}{dx} = 2 \times \left(-\frac{1}{2}\right)(x^2-2)^{-3/2} \times 2x$$

$$= \frac{-2x}{(x^2-2)^{3/2}}$$

$$= \frac{-2x}{\sqrt{(x^2-2)^3}}$$

12) $f(x) = (2 + x^{1/2})^{1/2}$

$$f'(x) = \frac{1}{2}(2 + x^{1/2})^{-1/2} \times \frac{1}{2}x^{-1/2}$$

$$= \frac{1}{4\sqrt{x}(2+\sqrt{x})^{1/2}}$$

$$= \frac{1}{4\sqrt{x(2+\sqrt{x})}}$$

13) Let $y = \cos 4x$

$$\frac{dy}{dx} = -4\sin 4x$$

14) $\frac{dy}{dx} = -6e^{-6x}$

15) $u = x \qquad v = (x+3)^4$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = 4(x+3)^3 \times 1$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= x\,4(x+3)^3 + (x+3)^4 \times 1$$

$$= (x+3)^3(4x + (x+3))$$

$$= (x+3)^3(5x+3)$$

16) $u = 3x \qquad v = \sin 2x$

$\frac{du}{dx} = 3 \qquad \frac{dv}{dx} = 2\cos 2x$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= 3x(2\cos 2x) + 3\sin 2x$$

$$= 3(2x\cos 2x + \sin 2x)$$

17) $u = 3x-1 \qquad v = 2x+3$

$\frac{du}{dx} = 3 \qquad \frac{dv}{dx} = 2$

$$\frac{dy}{dx} = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$$

$$= \frac{1}{(2x+3)^2}\left[(2x+3)3 - (3x-1)2\right]$$

$$= \frac{6x+9-6x+2}{(2x+3)^2}$$

$$= \frac{11}{(2x+3)^2}$$

18) $u = (2x+1)^2 \qquad v = (3x-2)^3$

$\frac{du}{dx} = 2(2x+1)\times 2 \qquad \frac{dv}{dx} = 3(3x-2)^2 \times 3$

$= 4(2x+1) \qquad = 9(3x-2)^2$

$$\frac{dy}{dx} = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$$

$$= \frac{1}{(3x-2)^6}\left[(3x-2)^3 4(2x+1) - (2x+1)^2 9(3x-2)\right]$$

$$= \frac{(3x-2)^2(2x+1)\left[4(3x-2) - 9(2x+1)\right]}{(3x-2)^6}$$

$$= \frac{(2x+1)}{(3x-2)^4}(12x - 8 - 18x - 9)$$

$$= \frac{(2x+1)(-6x-17)}{(3x-2)^4}$$

$$= -\frac{(2x+1)(6x+17)}{(3x-2)^4}$$

19) $\frac{dy}{dx} = 3x^2\cos(x^3-1)$

20) Let $y = \sec^3 x$

$\frac{dy}{dx} = 3\sec^2 x \sec x \tan x$

$= 3\sec^3 x \tan x$

21) $f'(x) = 5\sin^4 6x \times 6\cos 6x$

$= 30\cos 6x \sin^4 6x$

22) $u = e^x \qquad v = \sin x$

$\frac{du}{dx} = e^x \qquad \frac{dv}{dx} = \cos x$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= e^x \cos x + e^x \sin x$

$= e^x(\cos x + \sin x)$

23) $u = e^{2x}$ $\quad v = \tan 2x$

$\frac{du}{dx} = 2e^{2x}$ $\quad \frac{dv}{dx} = 2\sec^2 2x$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= e^{2x} 2\sec^2 2x + \tan 2x \; 2e^{2x}$

$= 2e^{2x}(\sec^2 2x + \tan 2x)$

24) $f'(x) = -2 \times \frac{1}{x}$

$= -\frac{2}{x}$

25) $f(x) = \ln|x+1| - \ln|x-1|$

$f'(x) = \frac{1}{(x+1)} - \frac{1}{(x-1)}$

$= \frac{(x-1)-(x+1)}{(x+1)(x-1)}$

$= \frac{-2}{(x+1)(x-1)}$

26) $u = \ln x$ $\quad v = e^{x^2}$

$\frac{du}{dx} = \frac{1}{x}$ $\quad \frac{dv}{dx} = 2x e^{x^2}$

$\frac{dy}{dx} = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$

$= \frac{1}{(e^{x^2})^2}\left[\frac{e^{x^2}}{x} - 2x e^{x^2}\ln x\right]$

$= \frac{e^{x^2}}{x(e^{x^2})^2}\left[1 - 2x^2\ln x\right] = \frac{(1-2x^2\ln x)}{x e^{x^2}}$

