Differentiation Sheet 5 Solutions

1) $y = x \cos x$

$u = x \qquad v = \cos x$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = -\sin x$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= -x\sin x + (1)\cos x$$

$$= \cos x - x\sin x$$

2) $y = \frac{\sin x}{x^3}$

$u = \sin x \qquad v = x^3$

$\frac{du}{dx} = \cos x \qquad \frac{dv}{dx} = 3x^2$

$$\frac{dy}{dx} = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$$

$$= \frac{1}{(x^3)^2}\left[x^3\cos x - 3x^2\sin x\right]$$

$$= \frac{x^2}{x^6}(x\cos x - 3\sin x)$$

$$= \frac{x\cos x - 3\sin x}{x^4}$$

3) $f(x) = \sin 8x$

$$f'(x) = \cos 8x \times 8$$

$$= 8\cos 8x$$

4) $f(x) = \tan 7x$

$$f'(x) = \sec^2 7x \times 7$$

$$= 7\sec^2 7x$$

5) $\frac{d}{dx}(x^3 \ln x)$

$u = x^3 \qquad v = \ln x$

$\frac{du}{dx} = 3x^2 \qquad \frac{dv}{dx} = \frac{1}{x}$

$$= u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= x^3\left(\frac{1}{x}\right) + 3x^2\ln x$$

$$= x^2 + 3x^2\ln x$$

$$= x^2(1 + 3\ln x)$$

6) $\dfrac{d}{dx}\left[(3x+1)^7\right]$

$= 7(3x+1)^6 \times 3$

$= 21(3x+1)^6$

7) $y = \sqrt{5x-2}$

$y = (5x-2)^{1/2}$

$\dfrac{dy}{dx} = \dfrac{1}{2}(5x-2)^{-1/2} \times 5$

$= \dfrac{5}{2\sqrt{5x-2}}$

8) $f(x) = \dfrac{e^x}{x^3}$

$u = e^x \qquad v = x^3$

$\dfrac{du}{dx} = e^x \qquad \dfrac{dv}{dx} = 3x^2$

$f'(x) = \dfrac{1}{v^2}\left[v\dfrac{du}{dx} - u\dfrac{dv}{dx}\right]$

$= \dfrac{1}{(x^3)^2}\left[x^3e^x - 3x^2e^x\right]$

$= \dfrac{x^2e^x(x-3)}{x^6}$

$= \dfrac{e^x(x-3)}{x^4}$

9) $\dfrac{d}{dx}\left[\dfrac{(2x-1)}{(3x+2)}\right]$

$u = 2x-1 \qquad v = 3x+2$

$\dfrac{du}{dx} = 2 \qquad \dfrac{dv}{dx} = 3$

$\cancel{d} \quad = \dfrac{1}{v^2}\left[v\dfrac{du}{dx} - u\dfrac{dv}{dx}\right]$

$= \dfrac{1}{(3x+2)^2}\left[2(3x+2) - 3(2x-1)\right]$

$= \dfrac{6x+4-6x+3}{(3x+2)^2}$

$= \dfrac{7}{(3x+2)^2}$

10) $y = \dfrac{1}{(3x+1)^2}$

$y = (3x+1)^{-2}$

$\dfrac{dy}{dx} = -2(3x+1)^{-3} \times 3$

$= -\dfrac{6}{(3x+1)^3}$

11) $f(x) = \dfrac{2}{\sqrt{5-2x}}$

$f(x) = 2(5-2x)^{-1/2}$

$f'(x) = 2(-\tfrac{1}{2})(5-2x)^{-3/2} \times (-2)$

$= 2(5-2x)^{-3/2}$

$= \dfrac{2}{\sqrt{(5-2x)^3}}$

12) $y = \ln|5-2x|$

$\dfrac{dy}{dx} = \dfrac{1}{(5-2x)} \times (-2)$

$= \dfrac{-2}{(5-2x)}$

$= \dfrac{2}{(2x-5)}$

13) $y = x^2 \sin 3x$

$u = x^2$ $\quad \dfrac{du}{dx} = 2x$ $\qquad v = \sin 3x$ $\quad \dfrac{dv}{dx} = 3\cos 3x$

$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$= 3x^2\cos 3x + 2x\sin 3x$

$= x(3x\cos 3x + 2\sin 3x)$

14) $f(x) = 2x(3x+1)^3$

$u = 2x$ $\qquad$ $v = (3x+1)^3$

$\frac{du}{dx} = 2$ $\qquad$ $\frac{dv}{dx} = 3(3x+1)^2 \times 3$

$= 9(3x+1)^2$

$f'(x) = u\frac{dv}{dx} + v\frac{du}{dx}$

$= 2x(3x+1)^2 \times 9 + 2(3x+1)^3$

$= 2(3x+1)^2 [9x + (3x+1)^1]$

$= 2(3x+1)^2 (9x + 3x + 1)$

$= 2(3x+1)^2 (12x+1)$

15) $\frac{d}{dx}\left(\frac{3x^2}{e^x}\right)$

$u = 3x^2$ $\qquad$ $v = e^x$

$\frac{du}{dx} = 6x$ $\qquad$ $\frac{dv}{dx} = e^x$

$= \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$

$= \frac{1}{(e^x)^2}\left[6xe^x - 3x^2e^x\right]$

$= \frac{3xe^x(2-x)}{(e^x)^2}$

$= \frac{3x(2-x)}{e^x}$

16) $y = \ln\left|\frac{2x+1}{3x-1}\right|$

$y = \ln|2x+1| - \ln|3x-1|$

$\frac{dy}{dx} = \frac{1}{(2x+1)} \times 2 - \frac{1}{(3x-1)} \times 3$

$= \frac{2}{(2x+1)} - \frac{3}{(3x-1)}$

$= \frac{2(3x-1) - 3(2x+1)}{(2x+1)(3x-1)}$

$= \frac{6x - 2 - 6x - 3}{(2x+1)(3x-1)}$

$= \frac{-5}{(2x+1)(3x-1)} = \frac{5}{(2x+1)(1-3x)}$

17) $f(x) = e^{x^2}$

$f'(x) = e^{x^2} \times 2x$

$= 2x e^{x^2}$

18) $\frac{d}{dx}\left[(3-2e^{3x})^4 \right]$

$= 4(3-2e^{3x})^3 \times (-2e^{3x} \times 3)$

$= -24e^{3x}(3-2e^{3x})^3$

19) $y = (x^2-3)(2x+1)^2$

$u = x^2-3$ $\quad$ $v = (2x+1)^2$

$\frac{du}{dx} = 2x$ $\quad$ $\frac{dv}{dx} = 2(2x+1)^1 \times 2$

$= 4(2x+1)$

$\frac{dy}{dx} = \frac{u\,dv}{dx} + \frac{v\,du}{dx}$

$= 4(x^2-3)(2x+1) + 2x(2x+1)^2$

$= 2(2x+1)\left[2(x^2-3) + x(2x+1)\right]$

$= 2(2x+1)\left[2x^2-6+2x^2+x\right]$

$= 2(2x+1)(4x^2+x-6)$

20) $f(x) = e^{\ln x}$

$f'(x) = e^{\ln x} \times \frac{1}{x}$

$= \frac{e^{\ln x}}{x}$

21) $y = 5^x$

$\frac{dy}{dx} = (\ln 5)5^x$

$= 5^x \ln 5$

22) $f(x) = 6^{3x}$

$f'(x) = (\ln 6)\, 6^{3x} \times 3$

$= 6^{3x}\, 3\ln 6$

$= 6^{3x} \ln 6^3$

$= 6^{3x} \ln 216$

23) $f(x) = \sin^2 x$

$f'(x) = 2\sin x \times \cos x$

$= \sin 2x$

Double Angle Identity
$\sin 2x = 2\sin x \cos x$

24) $y = \cos^2 5x$

$\frac{dy}{dx} = 2\cos 5x \times (-\sin 5x \times 5)$

$= -10 \sin 5x \cos 5x$

$= -5\,(2\sin 5x \cos 5x)$

$= -5 \sin 10x$

25) $y = x \sin^{-1} x$

$u = x$ $\qquad$ $v = \sin^{-1} x$

$\frac{du}{dx} = 1$ $\qquad$ $\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= x\left(\frac{1}{\sqrt{1-x^2}}\right) + (1)\sin^{-1} x$

$= \frac{x}{\sqrt{1-x^2}} + \sin^{-1} x$

26) $f(x) = \ln|\sin x|$

$$f'(x) = \frac{1}{\sin x} \times \cos x$$

$$= \frac{\cos x}{\sin x}$$

$$= \cot x$$

27) $\frac{d}{dx}\left[\cos^{-1} 2x\right]$

$$= -\frac{1}{\sqrt{1-(2x)^2}} \times 2$$

$$= \frac{-2}{\sqrt{1-4x^2}}$$

28) $y = \ln|\cos x|$

$$\frac{dy}{dx} = \frac{1}{\cos x} \times (-\sin x)$$

$$= -\tan x$$

29) $f(x) = x^3 \tan^{-1} 2x$

$u = x^3$ $\quad \frac{du}{dx} = 3x^2$

$v = \tan^{-1} 2x$ $\quad \frac{dv}{dx} = \frac{1}{1+(2x)^2} \times 2 = \frac{2}{1+4x^2}$