27) Let $y = \tan^{-1} 3x$

$$\frac{dy}{dx} = \frac{1}{1+(3x)^2} \times 3$$

$$= \frac{3}{(1+9x^2)}$$

28) $\frac{dy}{dx} = -4\text{Cosec}^2 4x$

29) $f'(x) = 6 \times 3\text{Cosec}\,6x\,\text{Cot}\,6x$

$= 18\text{Cosec}\,6x\,\text{Cot}\,6x$

30) $y = 8e^{-\frac{1}{2}x}$

$$\frac{dy}{dx} = 8e^{-\frac{1}{2}x} \times \left(-\frac{1}{2}\right)$$

$$= -4e^{-\frac{1}{2}x}$$

$$= \frac{-4}{e^{\sqrt{x}}}$$

31) $\frac{dy}{dx} = 4 \times \frac{1}{\sqrt{1-4x^2}} \times 2$

$$= \frac{8}{\sqrt{1-4x^2}}$$

32) Let $y = \ln|\text{Cos}\,3x|$

$$\frac{dy}{dx} = \frac{1}{\text{Cos}\,3x} \times (-3\text{Sin}\,3x)$$

$$= -3\tan 3x$$

33) $f'(x) = \frac{1}{(x-3)} + \frac{1}{(x-3)^2}$

$$= \frac{(x-3)+1}{(x-3)^2}$$

$$= \frac{(x-2)}{(x-3)^2}$$

34) $y = \ln|3x| - \ln|4x-1|$

$$\frac{dy}{dx} = \frac{1}{x} - \frac{4}{(4x-1)}$$

$$= \frac{1(4x-1) - 4x}{x(4x-1)}$$

$$= \frac{-1}{x(4x-1)}$$

$$= \frac{1}{x(1-4x)}$$

35) Let $y = \tan^{-1}x \ln x$

$u = \tan^{-1}x \qquad v = \ln x$

$\frac{du}{dx} = \frac{1}{(1+x^2)} \qquad \frac{dv}{dx} = \frac{1}{x}$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= \frac{1}{x}\tan^{-1}x + \frac{\ln x}{(1+x^2)}$$

$$= \frac{1(1+x^2)\tan^{-1}x + x\ln x}{x(1+x^2)}$$

36) $u = x \qquad v = \operatorname{Cosec} 2x$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = -2\operatorname{Cosec} 2x \operatorname{Cot} 2x$

$$\frac{dy}{dx} = -2x\operatorname{Cosec} 2x \operatorname{Cot} 2x + \operatorname{Cosec} 2x$$

$$= \operatorname{Cosec} 2x\,(1 - 2x\operatorname{Cot} 2x)$$

37) $f'(x) = \frac{1}{\sin x} \times \cos x$

$= \cot x$

38) $\frac{dy}{dx} = \frac{-1}{\sqrt{1-(x^2-1)^2}} \times 2x$

$= \frac{-2x}{\sqrt{1-(x^4-2x^2+1)}}$

$= \frac{-2x}{\sqrt{1-x^4+2x^2-1}}$

$= \frac{-2x}{\sqrt{2x^2-x^4}}$

$= \frac{-2x}{\sqrt{x^2(2-x^2)}}$

$= \frac{-2x}{x\sqrt{2-x^2}}$

$= \frac{-2}{\sqrt{2-x^2}}$

39) Let $y = x \sin^{-1} mx$

$u = x \qquad v = \sin^{-1} mx$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \frac{1}{\sqrt{1 - M^2x^2}} \times m$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= \frac{x \times m}{\sqrt{1 - M^2x^2}} + (1)\sin^{-1} mx$$

$$= \frac{mx}{\sqrt{1 - m^2x^2}} + \sin^{-1} mx$$

40) Let $y = \tan^{-1}(1 - 4x)$

$$\frac{dy}{dx} = \frac{1}{1 + (1-4x)^2} \times (-4)$$

$$= \frac{-4}{1 + (1 - 8x + 16x^2)}$$

$$= \frac{-4}{2 - 8x + 16x^2}$$

$$= \frac{-4}{2(1 - 4x + 8x^2)}$$

$$= \frac{-2}{8x^2 - 4x + 1}$$