$$f'(x) = \frac{u\,dv}{dx} + \frac{v\,du}{dx}$$

$$= x^3\left(\frac{2}{1+4x^2}\right) + 3x^2 \tan^{-1} 2x$$

$$= x^2\left(\frac{2x}{1+4x^2} + 3\tan^{-1} 2x\right)$$

30) $y = \ln|(2x+1)(3x-2)|$

$y = \ln|2x+1| + \ln|3x-2|$

$$\frac{dy}{dx} = \frac{2}{(2x+1)} + \frac{3}{(3x-2)}$$

$$= \frac{2(3x-2) + 3(2x+1)}{(2x+1)(3x-2)}$$

$$= \frac{6x-4+6x+3}{(2x+1)(3x-2)}$$

$$= \frac{12x-1}{(2x+1)(3x-2)}$$

31) $\frac{d}{dx}\left[\frac{1}{3x-1}\right]$

$$= \frac{d}{dx}\left[(3x-1)^{-1}\right]$$

$$= -1(3x-1)^{-2} \times 3$$

$$= \frac{-3}{(3x-1)^2}$$

32) $y = \frac{-5}{(4-3x)}$

$y = \frac{5}{(3x-4)}$

$y = 5(3x-4)^{-1}$

$$\frac{dy}{dx} = -5(3x-4)^{-2} \times 3$$

$$= -\frac{15}{(3x-4)^2}$$

33) $y = e^{1/x^2}$

$y = e^{x^{-2}}$

$\frac{dy}{dx} = e^{x^{-2}} \times -2x^{-3}$

$= \frac{-2e^{1/x^2}}{x^3}$

34) $f(x) = e^{2x} \ln|2x+1|$

$u = e^{2x}$ $\quad$ $\frac{du}{dx} = 2e^{2x}$

$v = \ln|2x+1|$ $\quad$ $\frac{dv}{dx} = \frac{2}{(2x+1)}$

$f'(x) = u\frac{dv}{dx} + v\frac{du}{dx}$

$= e^{2x}\left(\frac{2}{2x+1}\right) + 2e^{2x}\ln|2x+1|$

$= 2e^{2x}\left(\frac{1}{2x+1} + \ln|2x+1|\right)$

35) $y = \frac{3x^2}{e^x}$

$u = 3x^2$ $\quad$ $\frac{du}{dx} = 6x$

$v = e^x$ $\quad$ $\frac{dv}{dx} = e^x$

$\frac{dy}{dx} = \frac{1}{v^2}\left[v\frac{du}{dx} - u\frac{dv}{dx}\right]$

$= \frac{1}{(e^x)^2}\left[6x e^x - 3x^2 e^x\right]$

$= \frac{3x e^x (2-x)}{(e^x)^2}$

$= \frac{3x(2-x)}{e^x}$

$= 3x e^{-x}(2-x)$

36) $f(x) = \ln|x+3| + \frac{1}{(x+3)}$

$f(x) = \ln|x+3| + (x+3)^{-1}$

$f'(x) = \frac{1}{(x+3)} - 1(x+3)^{-2} \times 1$

$= +\frac{1}{(x+3)} - \frac{1}{(x+3)^2}$

$= \frac{1(x+3) - 1}{(x+3)^2}$

$= \frac{x+3-1}{(x+3)^2}$

$= \frac{x+2}{(x+3)^2}$

37) $\frac{d}{dx}\left(\ln\sqrt{x+1}\right)$

$= \frac{d}{dx}\left[\ln(x+1)^{1/2}\right]$

Let $u = (x+1)^{1/2}$ $\qquad y = \ln u$

$\frac{du}{dx} = \frac{1}{2}(x+1)^{-1/2} \times 1$ $\qquad \frac{dy}{du} = \frac{1}{u}$

$= \frac{1}{2\sqrt{x+1}}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{1}{u} \times \frac{1}{2\sqrt{x+1}}$

$= \frac{1}{\sqrt{x+1}} \times \frac{1}{2\sqrt{x+1}}$

$= \frac{1}{2(x+1)}$

38) $f(x) = \ln|e^x + e^{-x}|$

Easiest to let $y = \ln|e^x + e^{-x}|$

Now $u = e^x + e^{-x}$

$\frac{du}{dx} = e^x - e^{-x}$

Also $y = \ln u$

$\frac{dy}{du} = \frac{1}{u}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \frac{1}{u} \times (e^x - e^{-x})$$

$$= \frac{1}{(e^x + e^{-x})} \times (e^x - e^{-x})$$

$$= \frac{(e^x - e^{-x})}{(e^x + e^{-x})}$$

$$= \frac{e^{-x}(e^{2x} - 1)}{e^{-x}(e^{2x} + 1)}$$

$$= \frac{(e^{2x} - 1)}{(e^{2x} + 1)}$$